Binary Based Fresco Restoration*

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The Neidhart frescoes (located in Tuchlauben 19, 1010 Vienna) from the 14th century are depicting a cycle of songs of the 13th century minnesinger Neidhart von Reuental. The picture on the left shows a part of the Neidhart frescoes. The white holes in the fresco are due to the wall which covered the fresco until a few years ago. They arised when the wall was removed.

In the following we want to apply digital restoration (inpainting) methods to these frescoes. Thereby the main challenge is to capture the structures in the preserved parts of the fresco and transport them into the damaged parts continuously. Due to their great age and almost 600 years of living by owners and tenants in the apartment, saturation, hue and contrast quality of the colors in the frescoes suffered. Digital grayvalue, i.e., color interpolation, in the damaged parts of the fresco therefore demands sophisticated algorithms taking these lacks into account.

In our previous Bridges paper [1] we presented two inpainting methods based on partial differential equations (PDEs) and their application to the reconstruction of the Neidhart frescoes. Because of the lack of space in this paper we refer to Burger et al. [2] for a motivation of the PDE approach and relevant references. The first method in [1] constitutes in a modified Cahn-Hilliard equation for the restoration of the binary structures of the frescoes [2, 3, 4], see Figure 1 for a specific example. The second method is a generalization of this binary inpainting approach for grayvalue images and is called TV-H^{-1} inpainting [2]. The grayvalue approach produced fairly good results for the inpainting of homogeneous areas in the frescoes, but not very satisfying results for the continuation of edges into the holes [1].

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Figure 1: from left to right: Part of the fresco; binary selection; binarized selection of the fresco; initial condition for the inpainting algorithm where the inpainting region is marked with a gray rectangle; Cahn-Hilliard inpainting result after 200 timesteps with a large diffusion coefficient; Cahn-Hilliard inpainting result after additional 800 timesteps with less diffusion for sharpening the edges

In this work we propose a combined strategy for the restoration of the frescoes, which we call *binary based restoration* method. This method is motivated by the previous work of Fornasier [5], and Fornasier and March [6]. More precisely we propose an inpainting model for grayvalue images which uses a given (or previously obtained) binary structure inside the missing domain D. Thereby the binary structure of the image is usually obtained by a preprocessing step with Cahn-Hilliard inpainting [3, 4] and Figure 1. This new approach is described in the following paragraph and in Figure 2.

Let $f \in L^2(\Omega)$ be a given image which is a grayvalue image in $\Omega \setminus D$ and binary in D. We want to recover the grayvalue information in D based on the binary structure given by f by means of the following minimization problem:

$$u^* = \operatorname{argmin} \Big\{ \quad \frac{\mu}{2} \int_{\Omega \setminus D} |u(x) - f(x)|^2 \; dx + \frac{\lambda}{2} \int_D |\mathscr{L}_{bin}(u(x)) - f(x)|^2 \; dx + |Du|(\Omega), \; u \in L^2(\Omega, \mathbb{R}_+) \Big\} .$$

In our case $\mathscr{L}_{bin}$ is a transformation which projects the grayvalue range of u, e.g., $[0,1]$, on the binary range $\{0,1\}$. For our purpose $\mathscr{L}_{bin}$ is modeled by a relaxed version of the Heaviside function depending on a (presumably) given threshold τ.

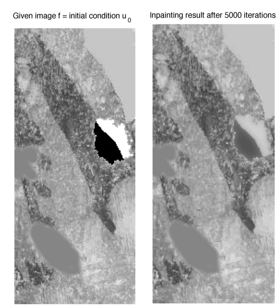

Figure 2: Binary based grayvalue inpainting: (l.) initial condition; (r.) inpainting result with $\mu = \lambda = 10^2$ and 5000 iterations.

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