Geometric Tools for the Magic Woodcarver

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Abstract

A magician is a person who can make you disbelieve your own eyes by doing something you believed to be impossible. The magic woodcarver does that by carving things that seem (almost) impossible. Traditionally wooden chains and balls in cages have been favored projects, also for me at the start of my carrier. Over the years, however, I have managed to expand the range of projects by introducing entirely new and vastly more complicated types of models to carve. I have used creative geometric tools to achieve this goal. This paper explains two of my most successful geometric methods.

Magic Woodcarving

If you were a country boy in this country 200 years ago and wanted to impress the dairymaid, a very clever strategy would be to carve her a wool winder with one or two loose balls inside the handle that would rattle when she used it. In Wales or in Norway, tradition would have suggested a love spoon, which might also have a ball inside the handle, or even a pair of spoons connected by a chain. Wooden chains were also a favorite pastime among American hoboes a few generations ago.

Figure 1: *Examples of traditional magic woodcarving.*

The magic of these objects comes from the obvious fact that it is impossible to get the balls inside the handles without breaking the surrounding bars, and it is equally impossible to assemble the links of the chain without breaking some of them. The inescapable conclusion is that the balls must have been carved inside the handles, and the links of the chains must have been already joined as they were carved from a single bar of wood. This is of course how it is done, and this is what I call Magic Woodcarving.

My contribution to the field has been to look for and find new and more sophisticated models to carve. This search has taught me many geometric tricks and methods. The purpose of this paper is to present two of the best and most productive of these methods. Great pleasure and wonderful aha-experiences have been involved in my work. If just a little of this could come through to you, my efforts would be more than justified.

It is of the utmost importance when working with creative geometry to get your mind into the proper geometric mode, and to keep it open to any impulse that may arrive. The methods and techniques I am discussing here may help you get into the right geometric mode, but how to keep an open mind is a totally different matter that I will not try to analyze here.

Themes and Variations

The first technique I discuss concerns the way one idea may lead to another through a process resembling the musical concept of theme and variations.

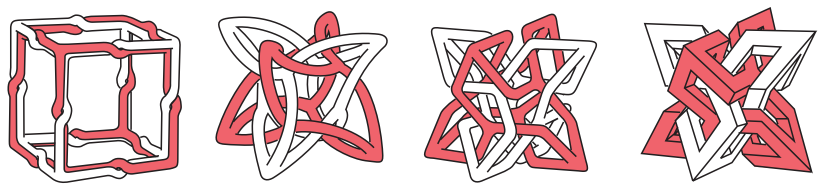

Figure 2: *My Double Star is essentially two cubic edge frames.*

We will find our first theme in a model I call Double Star which is derived from two cubic edge frames with a half twist applied to each pair of edges, so that the vertices of the two cubes are found alternately inside and outside the structure. As the structure is deformed to form a double star, the inside vertices become flat nodes. These internal nodes will be the theme of our first variation (figure 3, top row). To vary the theme

Figure 3: *Five typical examples of themes with variations.*

we split the outer vertices open and straighten the edges. This leaves us with eight three-branched propeller-like objects arranged around a central octahedral hollow which I have made visible to give a clearer picture of the situation. We will now join the arms of each pair of propeller-objects lying on opposite faces of the central octahedron, and thereby obtain an compound of four three-edged hosohedra.

For our next theme we may take the idea of propeller-like objects lying on the faces of a central polyhedron. This time let the polyhedron be a cube (figure 3, second row). The propellers should then be four-branched, and the result of joining the arms of opposite propeller-objects will be three four-edged hosohedra.

Now let us keep the outer arcs of this model steady and vary the way they are connected internally (figure 3, third row). By splitting open the inside nodes and curving the arms a little more, we can make each arm meet with one of its neighbors in a uniform manner to form four coiled rings linked together in an interesting way.

Keeping the outer arcs as the theme, we now split the internal nodes only partially keeping the arms together two by two in parallel courses (figure 3, fourth row). By alternating the directions of the splits we get six twisting rings each linking with four of the others and twisting in parallel with the sixth.

Let us finally revert to our first theme, the internal nodes of the Double Star (figure 3, fifth row). Again we split the outer vertices open and straighten the edges, but this time we bend each edge over to the nearest neighboring vertex. Reassembling the edges this way we get another kind of double star, whose individual parts are strangely enlacing themselves.

