Fibonacci Tornado
Phyllotaxy spirals consisting of all similar triangles

Akio Hizume
Star Cage Institute of Geometry
Chiba, Japan
E-mail: akio@starcage.org
http://www.starcage.org

Abstract

"Fibonacci Tornado" is a generalization of the classical phyllotaxy spirals, which consist of all similar triangles. The result is a computer graphic design for a public sculpture. These spirals based on phyllotaxy are only possible for a restricted set of numbers -- the Fibonacci numbers!
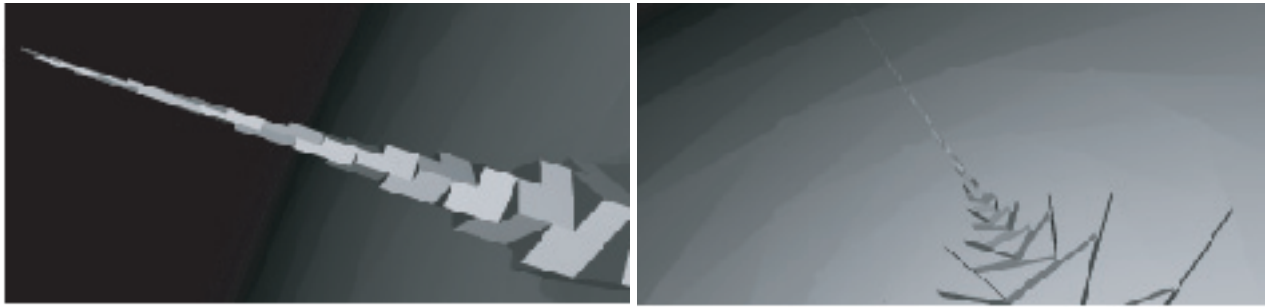
Fibonacci Tornado

Figure 1: *Fibonacchi Tornado mod 2 as a computer graphic design for a public sculpture*

You can choose any Fibonacci Number $F_{(j)}$.
where $F_{(0)}=0$, $F_{(1)}=1$, $F_{(j)}=F_{(j-1)}+F_{(j-2)}$, integer $j \geqq 2$.
On the recurrence diagram as shown Figure 2, it should require the following formula.

$$\delta = F_{(j+1)} * 2\pi / \tau$$
$$\phi = F_{(j+2)} * 2\pi / \tau$$
where $\tau = (1+\sqrt{5})/2$, that is the golden ratio.

The ratio of similarity s which is represented as
$a_{k+1}/a_k = b_{k+1}/b_k = c_{k+1}/c_k$ must accord the following formula.

$$bs^{F_{(j+2)}} + cs^{F_{(j+1)}} = a$$

in case of $j=3$,
$\qquad a=1$
$\qquad b = \sin\delta / \sin(\delta + \phi) = 0.7966510959\cdots$
$\qquad c = \sin\phi / \sin(\delta + \phi) = 0.5387910616\cdots$
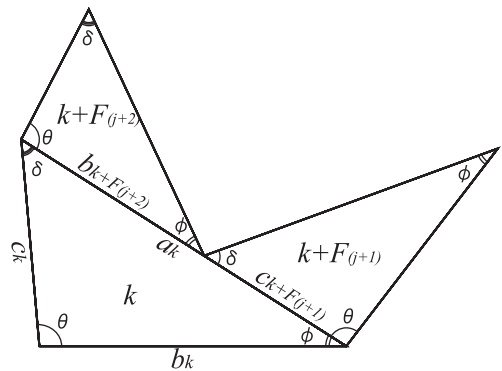

Figure 2: *Recurrence Diagram*

We can decide one logarithmic spiral which contains all vertexes of triangles.
It should be represented as ;

$r=G^{\omega}$

The logarithmic spiral can be clockwise or counterclockwise.
We should get value of the G which makes $r=s$ under each case of $\omega=2\pi(\tau-1)$ or $\omega=2\pi(2-\tau)$.

