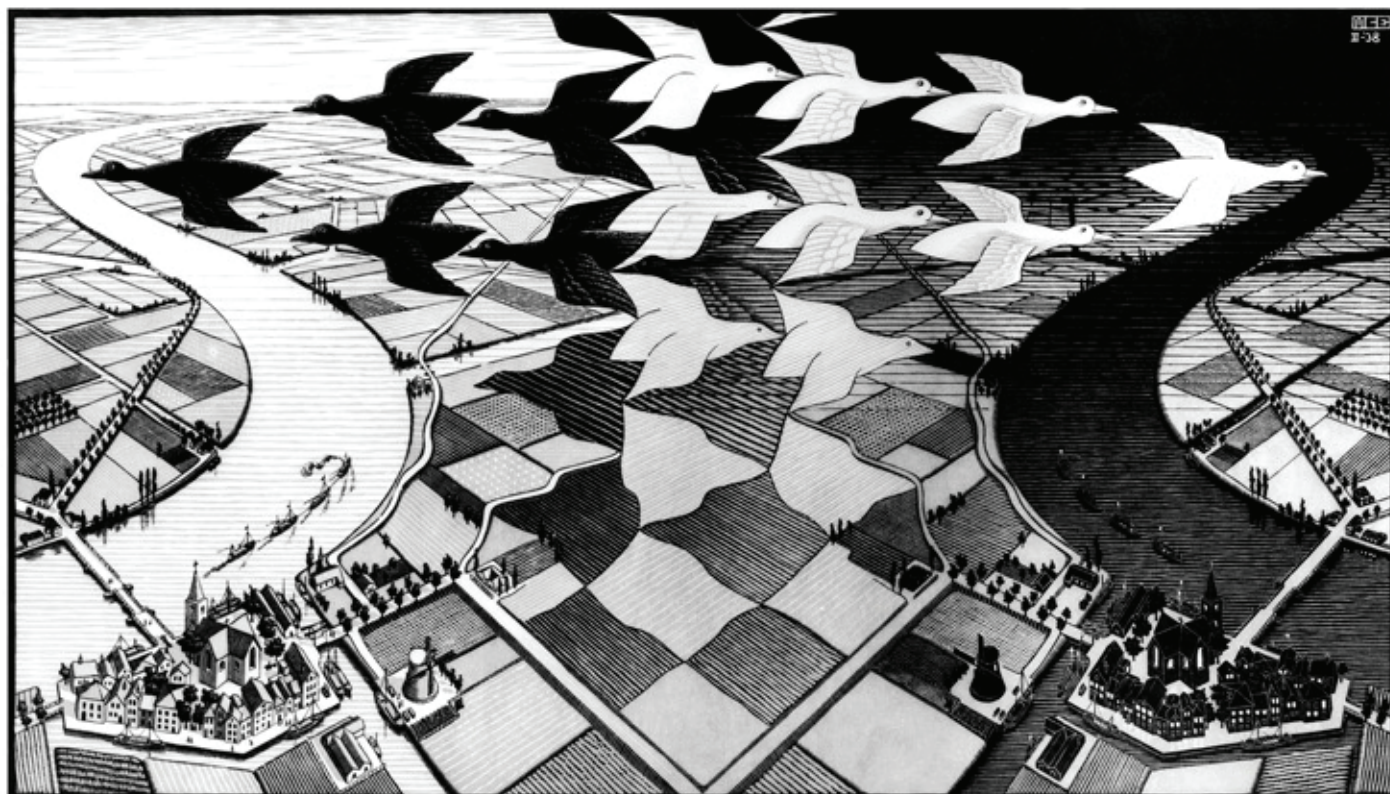


Celebrating the 110th Birth Year of M.C. Escher  
During the Eleventh Annual Bridges Conference

# BRIDGES LEEUWARDEN



Mathematical Connections  
in Art, Music, and Science

**PROCEEDINGS 2008**

Reza Sarhangi and Carlo Séquin, Chief Editors

Municipality of Leeuwarden, The Netherlands

Stenden University  
The Foundation Passages  
Eindhoven University of Technology  
University of Twente

# **Bridges Leeuwarden**

*Mathematics, Music, Art, Architecture, Culture*



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## Preface

For this eleventh year of the Bridges Conference, no city could be more appropriate than Leeuwarden, The Netherlands, the birthplace of M. C. Escher to host this event. The Bridges Conferences are annual multidisciplinary gatherings celebrating the connections between mathematics and art. In the perpetual partnership of mathematics and art, one of the most influential artists was certainly M. C. Escher.

