30 Cubes on a Rhombic Triacontahedron

Investigation of Polyhedral Rings and Clusters with the Help of Physical Models and *Wolfram Mathematica*

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Abstract

A cube is placed on each face of a rhombic triacontahedron (RT). In the cluster of 30 cubes produced in this manner, the cubes are connected at their vertices. With the use of physical models and the computer software Wolfram *Mathematica,* we study the possible geometrical features and relationships that can be associated with this geometrical sculpture. The purpose of this article is to introduce a method which is suitable for teaching a number of concepts by association to a single object.

The Golden Rhombus

A method of teaching/learning spatial geometry could be based on a selected object having simple geometrical features. The objective is to associate as much knowledge as possible with the use of simple relationships. Later, the associated concepts can be recollected by thinking of the object and using the simple relationships. Here, the relationship of a golden rhombus and an inscribed square is used. If half of the large diagonal is assumed as unity, then the small diagonal of the rhombus is $2/\phi$, and the cube that can be fitted into such a rhombus has an edge length of $2/\phi^2$, where ϕ is the golden ratio. The cube edges divide the sides of the golden rhombus in proportion to the golden ratio. If such a cube is placed on each of the 30 faces of a rhombic triacontahedron (RT), then a cluster of 30 cubes is produced, in which the cubes are in contact with their neighbors' vertices.

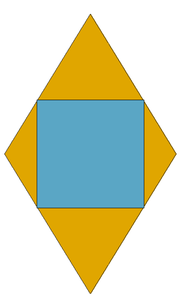

Fig. 1*: A cube fitted in a golden rhombus*

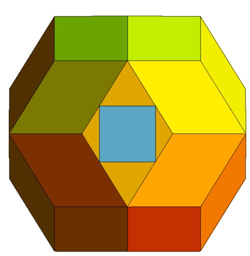

Fig. 2*: A cube placed on one face of a RT*

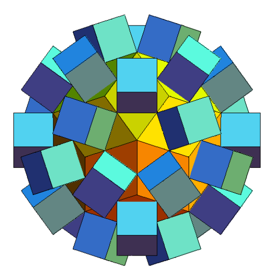

Fig. 3*: Cubes placed on 30 faces of a RT*

Relationship of Cube and other Polyhedra

In order to establish further polyhedral clusters let us try to substitute the cubes with different kinds of matching polyhedra (e.g. regular polyhedra or Platonic solids, namely tetrahedron, octahedron, dodecahedron, icosahedron, or semi-regular polyhedra, such as the rhombic triacontahedron (RT) or rhombic dodecahedron. For this purpose let us investigate the relationship of cube to other polyhedra.

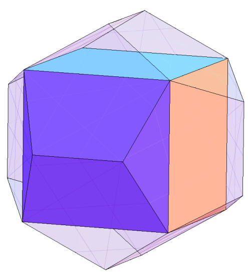

***Fig. 4**: Relationship of cube and pentagonal dodecahedron (PD)*

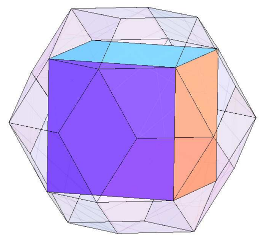

***Fig. 5**: Relationship of cube RT.*

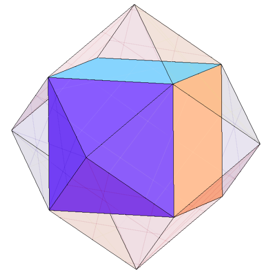

***Fig. 6**: Relationship of cube and rhombic dodecahedron (RD).*

In such assemblies, where the cube is used as a guide polyhedron, triacontahedra are in facial contact while the dodecahedra are connected along their edges.

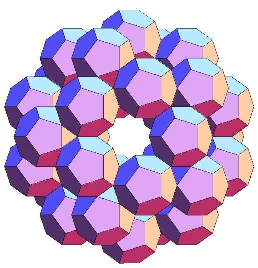

***Fig. 7**: Cluster of 30 PDs*

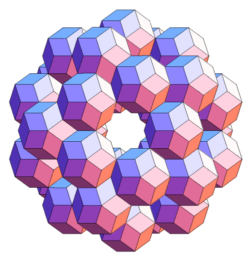

***Fig. 8**: Cluster of 30 RTs*

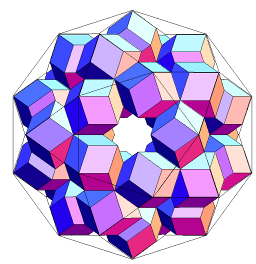

***Fig. 9**: Cluster of 30 RDs*

Rings of Polyhedra

The 30 units are placed on the face centers of RT, therefore the centers of units coincide with the vertices of an icosidodecahedron (ID), which is the dual of RT. In case of the rhombic dodecahedra, the tips of RDs coincide with the vertices of another ID. The edges of the ID can be interpreted as intersecting regular decagons. As a face of an RT is a distance of ϕ from its centre, one edge of a decagon is 2ϕ, which is equivalent to the line connecting the centers of two adjacent RTs.