$G=e^{(log(s)/\omega)}$

in case of $\omega=2\pi(\tau-1)$ and $j=3$, $G=0.9822502411\cdots$

in case of $\omega=2\pi(2-\tau)$ and $j=3$, $G'=0.9714381720\cdots$
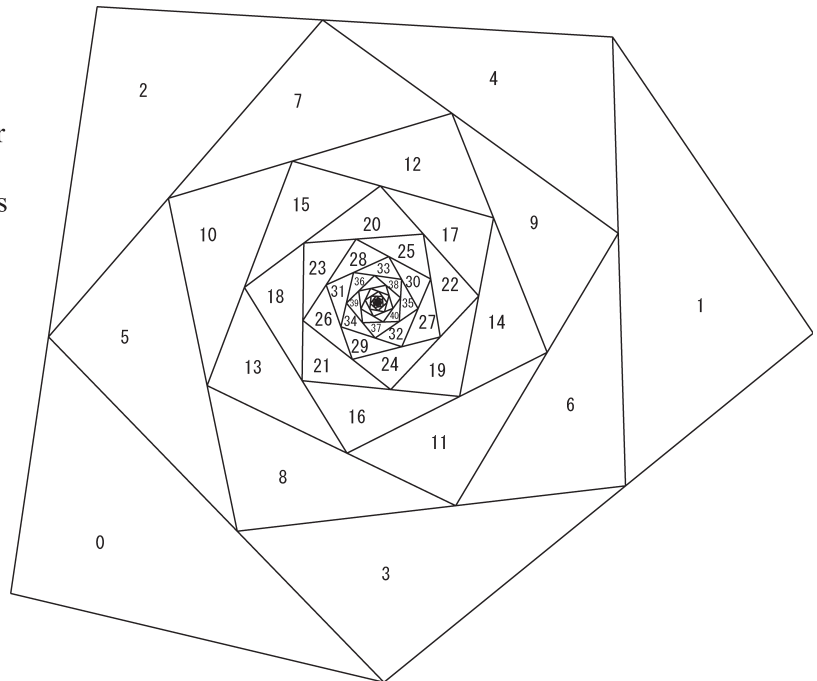
Then we get the figure right.

Figure 3: *mod 2 (j=3) spiral*

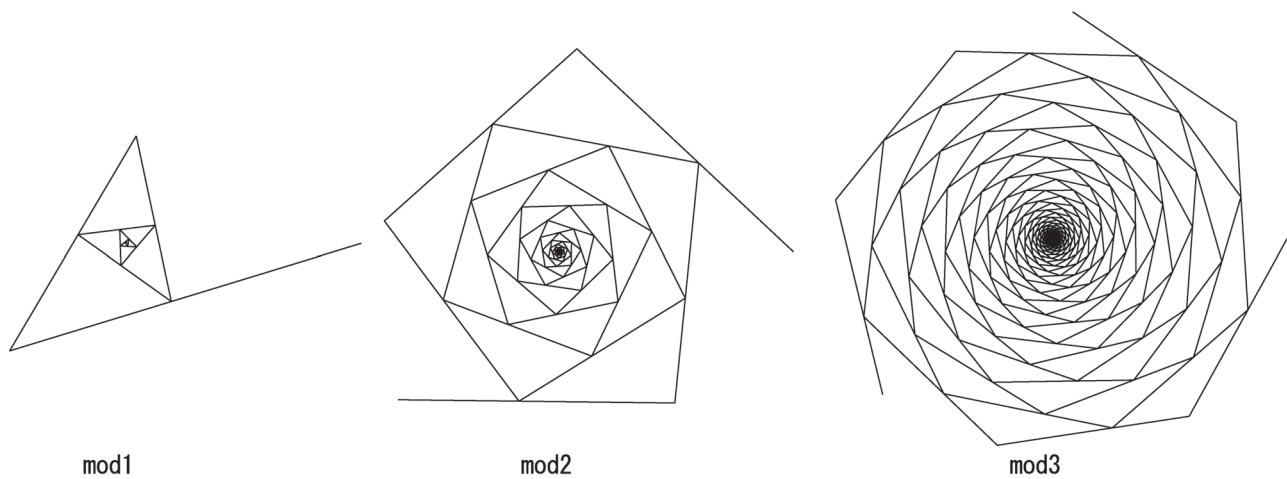Let me show the solution from *mod 1 (j=2)* to *mod 3 (j=4)* on Figure 4.

mod1　　　　　　mod2　　　　　　mod3

Figure 4: *The Fibonacci Tornado from mod1 to mod3*

References

[1] This article was published in Japanese on MANIFOLD #11, pp. 7-8. 2005.
[2] Akio Hizume, inter-native architecture OF music, ISBN978-4-9902966, pp. 117-118. 2006.
[3] See more images in http://www.starcage.org/dragon/tornado.html.