In the introduction of the book, *M. C. Escher, The Graphic Work*, republished in 1994, Escher describes himself: "The ideas that are basic to them often bear witness to my amazement and wonder at the laws of nature which operate in the world around us. He who wonders discovers that this is in itself a wonder. By keenly confronting the enigmas that surround us, and by considering and analyzing the observations that I had made, I ended up in the domain of mathematics. Although I am absolutely without training or knowledge in the exact sciences, I often seem to have more in common with mathematicians than with my fellow artists."

This year we are in Leeuwarden, where Escher was born 110 years ago, so it is appropriate that the four days of talks includes an Escher Day in celebration of his contributions.

The conference also includes a day for an excursion to several churches built in the middle Ages that also are the exhibit place of artwork of several modern European artists, to the National Ceramics Museum Het Princessehof, (located in the birth house of M.C. Escher), and to the Fries museum. In addition, the winners of the *Gateways to Fryslân* art competition will be announced at the conference, with € 30,000 in prizes to be awarded by the Province of Friesland. We are very grateful to the Christelijke Hogeschool and the local organizers, Andrew Borgart, Arno Pronk, Rinus Roelofs, Zsófia Ruttkay, and Cor Wetting, for the many excellent arrangements.

This is the first year that we have used a web-based paper submission system. We received a record number of submitted papers, and their contents and styles were spread over a particularly wide spectrum. This made it difficult to prune down the count to a manageable number of standard-length papers. Many authors had to be told to compress their material into a shorter presentation. The electronic submission and reviewing system caused some difficulties with many people who encountered such a system for the first time. On the positive side, the *EasyChair* system allowed us to distribute the reviewing process among a sizable international program committee. Our goal was to aim for an ever higher quality content of the conference; subjecting the full-length papers to a more rigorous reviewing process was a step in that direction.

Carlo Séquin, who served as the chair of the program committee, provided overall management, and distributed the papers among the associate editors who coordinated with the individual reviewers. After a careful reviewing process, the program committee selected 49 regular papers, and 37 short papers. As a result, this year's proceedings will be another wonderful source of inspiration, as well as a handy reference for many key ideas.

The authors are artists, mathematicians, computer scientists, musicians, architects, teachers, and others, who are coming to the conference from twenty-five different countries. The conference participants are not only diverse in their fields of studies, but they also bring different cultures and backgrounds. The conference promises to be a most interesting event, full of rich, interdisciplinary experiences.



The Exhibition of Mathematical Art is a highlight of the Bridges Conference. This year, the work of more than 60 artists will be represented. The exhibition features metal, wood, and stone sculpture, computer art, beadwork, paper folding, quilts, paintings, etchings, and photography. Mathematical topics explored in the art include fractals, polyhedra, tiling, knots, non-orientable surfaces, and hyperbolic geometry. Some of the submitted artwork is shown on the proceedings back cover. The complete exhibit can be seen at the Bridges website ([www.bridgesmathart.org](http://www.bridgesmathart.org)) and on the accompanying CD-ROM.

The cover of the proceedings book includes “*Day and Night*” by M.C. Escher, which Jeffrey Rutzky beautifully embedded into the overall design.

Hofstadter, the author of *Gödel, Escher, Bach*, in an article in the *M.C. Escher’s Legacy* book (Edited by D. Schattschneider and M. Emmer), writes about the first time that he saw an Escher print, the time when he was twenty years old, in January of 1966, in the office of Otto Frisch, who played a major role in unraveling the secrets of nuclear fission. He was mesmerized by “*Day and Night*”, showing two flocks of birds, one in white and one in black, flying in opposite directions through each other. He asked Frisch “What is this?” Frisch replied, “It is a woodcut by a Dutch artist, and I call it ‘Field Theory’...”

The young man ponders the relationship between this artwork and the domain of physics that Frisch is referring to: “I knew that one of the key principles at the heart of field theory is the so-called CPT theorem, which says that the laws of relativistic quantum mechanics are invariant when three “flips” are all made in concert: space is reflected in a mirror, time is reversed, and all particles are interchanged with their antiparticles. This beautiful and profound principle of physics seemed deeply in resonance with Frisch’s Escher print...”

The Bridges Conferences are run by the non-profit Bridges Organization, with a small board of directors and many enthusiastic volunteers. We are proud and grateful that our organization has attracted many countries and universities from around the world to open their doors to our conference.

*The Bridges Organization Board of Directors*  
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