Honors Seminar: A Creative Interdisciplinary Approach for Student Exploration

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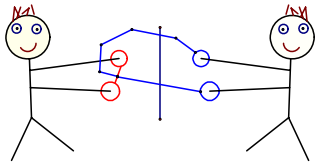
Abstract

This paper describes how atypical assignments in a mathematics seminar encourage students to explore mathematical ideas through multidisciplinary lenses. Students enjoy an extraordinary journey on a remarkable mathematical road with astonishing curves and twists from art, music, writing, and more.

Philosophy of the Seminar

The focus of the seminar is on classroom strategies including seminar discussions and small group work that encourage students to creatively engage in mathematical investigations. The student is expected to take a dynamic role in all parts of the class, including classroom discussions and presentations. The assignments and activities both in and out of class are designed for active student participation and to enhance each student's understanding, appreciation, and practical experience of the bridges connecting mathematics to the world around us. The role of the instructor is merely to provide questions and suggestions to facilitate a student centered classroom. The course focuses on classroom strategies that encourage students not only to enjoy exploring mathematical connections to other disciplines, but also to learn how to creatively engage in investigations linking mathematics to those disciplines. In this article we will explore a selection of activities and projects to achieve this goal. We only begin the journey; it is never ending.

The First Class



Tie a piece of yarn around each of your wrists. Tie another piece of yarn around your twin's wrist and overlap your yarn as shown in the picture. Separate yourselves without taking the string off of your wrists, without breaking the string, and without untying the knots.

Figure 1: *Siamese Twin Challenge*

As students enter the room the first day of class, they are immediately given pieces of yarn and presented with the above challenge. As they contort their bodies, searching for a solution, the atmosphere vibrates with laughter and discussions of how to find the solution. Sometimes a pair of students shouts with glee as they solve the puzzle. Sometimes frustration mounts, and after a reasonable period of time, I lead the students to the solution which is rather simple. Person A takes their yarn and pulls it through the wrist yarn of person B from the inside to the outside. It is really neat to see that, almost magically, the yarn separates! Of course, there is no magic involved, just knot theory. The class begins to hear about an unusual area of mathematics and a particular application.

Toothpick challenges are next. Students are given six toothpicks and must create designs you consisting only of **equilateral** triangles. The **entire** toothpick must be the side of the triangle. Design 1: Use five toothpicks to make two identically sized equilateral triangles. Design 2: Use six toothpicks to make two identically sized equilateral triangles. Design 3: Use six toothpicks to make four identically sized equilateral triangles. The first two designs are readily found. Design 3 is impossible in two dimensions, but easily solved in three by a triangular pyramid. As one student commented "I learned about people's natural assumption of "flatland." As we discuss the problem, we talk about how the problem itself was not particularly hard, but many made it not only harder, but impossible, to solve because they added an assumption that was not there. This helps students to recognize that we need to keep ourselves from making such assumptions.

In a reflection, one student commented that he had always thought "that all math was a series of steps to obtain a solution. That is, until I reached this class. The first class we were already forced to think outside the box." This sets the tone for the semester. Students anticipate an intriguing adventure to examine mathematics in an unusual fashion.

The Mathematical Autobiography

Good teaching orchestrates the learner's experience so that all aspects of brain operation are addressed such as analytical thinking, imagination, and emotions. The first assignment in the seminar is to write a mathematical autobiography. The goal of the project is to share glimpses of one's life through the lens of mathematics. There are many options for this project ranging from a writing an essay to composing a song. Students are limited only by their imagination. The work must be **creative**, **lively**, and **engaging**. Interestingly, students are generally uncomfortable with the freedom to make interesting choices. Yet the results are utterly amazing. The words of one student illustrate how many view the world of mathematics, "My first reaction was, "is this even possible?" On top of having to write an autobiography we were told to be creative. This is way outside my realm. I always thought there was no room for creativity in math." As you consider the following examples, each unique in its own way, observe how the students created links from their mathematical lives to literature, geometric shapes, and music.

Example #1 is a wonderful illustration of using literature to describe the nature of mathematics.

> Imagine a vast nation that stretches from sea to sea, with terrain as varied as the personalities of its citizens. Most people spend at least thirteen years of their life here, sprinting for the borders as soon as their paperwork clears. Others, however, are willing to give everything they have to stay, fearing what lies abroad. This country is made up of an almost uncountable number of states; some with mountains that stretch past the capacity of most climbers, others with barren uninteresting desert. This country, of course, is the United States of Academia.
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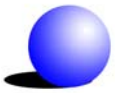
> Although I currently live in the state of Mathematics, I have spent time all over the country. As a young man, I moved around a lot. Many of my favorite places included my current home state, English, Physics, History, Chemistry, and even Music Theory. At one time or another, I thought I could find a home in each one of these states, going from one to another on a daily basis (gas was cheaper back then). After spending thirteen years as a vagabond in the USA, it was about time for me to find a permanent residence.

Example #2 shows how one student uses geometric shapes as symbols of her personality.

I am a square. With its four right angles and equal sides it is truly symbolic of my perfectionist nature.



I am a sphere. It takes great artistic talent to be able to shade a drawn circle properly so that it appears to the viewer as a sphere. Therefore, it is symbolic of my creative talent.

I am a rectangle, standing proud and confident. Once becoming comfortable with those around me, I tend to voice my opinion without concerns of what those around me will think of me because of it. . . . However, I am not a towering rectangle looking to be in the spotlight, but one that prefers to get silently recognized for my accomplishments.