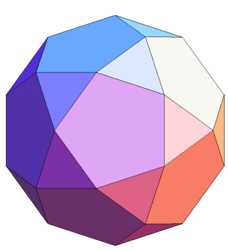

Fig. 10: Icosidodecahedron (one of the Archimedean polyhedra)

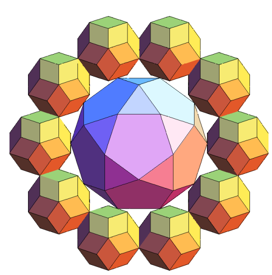

Fig. 11: Ten RTs in a ring, the centers of which coincide with the vertices of a regular decagon.

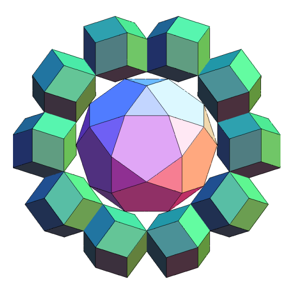

Fig. 12: Ten RDs in a ring.

Further polyhedral rings could be established by substituting other fitting polyhedra.

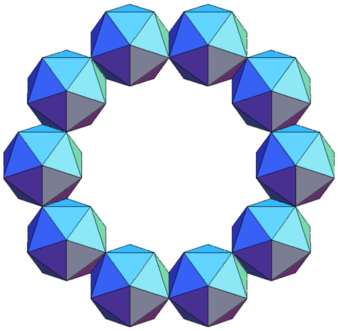

Fig. 13: Ten icosahedra in a ring connected with edges

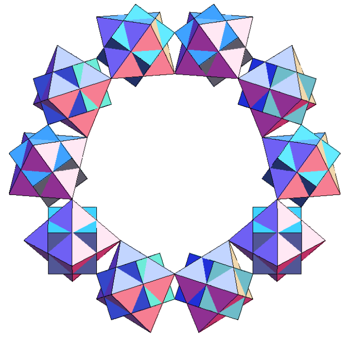

Fig. 14: A ring of ten units, where each unit contains one cube and its dual, the octahedron.

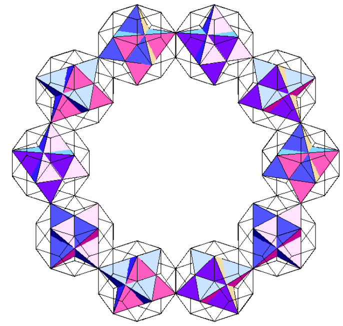

Fig. 15: A unit consisting of two tetrahedra (stella octangula) is fitted in each of the ten RTs arranged in a ring.

Clusters of Polyhedra

The rings of ten units, that can be found within the cluster of 30 polyhedra, are aligned in parallel with the pentagonal faces of ID, as well as in parallel with the faces of a pentagonal dodecahedron, because an ID is a truncated PD. Utilizing this feature let us place one ring on each of 12 faces of a dodecahedron. If the size of the dodecahedron, or the distance of ring from the origin is selected properly (i.e. 11.1352, calculated with simple trigonometric relationships), then the rings contact each other at their faces in the case of RTs, at their edges in the case of dodecahedra and icosahedra, and at their vertices in case of cube, octahedra and tetrahedra, the same way as within the rings.

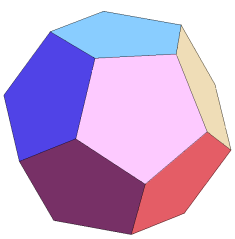

Fig 16: Pentagonal dodecahedron (PD)

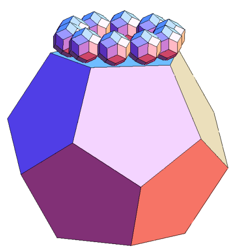

Fig. 17: Ring of ten RTs placed on a face of PD.

