A Workshop to Build Three Simple Tensegrity Models for K-12 Mathematics Classrooms

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Abstract

In this workshop, participants will build two styles of models. One style of model is built from sticks, and vinyl tubing. This first model will determine lengths of struts and lines, and will also be extensively labeled to help in building finished tensegrity models. A second style of model will be built from PVC pipe, PVC sleeves, and bungee cord. Additional printed information will be provided to help build the finished tensegrity models.

Building physical models [1], [2], [3], and writing about the building process, continues to be my focus. Most recently, I have chosen to build tensegrity models, and to provide clear instructions to help build them from PVC pipes, PVC sleeves, and bungee cords. I have constructed preliminary models from sticks and vinyl tubing, and labeled them extensively in order to assist in the building of the final PVC models. The final models take a considerable amount of time and resources to build. It has been important that the models be durable so they can be handled by students through a number of years, and be of a reasonable size so that they can be seen by 20-30 students in a classroom. Each tensegrity model has an interesting set of spatial properties and is engaging as a physical object.

1. Introduction

Physical models for use in K-12 classrooms can be built from paper [1], from sticks and vinyl tubing [2] or from PVC pipe, PVC sleeves and bungee cord [3]. These models are simple and use inexpensive materials, yet convey meaningful spatial concepts to students who manipulate them in their hands. The models described in this paper are tensegrities which combine both compression members and tension members. The compression members are struts realized with PVC pipe, and the tension members are realized with bungee cords. PVC sleeves interconnect compression members and tension members, **Figure 1**. Tensegrity models are stable, in that their compression members do not move relative to each other. In addition, compression members do not touch each other, giving the viewer the impression that the compression members are floating in space.

In my research of tensegrity models I encountered the work of Kenneth Snelson [4] with his initial discovery of tensegrity structures in 1948. Snelson was an art student at Black Mountain College in North Carolina. Snelson's work of 60 years ago continues to be inspirational for me and is the motivation for this paper. He is an artist with his work being exhibited in many prestigious art galleries.

I have produced two sets of physical models for this paper. The first set of models is built with sticks and vinyl tubing. These models are based on simple geometric shapes such as a cube, **Figure 3a,** a truncated tetrahedron, **Figure 4a,** and a twisted triangular anti-prism, **Figure 5a**. These models contain all the struts of the finished tensegrity models, and are extensively labeled in order to better understand and assist the building of the final tensegrity models. The stick models serve to support all the compression struts of the final tensegrity models.

A cube, **Figure 3a,** provides support for the struts in the building of a 1^{st} tensegrity model which is entitled an extended octahedron, **Figure 6a**. A truncated tetrahedron, **Figure 4a,** serves to support the struts of a 2^{nd} tensegrity model. A twisted triangular prism, **Figure 5a,** serves to support and build a 3^{rd} tower tensegrity model. The stick models provide both the lengths for compressions members of PVC pipe and the length of bungee cord for the tension members.

All struts in a single tensegrity models are the same length. Once there is an understanding of using PVC sleeves and line to connect two struts, all connections in a single tensegrity model are the same. The stick models have their struts labeled and their sleeves labeled and provide a map to follow in assembling them into a complete tensegrity structure. All individual tensegrity models are assembled in an identical manner.

Appendix I itemizes supplies for the individual tensegrity models. This table of supplies has both the length of struts, the length of lines, and the number of sleeves to interconnect compression members and tension members. Labeled figures in the paper provided a plan for assembling the tensegrity models. The labeled figures will hopefully make it easier to be convinced that building these models is a reasonable and manageable task to accomplish.

