Polyhedra Through the Beauty of Wood

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Abstract

This paper has been prepared to demonstrate how I have used the geometry of polyhedra and my chosen medium of lathe turned wood to present the models shown in an aesthetically pleasing and artistic way. Each of the five platonic solids (tetrahedron, hexahedron, octahedron, docedahedron and icosahedron) has been interpreted through the eyes of a woodturner allowing the viewers alternate perspectives of these classic polyhedra. The description of each model includes details such as the type of wood used, dimensions and the process of fabrication.

Introduction

My interest in geometry stems no doubt from a lifetime spent in the cabinet making industry. Initially I worked as a hands-on craftsman and then later in a supervisory position which consisted of interpreting designer/architectural concepts and turning them into practical and beautiful pieces. After my retirement in 1996, I turned my interest in geometry into a hobby, using wood as a medium and a lathe as one of my many tools. I started a second journey of discovery as a wood turner/artist.

My investigation and interpretation of the platonic solids has been influenced by Johannes Kepler, Luca Pacioli, Leonardo De Vinci, M. C. Escher, and later by Buckminster Fuller and Donald Coxeter. After exhibiting some of my work at the Fields Institute I was invited to share space in Donald Coxeter's showcase in the math department at University of Toronto where I currently have four pieces on display.

The five platonic solids are the tetrahedron, hexahedron (cube), octahedron, dodecahedron and icosahedron. The properties of the platonic solids are that each of the faces is regular, the faces are identical, and at each vertex the same number of faces meet. Each pair of adjoining faces meet at the same angle and the vertices lay on the surface of a sphere.

Tetrahedra

Figure 1 is a simple tetrahedron model, which has been made of pear wood. Each of the faces is an equilateral triangle of about 2 inches long. The faces were cut using a template on a table saw with the mitre angle being 35.264 degrees. The angle was originally obtained from Table 8 [1] and then adjusted if required based on dry fitting the pieces. The piece was then glued and the holes were cut only to show that it is a hollow form.

Figure 1: *A simple tetrahedron*

The two tetrahedra shown in Figure 2 have turned spindles representing the edges. This type of model can be referred to as the web of the polyhedral form. The spindles, which are 5/16" in diameter, have tenons turned on each end fitting into holes drilled into the ½ inch spheres representing the vertices.

The angle and the spacing of the three drilled holes is critical. The angles were approximated and then confirmed using cardboard models and a master block was produced. The holes were then drilled using the angled block and an indexing system to ensure equal spacing of the holes. This process was used to create other polyhedral spindle figures by producing other master blocks of appropriate angles and indexing.

Figure 2: *Two web tetrahedral*

The tetrahedron in Figure 3 was initially turned as a 6 inch walnut sphere. The four vertices were located using a compass. The compass is set using the diameter of the sphere divided by 1.25, which in this case is 4.8 inches. The first vertex was used as the starting point and a circle was scribed on the sphere. The second vertex is any point on the scribed circle. By rotating the compass left and right, the third and fourth vertices are established. If the setting on the compass is correct, this line will have been divided into three equal parts and you have located the four vertices of a tetrahedron.

The sphere was then hollowed out to a wall thickness of about ¾ inch, leaving enough wood to carve the double twists, which connect the vertices. The twists were cut freehand using a dremel burr.

Figure 3: *Artistic tetrahedral*

Hexahedra

The 6 inch cube in Figure 4 is a hexahedron designed to be a secret box. Made of applewood, its six faces are identical and one of them is also a screw-in panel. Each face is inscribed with two logarithmic spirals textured with a rotary burr prior to being highlighted with black acrylic paint. The logarithmic spirals were developed by repetitive division of a rectangle conforming to the proportion of the golden mean [2]. The spirals were transferred to the figure by rotating the template 180 degrees around the centre of each face.

Figure 4: *Hexahedron box*

Octahedra

The grouping shown in Figure 5 was one of my first attempts at working with platonic solids. Although they appear to be one octahedron and three spheres, all four started as octahedra each having eight equilateral triangular faces. The name "The Family" comes from the idea that all the boxes come from the same original form.

Three of the octahedra were lathe turned to their spherical shapes What sets them apart is the species of wood and the manner in which the lids are oriented. For the lower box I used becote and divided it at the equator; four sections for the lid and four sections for the bottom. For the middle box I used laurel and used two sections for the lid and six sections for the bottom. For the upper box I used wenge. Again, there were four pieces for the lid and four pieces for the bottom, the difference being the orientation of the triangles, which produces the zig-zag edge when the lid is removed

Figure 5: *The Family*

This octahedron in Figure 6 was inspired by a traditional quilting design known as tumbling blocks. The technique used is referred to as stickware or Tunbridgeware as it has been made up from a bundle of sticks. In this case, the sticks are rhomboids cut at 30 degrees with each of the faces being of equal length. A group of three made of three different colours of wood can be bundled together, the end view creating the optical illusion of a cube. The next step is to rebundle them together to make a triangular stick, which can then be cut into identically patterned slices. Eight of the resulting equilateral triangles are mitred and glued together to produce an octahedron. The woods in this case are holly, white oak and cocabola.