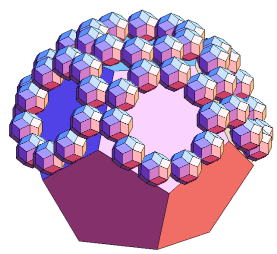

Fig. 18: Six rings of TRs on six faces of PD.

If a ring of ten units is placed on each of the 12 faces of a dodecahedron, then a cluster of 120 units is obtained. Essentially, the cluster has a dodecahedral arrangement. However, a detailed observation could reveal features that are characteristic of various shapes. For instance, in the cluster of 120 units, you can observe rings of six units, and rings of four units also, in addition to the rings of ten units. Now we can start searching for polyhedra having polygons with four, six and ten sides.

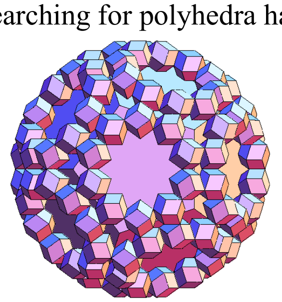

Fig. 19: Cluster of 120 RDs

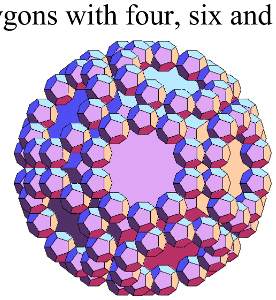

Fig. 20: Cluster of 120 PDs

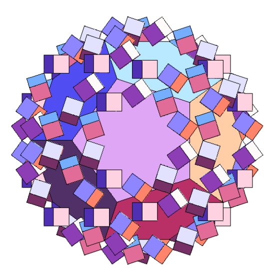

Fig. 21: Cluster of 120 cubes

For this reason, the whole cluster can be interpreted as if the units were placed at the vertices of a great rhombicosidodecahedron (GRID).

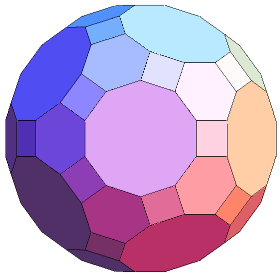

Fig. 22: The GRID.

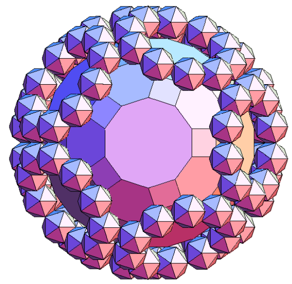

Fig. 23: Icosahedra at the vertices of a GRID

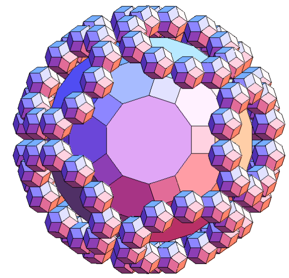

Fig. 24: RTs at the vertices of a GRID

Imagine now that we start to move all of the twelve rings in the direction of the origin. At a given distance (8.0574) certain units coincide, and the assembly seems to be consisting of 60 units, where the units are located at the vertices of a truncated dodecahedron.

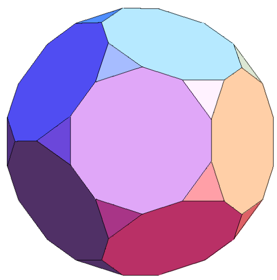

Fig. 25: Truncated pentagonal dodecahedron

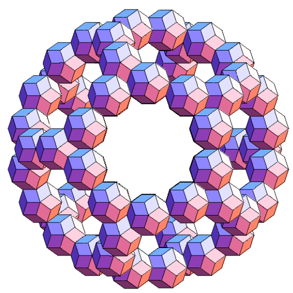

Fig. 26: 60 RTs at the vertices of a truncated dodecahedron.

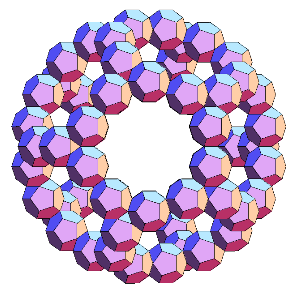

Fig. 27: 60 dodecahedra at the vertices of a truncated dodecahedron.

If the rings are moved further inwards (to a distance of 4.9798), then the assembly is rearranged, and the units take positions at the vertices of a small rhombicosidodecahedron (SRID). The coinciding RTs, icosahedra, and dodecahedra have the same orientations, but the coinciding cubes and RDs have different orientations.