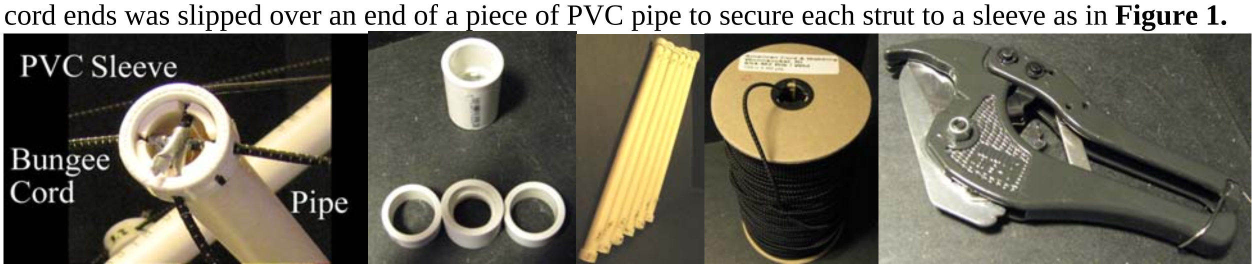
2. Supplies and Tools

Supplies for these models consist of PVC sleeves cut into short lengths for individual struts, (**Figure 2a**), ½" PVC pipe, (**Figure 2b),** and a roll of 7/16" bungee cord, (**Figure 2c**). A PVC pipe cutter is the principal tool (**Figure 2 d**) for producing the parts for the tensegrity models.

The ½" PVC pipe was chosen to balance with the 16" length of the struts, producing reasonably sized models for teachers and students to use in a classroom. The PVC connectors were used to secure the bungee cord to the PVC struts, as seen in **Figure 1.**

A tape measure was used to measure the length of each strut and each piece of bungee cord. The PVC pipe cutter was used to cut each strut with a straight 90° cut. It was also used to cut sleeves, **Figure 2a,** into pieces; 2 sleeves and a scrap piece. Cuts for stuts were made straight across the pipes in order to form 90° right angles when a piece of PVC pipe was joined together with the PVC connector sleeves. Small slots were cut lengthwise along pipes near the end of the pipes with a finish saw that was wide enough to tightly hold the bungee cord in place.

A solvent was used to clean printing from pipes & sleeves. A power drill with a 1/8" bit was used to drill holes in sleeves and pipes through which the bungee cord was threaded. A sleeve with 4 bungee cord ends was slipped over an end of a piece of PVC pipe to secure each strut to a sleeve as in **Figure 1.**

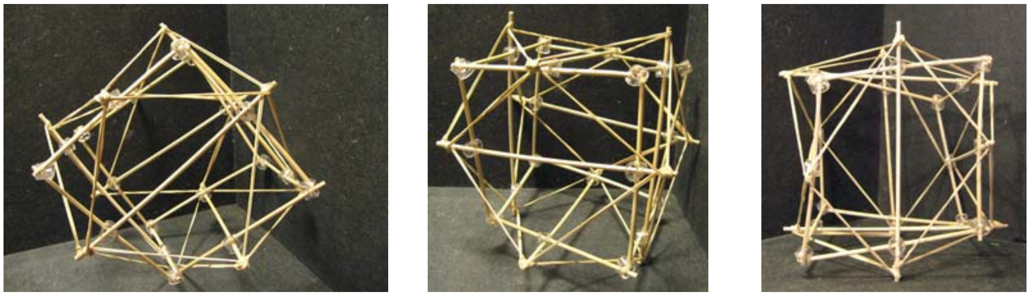


(a) **(b)** **(c)** **(d)** **(e)**

Figure 1: Pipe, sleeve, cord (a), Sleeves (a), Struts & Sleeves (b), Bungee Cord (c), and pipe cutter (d)

3. Tensegrities from a Cube, a Truncated Tetrahedron, and a Twisted Triangular Prism

3.1. A Tensegrity Model based on a Cube. A first tensegrity model can be built using a cube as a supporting structure as in **Figure 3a**. Struts of this tensegrity model can be seen tilted at a 45°angle to the horizon, **Figure 3a**, parallel to the horizon, **Figure 3b,** and perpendicular to the horizon, **Figure 3c.**