Figure 6: *Stickware octahedron*

Dodecahedra

The dodecahedron in Figure 7 came about when I made stickware for some spinning tops. Having enough left over for 12 faces, I decided to make this dodecahedron. I took the ten sided stick and clad it with five pieces of oak to construct the pentagonal stick. From this stick I cut 12 panels, each 3/16 inches thick, to make the dodecahedron. The woods used are holly, cocabola, pear, white oak, and maple, and it is about 7 inches in diameter.

Figure 7: *Stickware dodecahedron*

Figure 8 is titled Piece-Peace Suspended. This title is intended to be a play on words. As you can see, the dodecahedron box is suspended. Each face of the dodecahedron is a pentagon, which is a symbol of peace. The box was intended to symbolically contain George’s weapons of mass destruction but when you look inside, the box is empty. The web, made of ebony, represents the web of evil. The handle, which is a dove, but not the white dove of peace, is made from the same black ebony.

Figure 8: *Piece-Peace Suspended*

Figure 9 is a stellated dodecahedron. The term stellated was coined by Johannes Kepler [3]. Stellation is a process that allows us to derive a new polyhedron from an existing one by extending the faces until they re-intersect.

Figure 9: *Stellated dodecahedron*

The flat planes, which emerge from these figures, enable this piece to be produced using the lathe or the bandsaw; I opted for the latter. The first step is to create a sphere and then locate the 12 vertices. The compass setting is calculated using the diameter of the sphere times 0.526 [4] and an initial circle is scribed on the sphere. Choosing any point on the initial circle, a second circle is scribed creating two intersection points. Circles are drawn from each point where two circles intersect. Each point of intersection becomes a vertex. At the end of this process, the twelve vertices will have been located.

Holes were drilled at each of the vertices and a temporary rod was inserted to act as a pivot while cutting the faces on the bandsaw. A jig was used to control the depth of cut on all 12 axes and as the wood fell away a dodecahedron emerged. The holes were plugged using contrasting coloured wood for emphasis.

Figure 10: *Web icosahedron*

Icosahedra

The simple web of the icosahedron (Figure 10) emphasizes the symmetry and balance of all of the platonic solids. The web is created using spindles and spheres for the vertices as previously described with the only difference being the angles at which they are drilled and the number of holes drilled. The spindles are made from rose wood and the spheres at the vertices are boxwood. The piece is about 12 inches in diameter.

Figure 11: *Spherical icosahedron*

The sphere in Figure 11 was initially a faceted icosahedron made from curly maple. The joints have been emphasized with a laminate of black, white, and black veneer. The vertices were drilled out and replaced with black inserts. Centered on each insert are hand turned flowers of several designs and a variety of woods.

Figure 12 is a stellated icosahedron, which began as a solid sphere made from madrone. The faces were then cut on the bandsaw using a similar technique described for Figure 9. In this case there are 12 pivot points of rotation located in the valleys, which can be seen as the black caps of ebony. The base is made of walnut and was made on the lathe, using a swing pivot router, then painted black. The finished piece is 9 inches high.

Figure 12: *Stellated icosahedron*

Figure 13 is a combination of a stellated and a concave icosahedron. All of the panels used to make this piece are 1/8 inch thick. There are 120 flat panels required to construct the piece. It is about 8.5 inches from point to point. The panels are mitre cut using a template and glued together.

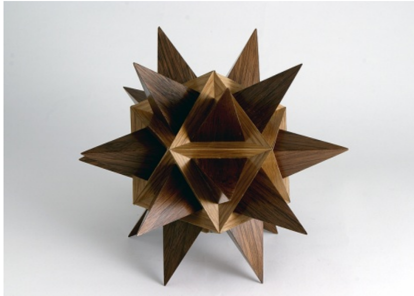

Figure 13: *Stellated and concave icosahedron*

Hybrids

In the following pieces I have used the solid faces of the platonic solids surrounded by web models. I call these designs hybrids.

In Figure 14 we have compounds of two tetrahedra constructed from eight minor tetrahedra; four of which are walnut, and the other four are maple. One of the vertices of each of the eight minor tetrahedrons becomes the corners of a hexahedron. I have joined these together using applewood spindles to reinforce the relationship.

Figure 14: *Tetrahedron-hexahedron hybrid*

This stellated dodecahedron in Figure 15 has been made up of 3/16 inch babinga panels; five panels for each pyramid with a total of 60 panels. The 12 vertices give us the framework for the web, which as you can see is an icosahedron. The 30 spindles and spheres are rosewood.

