Geometry and Geodesy: Estimating the Earth's Circumference with Prospective Elementary Teachers

Sandy Spitzer
Mathematics Department
Towson University
8000 York Road
Towson, MD, 21252, USA
E-mail: sspitzer@towson.edu

Abstract

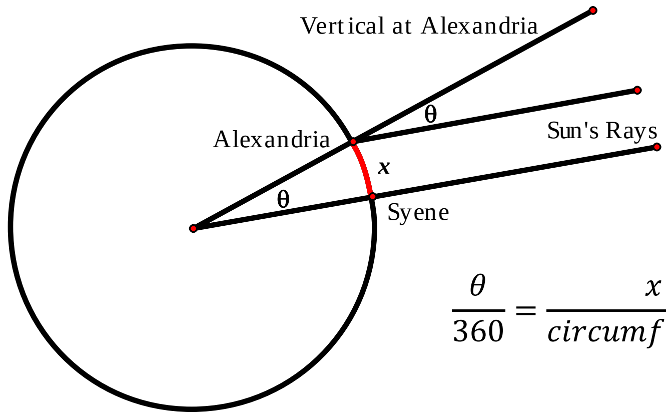
In order to stimulate interest in geometry, model the use of complex, real-world applications of mathematics for prospective teachers, and draw a connection between mathematics, history, and culture, a lesson was designed in which students replicate the experiment of Eratosthenes of Cyrene (c. 275-194 B.C.) to estimate the circumference of the earth. During the lesson, students measure the angle of inclination of the sun on a date near the spring equinox and build models of the situation using dynamic geometry software. This process helps prospective teachers develop knowledge about modeling situations, measurement, similar figures, and other geometric topics.

Introduction and Context

A central concern for those professors teaching mathematics content courses for prospective elementary school teachers is how to maintain a high level of mathematical inquiry while stimulating students' interest and confidence in doing mathematics. Recent recommendations by policy groups and literature analyses [1, 2] suggest that in these courses, a high priority should be put on developing prospective teachers' conceptual understanding of mathematics and modeling effective methods of mathematics instruction. In particular, the Conference Board of the Mathematics Sciences [1] emphasizes the importance of building visualization and modeling skills and encouraging mathematical communication in geometry courses for prospective teachers.

I designed the "Geometry and Geodesy" lesson in an attempt to fulfill these recommendations in a geometry class for prospective elementary teachers. Prior to the lesson, students had completed a three-week unit on basic geometric constructions and attributes as well as a one-day lesson on similar figures. Throughout the course, students are encouraged to describe geometric situations using increasingly precise language and investigate conjectures beginning with empirical methods (such as by using dynamic geometry software to test a conjecture for many examples) and moving toward deductive proof.

The problem under consideration is one that has interested humans since ancient times: how big is our planet? The study of geodesy (the science of measuring the size and shape of the earth) originated with Aristotle, who gave a value for the size of the earth but did not describe his procedure for obtaining this value [3]. The first scientific approach to finding the circumference of the earth was used by Eratosthenes of Cyrene (c. 275-194 B.C.), whose other cultural contributions include a prime number sieve and the invention of the leap day. According to legend, Eratosthenes knew (perhaps by looking into a well and seeing no shadow) that on the day of the summer equinox, the sun was directly overhead in Syene (present-day Aswan, Egypt). By measuring the angle of elevation of the sun on the same day in Alexandria (roughly on the same longitude as Syene) and calculating the over-land distance between the two cities, Eratosthenes was able to calculate the circumference of the earth using proportional logic (see Figure 1) [3]. Note the elementary geometrical concepts involved, including the ability to identify corresponding angles on parallel lines, the use of proportional reasoning with similar figures, and the ability to model a real-world situation with a condensed and abstract geometric diagram. These are all goals of conducting the activity with prospective teachers.



$$\frac{\theta}{360} = \frac{x}{circumference}$$

Figure 1: *Eratosthenes' method for calculating Earth's circumference*

Steps of the Activity

In order to carry out the estimation, students are divided into groups, with 3-4 students per group. Each group is given a yardstick and string to measure the length of a shadow. Because these students are unfamiliar with trigonometry, they cannot compute the angle of elevation directly from the measurements. Instead, they use dynamic geometry software to construct a triangle similar to the one formed by the vertical stake and its shadow. They can then use this similar triangle to determine the angle formed. Through an activity sheet, students are then directed to design a model of the earth and sun's rays using dynamic geometry software, determine which angles are congruent, and set up a proportion to solve for the circumference. In order to solve the proportion, students must then use the internet to research the north-south distance between Towson, MD (their location) and the equator. Once the distance is determined, students can solve the proportion. The different groups' estimates are compared and differences explored. This discussion can lead to a rich exploration of the sources of error in a measurement. In this way, students are able to participate in an interactive, engaging lesson which explores an accessible application of geometry to a serious and realistic problem from history.

References

[1] Conference Board of the Mathematical Sciences. *The Mathematical Education of Teachers.* Washington DC: American Mathematical Society and Mathematical Association of America, 2001.

[2] Wilson, Suzanne, Robert Floden, and Joan Ferrini-Mundy. *Teacher preparation research: Current knowledge, gaps, and recommendations.* Seattle: University of Washington, 2001.

[3] Smith, James. *Introduction to Geodesy: The History and Concepts of Modern Geodesy.* Hoboken: Wiley, 1997.