The Hidden Art in a Dynamic Geometry Software Program

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Abstract

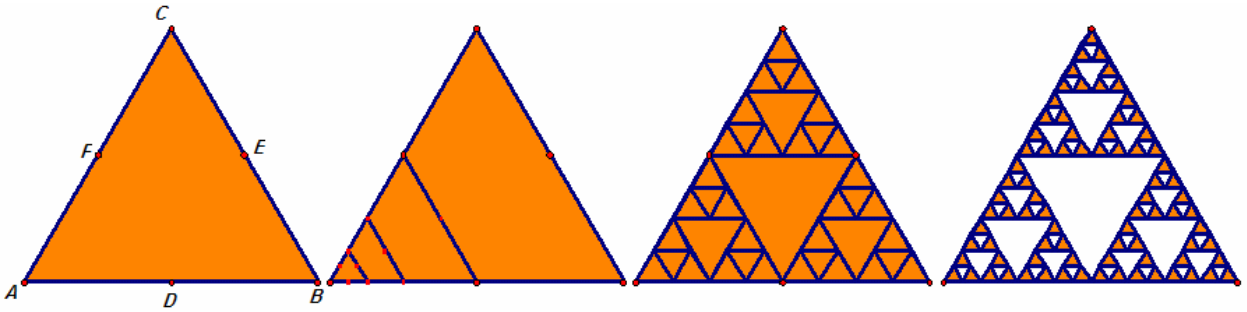
The article demonstrates the mathematics and art that is involved in the construction of some fractal geometry objects using the Geometer's Sketchpad.

Introduction

The Geometer's Sketchpad is a dynamic geometry software program, which has been developed solely for mathematics educational purposes. Nevertheless, we may use some of its functions to explore the realm of visual art. This software program is based on the rules of geometric constructions using compass and straightedge. A recent development in this software utility is the capability of creating some fractal geometry figures using the iteration function. An old fractal geometry object is the Sierpinski Gasket that can be created in Sketchpad. In this article we briefly show how we can make a Sierpinski Gasket in Sketchpad, and then we use the technique to create many new and pleasing images.

Constructing a Sierpinski Gasket Using Sketchpad

We construct the equilateral triangle ΔABC and its interior in a way that the third vertex C is constructed based on the other two given points of A and B. We then construct the midpoints of its sides, as is presented in Figure 1.a. Now we want to iterate on A and B. For this, we select A and B in order and click on "Iterate" on the ***Transform*** menu. Now in the dialog box we select A as its own image and select D as the image of B. With this action, we create a self-similar triangle with the scale factor of one-half to be placed on the lower left part of our original triangle (Figure 1.b). To complete the gasket we should add two new mappings by choosing A and B again and clicking on "Structure" and choosing **Add New Map.** Then we make similar triangles on the top and on the right side (Figure c).



(a) **(b)** **(c)** **(d)**

Figure 1

We now set our iterations to only show non-point images by going to "Structure" and clicking on **Non-Point Images.** On "Display", we click on **Final Iteration Only**. We only want to see the most recent iterated images and be able to increase the stages of iterations. For this, click on the center of the interior of the original triangle and "Hide Triangle". This will result in one of the stages of the Sierpinski Gasket. By selecting the entire figure and using the **Plus (+)** and **Minus (–)** keys, we can experiment with different stages of iterations. Now we can click on the **Point** tool, go to the ***Edit*** menu, and "Select All Points". Then go to the ***Display*** menu and "Hide Points". Our Sierpinski Gasket is ready! (Figure 1.d)

But how can we use this idea to create something new? One way is to change the gasket from its triangular shape to another shape, such as a square. However, if we construct the midpoints of the sides and connect them we will divide the square into four smaller squares. But then we cannot apply the idea of the Sierpinski Gasket, which is based on the creation of middle holes. One solution for the square shape gasket is to divide the polygon into twelve smaller versions. This will create a middle square. If we number the squares from left to right and from top to bottom, then the middle square will be 5. The following three images are based on the application of the same idea but on different sets of squares. Figure 2.a presents the result of applying the idea to squares numbered 1, 3, 5, 7, and 9. For Figure 2.b we used squares numbered 2, 4, 6, and 8. Figure 2.c illustrates the application of the idea to all but square number 5.

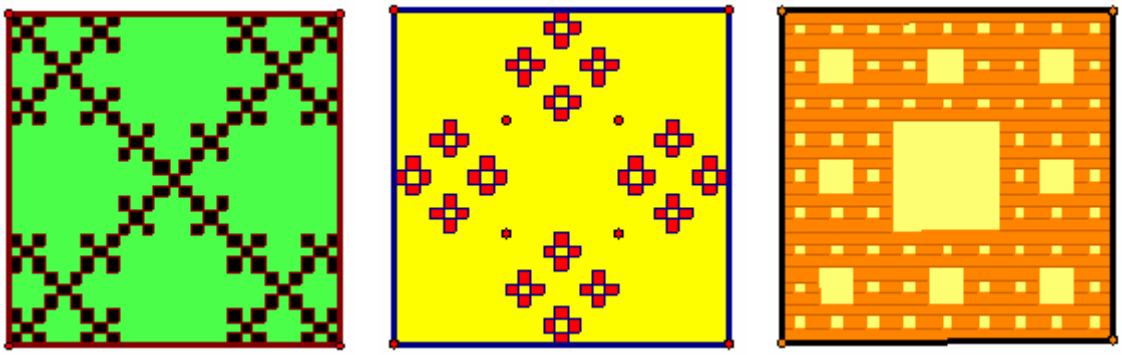


Figure 2: (a) *A Little Rug*, (b) *Love of Flowers*, (c) *Bamboo Mat*

The last two figures present the extension of the idea to a different dissection of a square, as well as to the pseudo-star polygon of the Octagram.

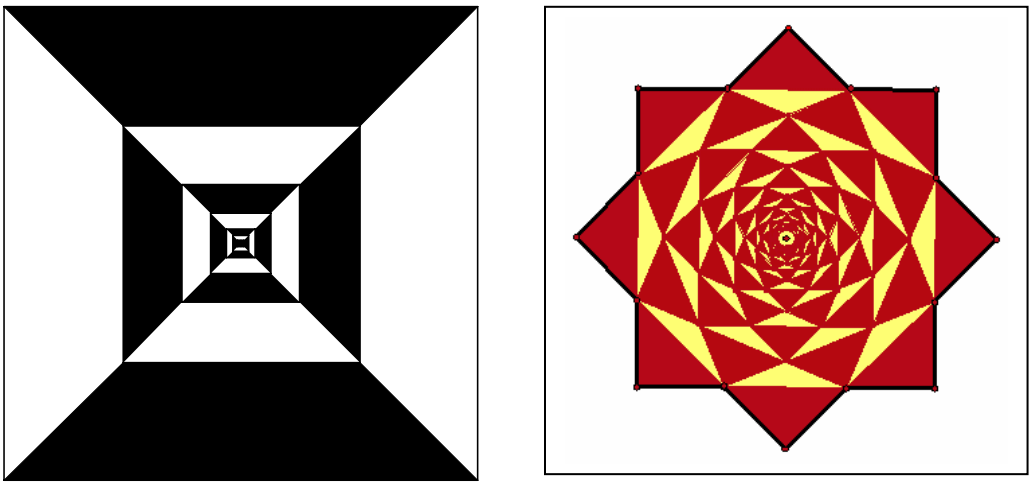


Figure 3: (a) *The Contemporary Yin-Yang*, (b) *The Octagram Gasket*