Figure 15: *Dodecahedron-icosahedron hybrid*

The piece shown in Figure 16 was completely turned on the lathe. It was inspired by Wenzel Jamnitzer who was a German etcher/goldsmith and drew this piece in 1568 [5]. I can only assume it was never made and that it was only a graphic design as the main body only hovered above the base. The wood for the core icosahedron is honey locust and the web and base were made in cocabola.

Figure 16: *Totally Turned*

Web Models

The following four pieces have been modelled using a web design. They consist of more artistic interpretations of the platonic solids and were done using webs so that the entire shape, as in line drawings, are visible from a single perspective.

This is a web model of a stellated icosahedron (Figure 17). It is interesting to see that when the 20 faces are stellated to create 20 new vertices we begin to see a relationship between the icosahedron and the dodecahedron. There are 12 planes where pentagons are formed. The vertices of the stellated icosahedron are created by extending the edges of these pentagons. The ratio of the length of the spindles to the length of the base conforms to the golden mean.

Figure 17: *Stellated icosahedron*

After completing the work shown in Figure 17, I saw further possiblilities and so created Metamorphosis (Figure 18). The relationship between the dodecahedron and icosahedron fascinated me. It began with a central dodecahedron. In the centre of each of the 12 faces, spindles were projected to create the 12 vertices of the icosahedron. Finally the icosahedron was stellated to create the 20 vertices of the outer dodecahedron. The web was then connected between these vertices completing the metamorphosis.

Figure 18: *Metamorphosis*

Johannes Kepler lived from 1571 to 1630, had a great interest in the platonic solids, and was the inspiration for this piece. It was his contention that the orbits of the planets are related to the platontic solids. In Figure 19, each of the platonic solids are contained within one another and each rotate on their own axis. From the centre working out we have the tetrahedron, hexahedron, octahedron, dodecahedron and then finally the icosahedron. Each rotates, as do the heavenly bodies. This piece has a total of 90 spindles and 40 vertices. The spindles and the circular frame are Brazilian rosewood, the vertices are boxwood and the base is black painted hardwood.

Figure 19: *Kepler Theory*

Figure 20 is an example of a progressive web transformation. The centre is an octahedron, which has been converted by making each of the eight faces into tetrahedrons. Four were made from ebony and four from holly. These not only express two major intersecting tetrahedrons, but also indicate the eight vertices of the hexahedron. The various woods used here are pear, ebony, holly, and satinwood. The circular frame is walnut and the base is black-painted hardwood. This piece has found a home in Donald Coxeter's showcase of models, at the University of Toronto's math department.

Figure 20: *Transformation*

The following two pieces have been inspired by Luca Pacioli's publication of De Divina Proportiona, published in 1509 in which Leonardo De Vinci drew the illustrations of the regular solids. Leonardo's drawings are probably the first illustrations of skeletonic solids.

Figure 21: *De Vinci model 1*

Figure 21 is similar to De Vince's model and my interpretation of the piece is constructed of 84 individual frames mitred together. The side angles had to be cut prior to assembly. After the first row of 12 frames had been glued together the other rows could be glued and assembled until the sphere was closed. All faces of the strips used for the frames were prefinished prior to assembly as all faces can be seen. The sphere was made from babinga wood and has a 10 inch diameter. The base is turned from walnut and has been weighted with lead shot to permit the piece to be shown in an offset position.

Figure 22: *De Vinci torus*

The second of Leonardo's pieces is a torus. This is his mazzocchio which he drew in solid edge form. It consists of 32 sections around the circumference (Figure 22). Each section contains eight frames for a total of 256 frames, each with specific side angles. This model is made from zebra wood and is 14 inches in diameter.

Figure 23 shows a web model of Buckminster Fuller's Vector Equilibrium [6], which is a polyhedron. It is a cuboctahedron with all the vertices connected to the centre.

Figure 23: *Vector equilibrium*

It has many unique properties; it has 12 vertices, which lie on the surface of a sphere. It contains 24 edges as well as 14 faces. Vector equilibrium is comprised of eight tetrahedrons and six half octahedrons. Finally, the distance from any two adjoining vertices is identical to the distance from any vertices to the central sphere. The piece rotates within its circular oak frame, which in turn pivots in the base.

This article has described many of the polyhedra and platonic solids I have worked on in the past few years and is also the result of much reading on the subject. The journey is endless and the variables infinite.

References

[1] J. Cox, *Beyond Basic Turning*, pp. 209. 1993.
[2] R. Lawlor, *Sacred Geometry*, pp. 64-70. 1982.
[3] P. Cromwell, *Polyhedra*, pp. 168-171. 1997.
[4] D. Sprigett, *Woodturning Wizardry*, pp. 20-22. 1993
[5] P. Cromwell, *Polyhedra*, pp. 288. 1997.
[6] R. Buckminster Fuller in collaboration with E.J. Applewhite, *Synergetics*, pp. 118. 1982.