Example #3 illustrates how one student uses music to show "a glimpse of musical, mathematical, authorial "me." She cleverly creates a miniature story book accompanied by an audio CD containing "musical interludes from . . . my favorite artists." For example, one segment contains an illustration of the Beatles Yellow Submarine and reads "The fun simplicity of 'All Together Now' with its 1, 2, 3, 4" lyrics reminds me (in hindsight) of the relaxed, happy math of elementary school. Math was no longer my address or my telephone number – it was counting the little girls in my Madeline book, timing my friends in races around . . ."

Brain Teaser Leads to a History/Mathematics Connection

Brain teasers delight students of all ages. The following helps to introduce students to mathematical history.

The Initial Question: Consider the first five entries in a sequence of letters. Can you determine the pattern? What are the next five letters? **O T T F F**
The Solution: Consider the numbers we count with, 1, 2, 3, 4, 5, etc. Take the first letter from each number to form the sequence. The next five letters come from 6, 7, 8, 9, and 10, S, S, E, N, T. Students enjoy this challenge and frequently work together in their search for the answer.

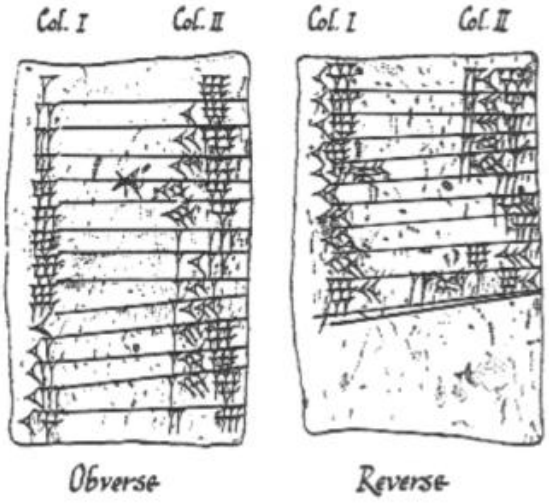


Figure 2: *An Ancient Clay Tablet*

After solving the brain teaser, students are informed that they will be playing the role of mathematical archeologist for the day by exploring the Babylonian cuneiform symbols found on a clay tablet from ancient Babylonia. The tablet was likely the equivalent of a worksheet from an elementary level classroom. The obverse is the front side of the tablet and the reverse is the back side of the tablet.

The clay was often well preserved so by putting pieces together, mathematical historians have been able to uncover and understand the world of ancient Babylonian mathematics.

Each group of students is given a copy of the tablet shown in figure 2. The first challenge is to examine only the first column and determine how the symbols represent numbers. As each group hunts for the solution, intense discussions on individual theories echo through the class. Since they have just solved the brain teaser, counting is in their minds. Quickly, almost all students conjecture that the vertical triangle represents the numeral 1 and the horizontal triangle represents the numeral 10. As the students decode the first column, a stumbling block frequently occurs after the numbers one through twenty. The next symbol consists of three horizontal triangles, which represents thirty, not the expected twenty-one. But further examination shows that the writer is now counting by tens until fifty is reached. Another curiosity – the last line doesn't fit the pattern. This is called the catch line and really belongs to the next topic to be covered. Thus, it does not need to be considered for our work.

Next, students are asked to find the relationship between the first column entry and the second column entry. After many conjectures and directed questions, students conjecture that the table appears to a multiplication table for 9. However, their original conjecture for the meaning of the symbols breaks down in line 7 because the symbols for 63 are one vertical triangle followed by a group of three vertical triangles. Why is this 63 and not 4? We then discuss that the vertical triangle represents a 1 or a 60 or a 60^2 or a 60^3, similar to how in the following numbers the digit 1 can stand for a unit, ten, hundred, or a thousand depending on its location. 1, 15, 132, 1987. The excitement in figuring out how the number system works arouses student curiosity. We use the Babylonian sexagesimal number system today when we refer to time; there are 60 seconds in a minute and 60 minutes in an hour.

This project connects mathematics, history, and archeology; it begins to help students recognize the interdisciplinary nature of mathematics.

More Mathematics Connections

During the seminar, students are expected to engage in scholarly discourse that focuses on expanding awareness of how mathematics relates to our lives. Here are brief descriptions of some of the other areas we explore. A colleague from the art department visits to provide a workshop on the use of the golden ratio in artwork. Students explore paintings using a ruler and a calculator to discover how the golden ratio appears when examining important locations on paintings. The world of art becomes intertwined with mathematics. Another colleague presents a workshop on mathematics and makeup, describing how makeup is applied to provide the illusion of the golden ratio for various facial measurements. Students are amused with the unexpected makeup and mathematics connection. Through a guided discovery project, students investigate the symmetries of a square to learn the basic concepts of group theory. Strangely enough, this abstract mathematical theory explains why particular items cause a cash register to bleep, how the machine knows a UPC code is not being read correctly, why passports have particular ID numbers, and how the ISBN number of a book is constructed. All of these experiences illustrate how mathematics is intricately woven into the fabric of our lives.

Conclusions

Students delight in the variety of mathematics used to explore a smorgasbord of disciplines in an engaging, hands on fashion. By the conclusion of the course, students see mathematics not only as numbers and symbols, but as a means to link disciplines as varied as music, art, literature, and more.