Diagrams and Kinship

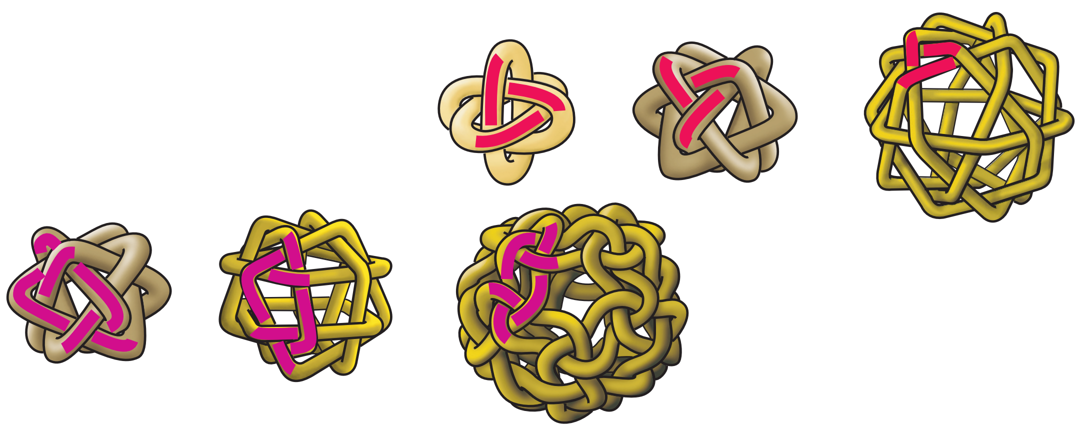

Figure 4: *Some models are related by the lace patterns formed by their rings.*

Another very powerful geometric method has to do with kinship among the models. When you look at ring models as the ones shown here you tend to focus on the shape of the individual rings and the number of rings. This is in effect to focus on the differences. Try instead to look at the pattern the rings form where they meet, and you will start finding characteristic similarities between models (figure 4). In the top row you see a basic pattern of three rings meeting in a vortex, which recurs in all three models. On the big model this basic pattern is repeated five times around pentagonal openings; in the middle it is repeated four times around square openings, and on the Borromean Rings it is repeated three times around triangular openings (which incidentally happens to be identical to the basic pattern - hence the mirror symmetry of this model). In the row below, vortices of three rings alternate with vortices of four rings. The way the patterns repeat is just like in the top row. A practical systematic way to expose such similarities might help us classify the models and give us a better understanding of their nature.

Regular Diagrams

A good way to focus attention on the patterns between rings is to project the rings onto a circumscribed regular polyhedron of the appropriate type (figure 5). As you can see from the example shown here we may have a choice of which polyhedron to use, and the resulting diagram may be a triangle or a square. We may also sometimes get pentagonal diagrams and the relation between the different diagrams may not always

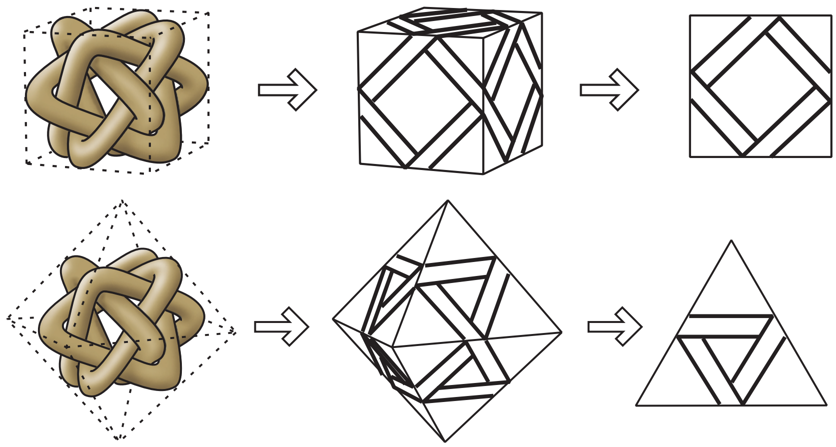

Figure 5: *Two different regular diagrams derived from the same model.*

be obvious. It would be convenient if we could have just one diagram that captured all relationships based on a single basic pattern. To see how this can be achieved we need to take a close look at the five Platonic solids, the prototypes of spatial symmetry.