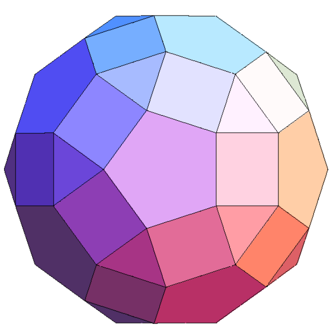

Fig. 28: The SRID.

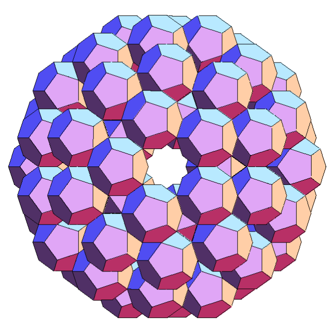

Fig. 29: 60 pentagonal dodecahedra at vertices of SRID

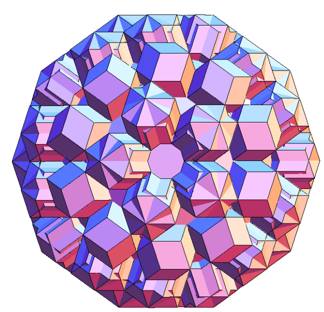

Fig. 30: Rhombic dodecahedra at vertices of SRID

Combined Polyhedra

If the rings are moved to the origin, then we are back at the original cluster of 30 polyhedra. These clusters could be depicted together with various polyhedra to establish interesting structures.

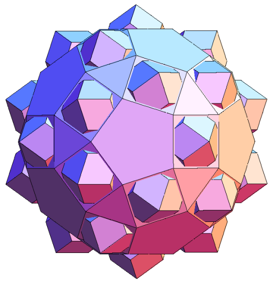

Fig. 31: Composition of 30 RD assembly and a SRID

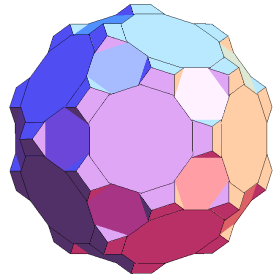

Fig. 32: Composition of 30 dodecahedra and a SRID

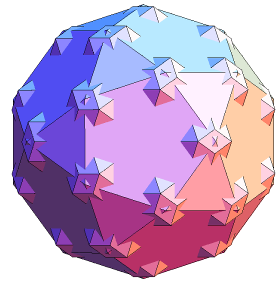

Fig. 33: Composition of 30 RTs and an ID

The reader might wish to investigate further structures using the relationships mentioned in this paper. One starting point could be the cluster of 120 units. If each ring of six units is moved simultaneously in the direction of the origin, then after a while every unit coincides with another, and a cluster of 60 units is established, where the units are located at the vertices of a truncated icosahedron. If the unit is a carbon atom, then we get the model of C60 molecule. Another path of further exploration is to find shapes, e.g. star polyhedra, which can be fit into the basic polyhedra mentioned here, and then insert such polyhedra in place of such assemblies. The clusters themselves can be fit into basic polyhedra, and then fractal polyhedra can be prepared (http://www.georgehart.com/rp/polyhedra-clusters/Polyhedra-Clusters.html). The computer software Wolfram Mathematica is very convenient for such explorations because, among many other beneficial functions, it includes data about 200 different polyhedra.

Physical Models

Physical models of polyhedral clusters can be made from Styrofoam blocks. These can be cut out with the hot wire method, where a wire is heated with electrical current passed through the material. This method is used in a machine constructed by the author. This kind of modeling can be used as a relatively inexpensive mode of exploring the possible arrangements for geometrical sculptures to be made from more durable materials.

The solid model of RT can be produced in a number of different ways starting from rhombohedra.

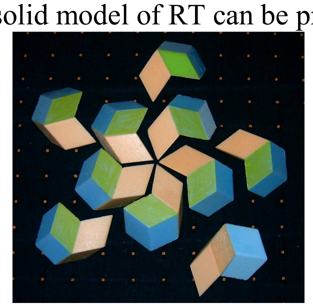

Fig. 34: A physical model of RT can be assembled from golden rhombohedra

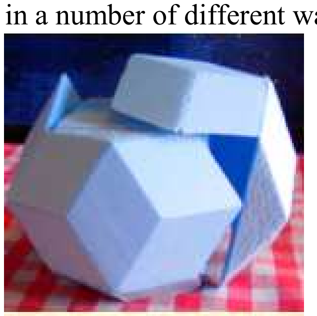

Fig. 35: Constructing an RT by placing six halved rhombic dodecahedra on a cube.

