Projecting Mathematical Curves with Laser Light: A Tribute to Ptolemy

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Abstract

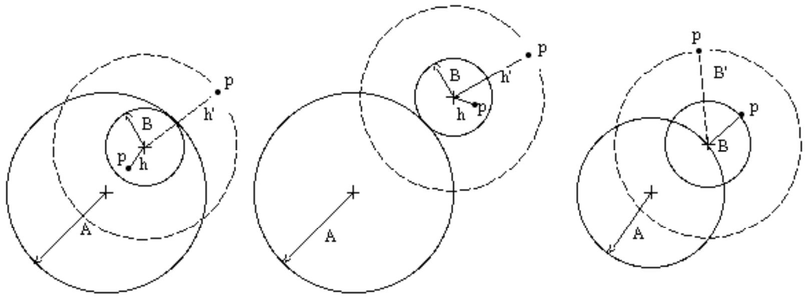
This paper describes the math and technology required to project a variety of mathematical curves with a computerized laser light control system. The focus is upon creating "large-scale" animated laser projections of roulette or spirograph shapes, such as those found in the epitrochoid, hypotrochoid, epicycloid, and hypocycloid families. The projection process described utilizes a geometric approach that was first presented by the Greek or Egyptian mathematician and astronomer Ptolemy, that of "epicycles." Examples of these laser images can be viewed at http://spot.colorado.edu/~lessley/.

Introduction

Many mathematical curves in the epitrochoid, hypotrochoid, epicycloid, and hypocycloid families are beautiful to view. These curves are usually graphed by incorporating such devices as a plotter, printer, or video device. The most common mechanical method for graphing such curves involves a spirograph tool in which a small trace wheel is rotated within a larger stationary wheel. While these techniques produce interesting images, the images are normally rather small and not animated. Creating very large-scale animated images with high-intensity lasers such as those encountered in laser light shows, concerts, or art installations require some unusual graphing and projection strategies.

The Problem: Moving from the Spirograph to the Laser

Forming a projected pattern of any mathematical curve with a moving laser "dot" is done by "scanning" that dot rapidly with X and Y axes galvanometers through an image path at least sixteen times a second. At this rate, our "persistence of vision" makes the image appear solid. Scanning a laser dot rapidly in a circular pattern creates the appearance of a solid circle in light. Furthermore, in moving the laser dot to draw a series of smaller circles that follow the path of a larger circle (a roulette shape), we see that--unlike the spirograph tool with its static base circle--the trace and base circles rotate simultaneously. When scanning this way, both circles usually possess individual frequency and diameter factors. In terms of geometric patterns, this is similar to what the ancient Greek or Egyptian astronomer/mathematician Ptolemy expressed in his attempt to explain the visual motion of the planets by developing the idea of "epicycles." Of critical importance in our project was how Ptolemy's approach could help us create roulette patterns in a slightly different manner than we would normally do with a conventional spirograph tool. His method has the trace circle rotating "on the circumference" of the base circle (not inside or outside of it). Both circles can also maintain differing rotational speeds (frequencies) and directions. In terms of translating the math to electromechanical scanning devices (galvanometers), through summing amplifiers and digital-to-analog converter circuits, this approach is simple and very flexible. Epitrochoid and hypotrochoid formulas can be modified to include special-case curves like the rose and ellipse.



(a) **(b)** **(c)**

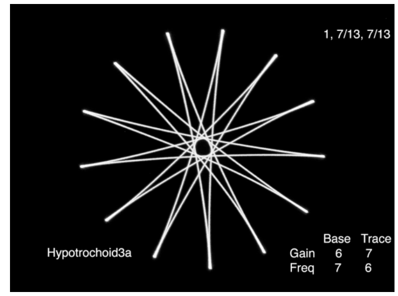


Figure 1: *Traditional spirograph device (a and b) compared to Ptolemy's epicycle approach (c).*

Figure 2: *Laser projection of a hypotrochoid curve.*

Items **(a)** and **(b)** in **Figure 1** follow the normal graphing strategy used by a spirograph device. Drawing **(c)** shows how our tracing strategy follows the epicycle approach in which the center of the trace circle moves on the circumference of the base circle and both circles rotate simultaneously.

This mathematical approach requires that the traditional spirograph or roulette equations be modified to accommodate the "dynamic" nature of having the base and trace circles move simultaneously. For example, the usual parametric equation for graphing a hypotrochoid curve is shown in **Figure 3**; our approach, **Figure 4**, uses base and trace oscillators to form the hypotrochoid images (where $\omega_0 = 2\pi f_{base}$ and the base frequency f_{base} is the number of times per second that the base oscillator completes a cycle). **Figure 2** is an actual laser projection of a typical hypotrochoid curve that was created with this mathematical approach.

$$x = (a-b)\cos(t) + h\cos\left((\frac{a}{b}-1)t\right).$$
$$y = (a-b)\sin(t) - h\sin\left((\frac{a}{b}-1)t\right)$$

Figure 3: *Traditional parametric equation for hypotrochoid curve.*

$$x = (a-b)\cos(\omega_0 t) + h\cos\left(\frac{a-b}{b}\omega_0 t\right)$$
$$y = (a-b)\sin(\omega_0 t) - h\sin\left(\frac{a-b}{b}\omega_0 t\right)$$

Figure 4: *Modified hypotrochoid equation.*

Conclusion and Future Work

In our research, we found that the "epicycle" approach to creating large-scale, high-powered laser projections of epitrochoid, hypotrochoid, epicycloid, and hypocycloid curves works very well. The math is easily replicated in computer code and electrical circuits. The overall flexibility and simplicity of the approach is refreshing. Future work will concentrate on software enhancements and reduced hardware. Laser images and art created this way can be seen on http://spot.colorado.edu/~lessley/.

References

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