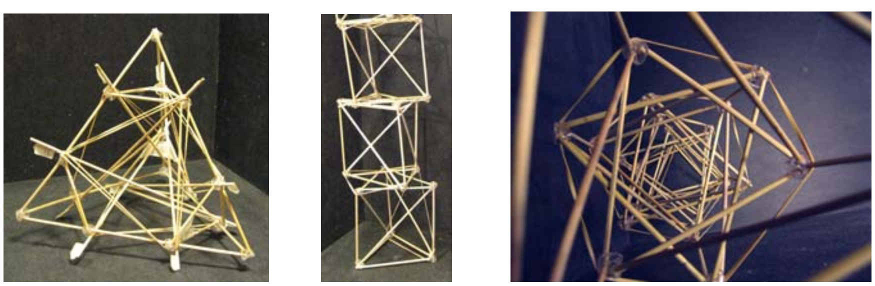


(a) **(b)** **(c)**

Figure 3: A cube with a 45° strut **(a)**, a more horizontal strut **(b),** a more vertical strut **(c).**

The cube was built from 12 sticks and 8 rings of vinyl tubing for its vertices. This tensegrity structure uses 6 struts that have additional vinyl tubing that are located at the middle of edges on each side of the cube. The 24 lines for the tensegrity are positioned in groups of 4 at the ends of the 6 struts. Views of more horizontal struts are seen in **Figure 3b,** and more vertical struts are in **Figure 3c.**

3.2. A Tensegrity Model based on a Truncated Tetrahedron. A second tensegrity model can be built using a truncated tetrahedron as a supporting structure as in **Figure 4a.** This tensegrity model has 6 struts and 24 lines. Initially, a tetrahedron can be built with 6 sticks and 4 rings of vinyl tubing for the vertices. This tetrahedron has 2 additional vinyl tubing rings added on each edge of the tetrahedron for the edges of the truncated tetrahedron, **Figure 4b.** Each strut has 4 lines connected to each end.



(a) **(b)** **(c)**

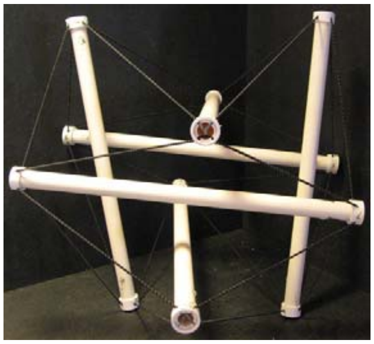
Figure 4: A truncated tetrahedron with tensegrity **(a)** **Figure 5**: Three triangular anti-prisms for a tensegrity tower **(b)**, Axial view of tower tensegrity with triangles alternating their direction **(c).**

3.3 A Tensegrity Model based on a Twisted Triangular Anti-Prism. A third tensegrity model can be built using three twisted triangular anti-prisms as supporting structures as in **Figure 5a.** Each twisted triangular anti-prism is rotated in either a clockwise or a counterclockwise direction, one above or below the other **Figure 5a.** Viewing the tower along its long axis, **Figure 5b**, yields a view of triangles that are pointing either mostly up or mostly down. Many of the sticks appearing in **Figure 5a** will be replaced by lines in the final tensegrity model.

4. Tensegrity Models of PVC pipes, PVC sleeves and Bungee Cord

4.1 A Tensegrity Model Based on a Cube. Six struts with 3 pairs of parallel struts **Figure 6,** 12 sleeves on the 6 struts and 24 bungee cords form an extended octahedron tensegrity model.