What matters here is their rotational symmetry (mirror symmetry is largely absent from my models). So let us look for axes of rotational symmetry (figure 6). All five solids have twofold axes through the midpoints

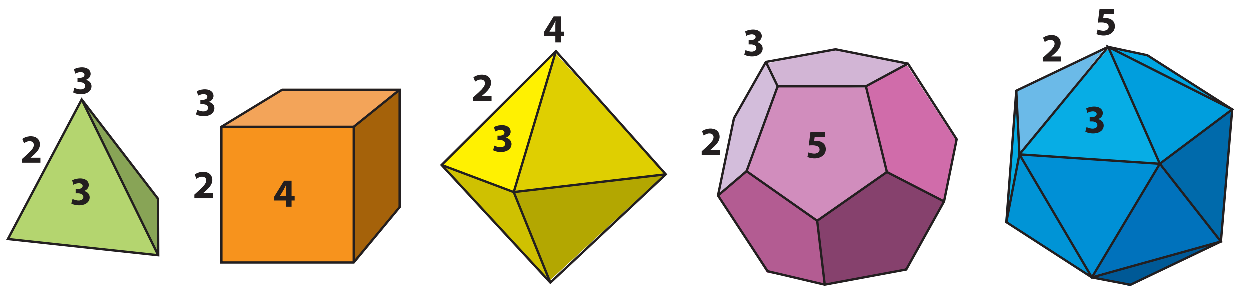

Figure 6: *The five Platonic solids marked with axes of rotational symmetry.*

of their edges and all five have threefold axes through either their vertices or the midpoints of their faces – the tetrahedron has both and they coincide. Then the cube and the octahedron have fourfold axes through the midpoints of faces and through vertices respectively. The dodecahedron and the icosahedron likewise have fivefold axes through the faces and vertices respectively.

A little further investigation will show that the five solids actually represent only three different types of symmetry, each type characterized by its highest order of rotational symmetry, which is three for the tetrahedron, four for the cube and octahedron, and five for the dodecahedron and icosahedron. These characteristic numbers are expressed in two different ways by the five solids: through faces and through vertices. The threefold symmetry is expressed through both faces and vertices of the tetrahedron; the fourfold symmetry is expressed through faces of the cube and vertices of the octahedron, and the fivefold symmetry is expressed by faces of the dodecahedron and vertices of the icosahedron.

Duality and Rhombic Solids

What we are discussing here is an instance of a more general geometric principle of duality. We say that the solids form dual pairs - the tetrahedron being its own dual – and these pairs can be brought together so that all symmetry elements coincide (figure 7): The edges intersect perpendicularly at their midpoints – the twofold axes; vertices of one solid align with midpoints of the other solid's faces – the threefold and maximum symmetry axes.

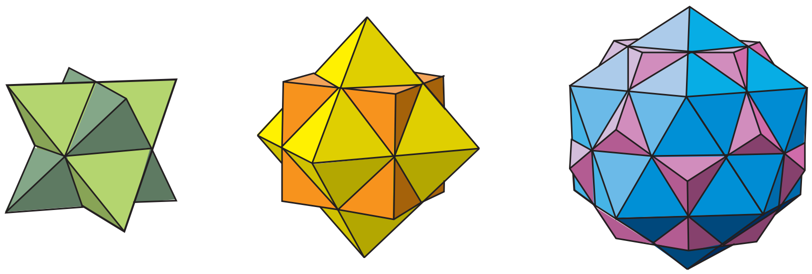

Figure 7: *The five Platonic solids arranged as compounds of dual pairs.*

These compounds elegantly exhibit what I need for systematizing my models, namely the three basic types of spatial symmetry expressed in two different ways. Luckily the same can be expressed by three solids having just one type of faces. Just wrap the compounds and you will get three rhombic solids (figure 8): the cube with six square faces (a square is a right angled rhombus), the rhombic dodecahedron with twelve rhombic faces, and the rhombic triacontahedron with thirty rhombic faces.

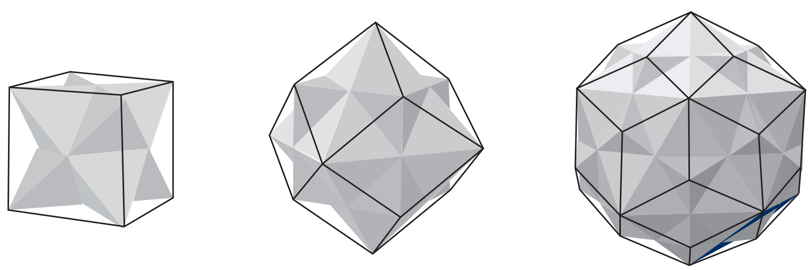

Figure 8: *Wrapping the dual compounds yields three rhombic solids.*

Rhombic Diagrams

You can now obtain five related models from a single rhombic diagram (which should have twofold symmetry). Start with the diagram as a right angled rhombus - a square. Apply six copies of the diagram to the cube (figure 9) so that equal corners of the diagram meet at all vertices. (There are two ways of doing this but they give identical results). Now squeeze the diagram to fit the faces of the rhombic dodecahedron and apply a copy to each of the twelve faces. There are two ways you can squeeze the diagram so you get two different models (unless your diagram has fourfold symmetry). Then squeeze the diagram a little more and apply it to the thirty faces of the rhombic triacontahedron. Again you get two models by squeezing one way or the other.