Fig. 36: An RT is assembled from twenty truncated prolate golden rhombohedra

The blocks cut out from Styrofoam (or expanded polystyrene) are adhered together to produce the final model. Adhering can hold the assembly together without the use of reinforcing bars. Where the blocks meet with their edges, supplementary rhombohedra can be used for support, without the need for jigs. With Zometool the clusters with edge connection and vertex connection can be built easily.

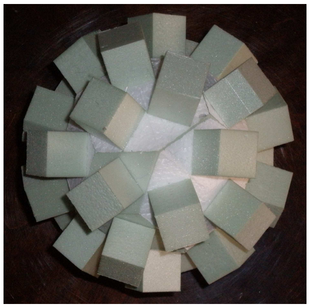

Fig. 37: Physical model of 30 cubes placed on an RT

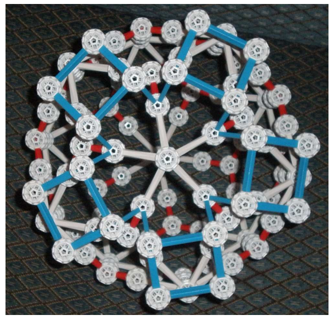

Fig. 38: Physical model of cluster of 30 cubes with Zometoool.

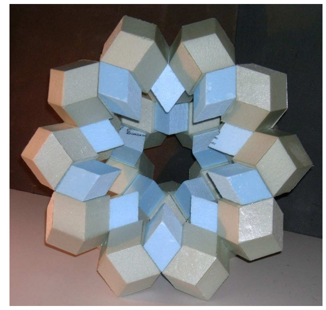

Fig. 39: The edge connected RDs are fastened with rhombohedra.

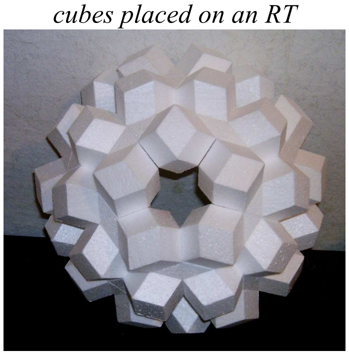

Fig. 40: Cluster of 60 RDs made of EPS blocks.

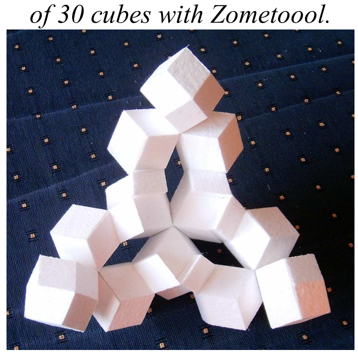

Fig. 41: A detail of cluster of 60 RDs seen from the inside.

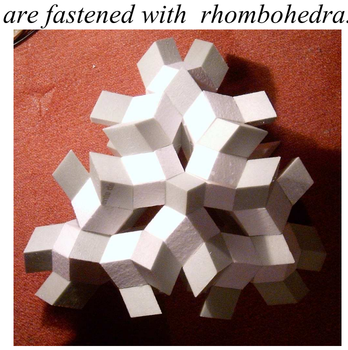

Fig. 42: A detail of cluster of 60 RDs seen from the outside

Data necessary for the production of oblate rhombohedra and the rhombic dodecahedron are detailed in the book Rhombic Structures [1], as well as in the book Mathematical Graphics with the Mathematica [2].

Summary

Based on the study of a geometrical model (sculpture), we have established associations with the following geometrical terms and features, as well as certain relationships among them: Golden ratio, Golden rectangle, Golden rhombus, Square root 2 rhombus, Oblate golden rhombohedron, Prolate golden rhombohedron, Oblate square root 2 rhombohedron, Prolate square root 2 rhombohedron, Regular (Platonic) polyhedra, i.e. cube, tetrahedron, octahedron, dodecahedron, icosahedron, Semi-regular polyhedra, i.e. rhombic dodecahedron, rhombic dodecahedron of the second kind (Bilinski polyhedron), rhombic triacontahedron, Archimedean polyhedra, i.e. icosidodecahedron, small rhombicosidodecahedron, great rhombicosidodecahedron, Stella octangula. Many of the dimensions and angles necessary for constructing such geometrical sculptures can be determined with simple calculations on the basis of essential relationships between the golden ratio and polyhedra.

References

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[8] Gerhard Kowalewski, Der Keplersche Korper und andere Bauspiele, Koehlers, Leipzig, 1938.