4.2 A Tensegrity Model Based on a Truncated Tetrahedron. Six struts where exterior of struts have a truncated tetrahedron with a small triangle and a larger triangle. The model has 12 shorter bungee cords and 12 longer bungee cord lines along edges of the truncated tetrahedron, **Figure 7.**



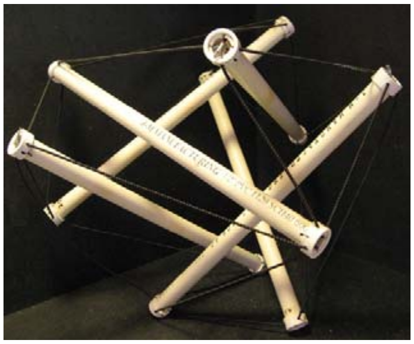
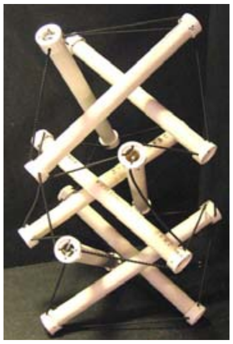


Figure 6: Tensegrity on extended octahedron. **Figure 7:** Tensegrity on truncated tetrahedron.

4.3 Tensegrity Model Based on 3 Anti-Prisms. A tower tensegrity model built from 9 struts with 3 sets of 3 parallel edges, with short and long lines for bungee cords, **Figure 8**. An axial view of the tower model yields a view of a central hexagon **Figure 9.**



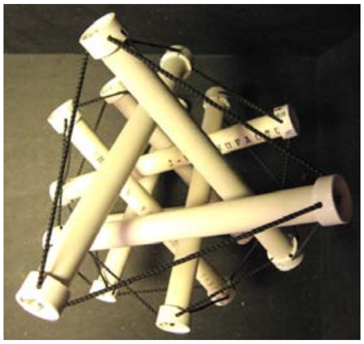


Figure 8: Tower tensegrity model **Figure 9:** Axial view of tower tensegrity model

5. Handling PVC Pipes, PVC Sleeves and Bungee Cord Tensegrity Models

I particularly like the 3 tensegrity models presented in this paper, because each stable model provides a very different spatial experience for the student. The experience gained by building each model helps a great deal toward building a second and third model. These three models allow students to observe structures which have struts that appear to float in space without touching each other.

6. Conclusion

During the past year I learned a very valuable lesson about the size of the physical models for the classroom and the size of the average student in that classroom. There is a relationship between the size of the model where the students are comfortably assembling and manipulating the models in their hands. A somewhat smaller model has emerged where the comfort for the students to manipulate the models has improved. It may be because I usually present the models to adult teachers, that my models have been somewhat larger. However, this year I used smaller models for 6^{th}, 7^{th}, and 8^{th} grade students and the presentations have improved for the students.

These PVC based models were built during the past year for the first time. Preliminary stick models provided me detailed building information and confidence to go ahead and build the tensegrity models, for subsequently presenting them in the classroom and at the conference.

I have given presentations with this paper and these models at the McGillis School in Salt Lake City for 3^{rd} through 8^{th} grades. These presentations have been invaluable for the students, who both listen well and anticipate the opportunity to handle the models. Models that can be thrown somewhat gently around the classroom help to entertain as well as inform the students.

In addition, the presentations with the physical models also help me to learn more about the effectiveness of a particular set of models and their associated building instructions in the classroom. The tactile experience of handling the models provides the students with a more engaging experience than merely viewing three-dimensional images on paper or on a computer monitor.

I have been presenting models in classrooms for over 20 years and they have proven to be well received, in addition to being valuable experiences for me. When I am building models I am imagining them being handled by students and presenting the associated concepts in a classroom for students and teachers. It takes time and effort to build models and I always hope that they will be effective in helping to clearly convey underlying geometric concepts. However, with each new set of models, the testing ground remains in the classroom. The models are only considered a success if the presentations and models are both thought provoking and engaging for students and teachers.

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References

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Appendix I – 3 Tensegrity Modls

	struts		sleeves		lines	
polyhedron	**number**	**length**	**#**		**length**	**#**
Cube	6	16”	12		10”	24
Trunc Tetra	6	16”	12	Short	5”	12
				Long	12”	12
Tri TwAnti Prism	9	10”	18	Long	6”	12
				Short	4”	16