The family shown here has been my most successful use of rhombic diagrams. I knew only the first and the fourth of the models, the Triple Whitehead and Halo (ten triangles which I have reshaped like clover leaves), and I was not even aware they were related. The other three models were a direct result of this powerful technique.

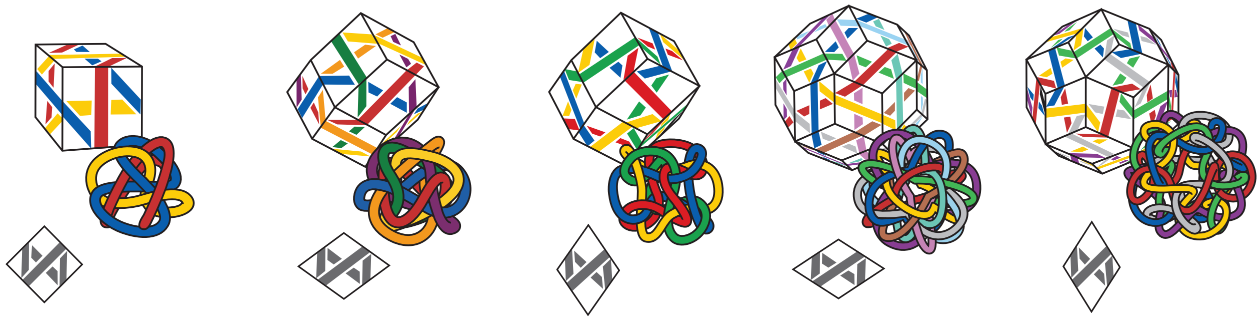

Figure 9: *Five related models based on a single rhombic diagram.*

Using the Diagrams

There are two tasks involved in working with these diagrams. One is to draw the diagram for a model at hand. The other is to construct or identify the different family members obtained from a given diagram. Each of these tasks has it own difficulties.

To draw the diagram for a given model you must first identify a region of the model that corresponds to a single rhombus (figure 10). Look for the rotation axes of the model. If your model has four or five fold axis, two of these will correspond to the acute vertices of the rhombus. The two obtuse vertices will correspond to threefold axis. If no four or five fold axes are present there should be two types of threefold axes. These correspond to pairs of opposite vertices of the rhombus (or square).

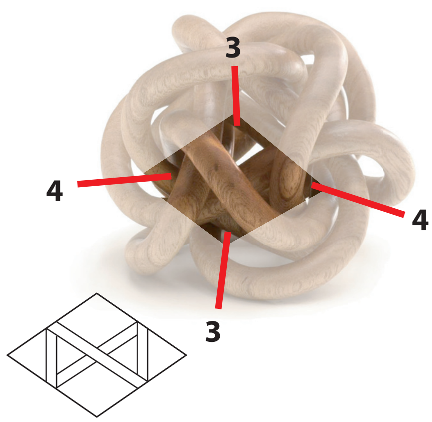

Figure 10: *The major symmetry axes of the model correspond to vertices of the rhombi.*

Now imagine a rhombic pyramid spanned between these four symmetry axes with its apex at the centre of the model and its base somewhere outside the model. The mental task is now to represent the part of the model contained within this pyramid as a simplified diagram at the rhombic base. A few hints may help you: Do parts of the model meet at the rotation axes (i.e. the vertices of the diagram)? How many parts of the model cross each face of the pyramid? Mark these crossings as points on the sides of the diagram. How are these crossing points connected? How do the parts cross one another, within the diagram or at its edges? The actual shape of the parts is not important; a schematic representation is to be preferred as the diagram is meant to represent five different models.

The task of constructing the models represented by a given diagram is more straightforward, at least in the initial stages. First you should prepare templates for the nets of rhombs and squares needed to fold and

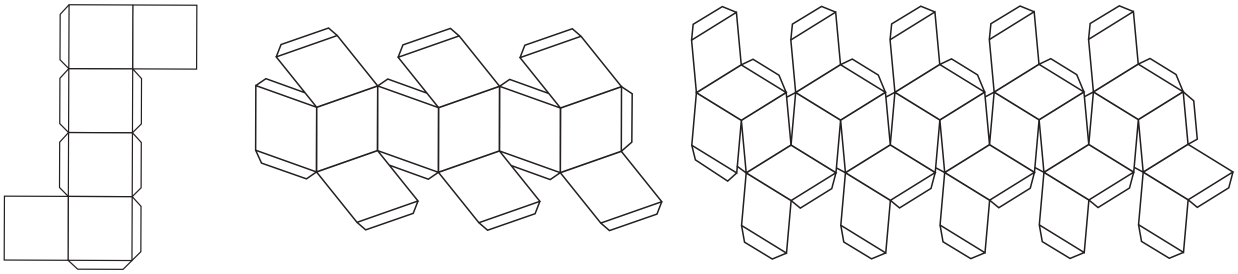

Figure 11: *Nets for the three rhombic solids.*

glue the three different rhombic solids (figure 11). I highly recommend doing this in a standard computer drawing application. This way it is easy to squeeze the rhombic diagram both ways and copy it to all faces before you print. The alternative would be to draw by hand on photocopies of the nets. After cutting folding and gluing the five solids, you are probably eager to study the structures represented by each of them. This is best done by tracing their individual parts with a color marker (figure 12). You should therefore always draw your diagrams in outlines.

Figure 12: *The result of applying a single rhombic diagram.*

If you are satisfied with the result you will probably want to make more realistic models. Pipe cleaners are ideal for this purpose, but the task is not always easy - both care, patience, concentration, and dexterity are required. If the pattern is just slightly complicated, you will soon find yourself in trouble keeping track of which pipe cleaner represent which strand, and which should cross over and which under. A useful trick is to make the pipe cleaner model with the rhombic paper model inside; then, once you have succeeded, you can cut up the paper model and pick it out with a pair of tweezers.

When I first attempted to arrange my models into families I discovered several unexpected relationships. More important were the missing family members. A few of these vacant family positions were waiting for some of my best models such as Wave Packet and Hexacoil (number 3 and 4 in figure 12). Others revealed interesting results that did not invite to be carved, either because they only had one single part, or because the parts did not support each other but would collapse into a mess. There were also slots that would reveal nothing more than a mirror image, or even a repetition. This was those with very simple diagrams.

Conclusion

You may have noticed that my approach is very practical and does not rely on sophisticated 3D computer software. I have tried such tools and am aware they can be of great assistance and save a lot of time, but with no institution behind me I have only had limited access to such facilities. Also, I quite like the hands-on approach; and if time were a big issue I should not be involved with magic woodcarving at all!

Others have studied regular links more thoroughly than I have. I still remember the feeling of amazement I felt back in the eighties when I first opened Alan Holden's book Orderly Tangles [1] and saw many of my own creations along with a lot of other regular polylinks, as he called them. Lately I learned that Robert Lang has expanded the scope of Holden's work into what he calls polypolylinks [2] of which he lists a total of 54 - again duplicating several of my own works. This is not surprising when you use mathematical methods and work in related fields. Holden and Lang, however, seem to focus more on the mathematical aspect, striving to enumerate an entire class of objects with certain properties. For such endeavors you have to impose certain constraints on the objects being studied in order to delimit their class. One such constraint could be a limitation to straight line segments, which seems to have been used very effectively by both Holden and Lang.

Being mainly interested in the aesthetic aspects of symmetry and regularity I never felt the urge to 'find them all'. I think what is probably new in my approach – if anything – is the emphasis on weaving patterns rather than the number and shapes of individual substructures. This has led me to structures whose parts curl around each other in complicated ways, sometimes even tying knots on themselves. Allowing such complications seems to open an infinite number of possibilities, so the idea of enumeration becomes irrelevant. What I am trying to do is merely to find some systematic ways of exploring this vast territory. Others artists have explored the territory of curved symmetric structures and knots - Bathsheba Grossman, Carlo Sequin, George Hart, and Rinus Roelofs [3] - [6], to mention just a few. As sculptors they tend to focus on structures that interlace themselves as a single connected structure, whereas I, as a magic woodcarver, am mainly interested in those structures that decompose into two or more interlaced parts. As Bathsheba Grossman puts it, she and I have complementary brains.

References

[1] Holden, Alan, *Orderly Tangles: Cloverleafs, Gordian Knots, and Regular Polylinks*, Columbia University Press, New York 1983

[2] Thomas Hull (ed.), *Origami^3*, A K Peters Ltd, 2002
http://www.langorigami.com/science/polypolyhedra/polypolyhedra.php4

[3] http://www.bathsheba.com/sculpt/

[4] http://www.cs.berkeley.edu/~sequin/ART/index.html

[5] http://www.georgehart.com/sculpture/sculpture.html

[6] http://www.rinusroelofs.nl/sculpture/sculpture-00.html