Imaging, Mathematics, and Art

Arjan Kuijper
RICAM *
Linz, Austria

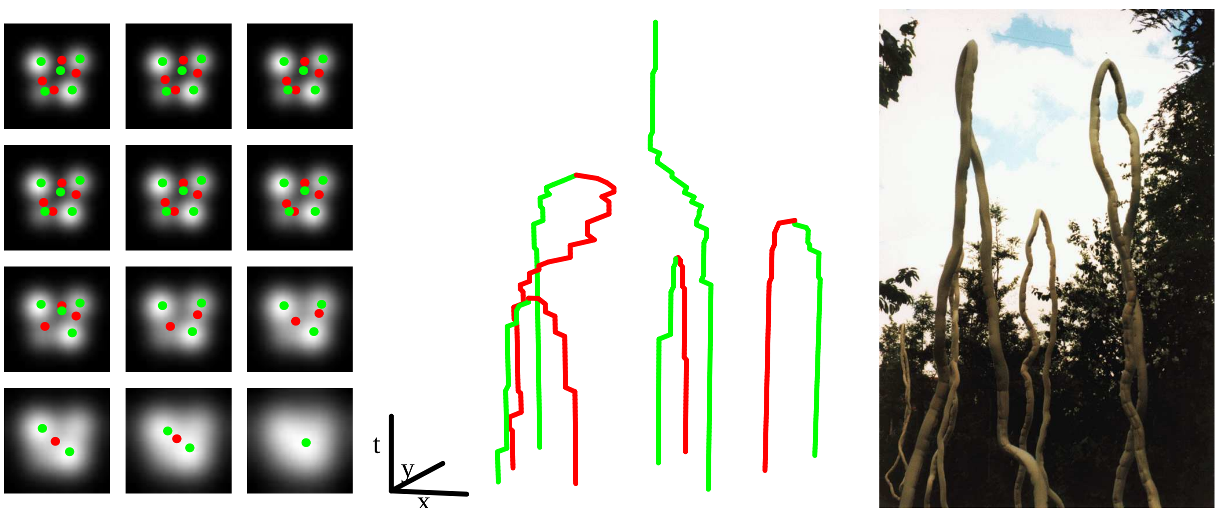
Helma Kuijper
Independent artist †
Alkmaar, the Netherlands

Abstract

In computer vision sophisticated mathematical methods are used. They range from partial differential equations via singularities to geometry. The work of the first author focusses on the analysis and use of these mathematical methods. The second author created several art works inspired by the same concepts and their visualisations. Two of such examples are shown and discussed.

1 Scale Space: Movement of Structure in an Extra Dimension

A key observation in image analysis is that objects have a scale and different objects most likely have different scales. Therefore, when investigating an image representing (unknown) objects automatically, one has to incorporate the notion of scale and use different scales simultaneously. This is actually what the human visual system does as well: We are able to see both large structures and fine details [2].



(a) (b) (c)

Figure 1: *a) When an image evolves and gets more and more blurred, structure disappears; The critical points disappear pairwise. b) The location of critical points in time t can be visualised by critical curves. c) Curved stapled foam elements hanging in the free nature.*

The scale space approach deals with the *continuous* evolution of the image under the heat equation [2], as shown in Figure 1a. Here the dots represent critical points that move as the time t evolves and disappear

*Johann Radon Institute for Computational and Applied Mathematics (RICAM) / Austrian Academy of Sciences, Linz, Austria. http://www.ricam.oeaw.ac.at/people/page/kuijper/ The WWTF grant "Mathematical Methods for Image Analysis and Processing in the Visual Arts" is gratefully acknowledged.

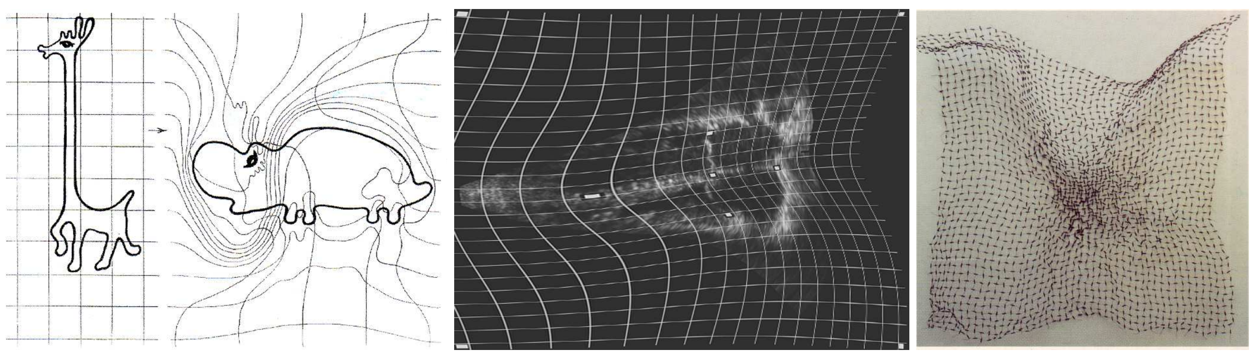
†See also the art work at the Bridges 2008 Art Exhibition, http://www.ricam.oeaw.ac.at/people/page/kuijper/helma and http://www.helmakuijper.nl/

pairwise in singularity points. Using *t* as *extra* dimension, these points are connected and form critical curves as shown in Figure 1b. The curves form a key element in an automated topological segmentation of the image [3]. When the underlying image is gradually changed, these curves move in the scale space.

This principle of growth of structure out of a basis surface into a new dimension, the pairwise connection of structure elements, and the impossibility to escape, inspired the second author to create the artwork in Figure 1c. The art work is made of foam elements, stapled to form cylindrical objects. They are exhibited in the free nature, hanging on thin wires. Each curve has a span of 80cm and a height of 250cm. Due to external forces (either the wind or a motor) the curves can move.

2 Grid transformations: Capturing Movement by Contractions

Image warping is a technique that searches for an optimal transformation from one image to another. An image can be thought of of living on a Cartesian grid, whereas the warping changes (or destroys when singularities appear) the grid layout. A typical way to represent such a warping is shown in Figure 2a: An object on a Cartesian grid is changed into a different object by changing the grid (taken from [1]). In most applications, the changes are not that rigorous. In medical imaging, changes occur mostly due to patient movement or the heartbeat. Here we often see small contractions of the grid, as in Figure 2b (from [4]).



(a) (b) (c)

Figure 2: *a) By warping the curvilinear coordinate system, one can change one image into another. b) This is used, for example, in medical imaging. c) The grid deformation in an artistic visualisation.*

This principle of visualising movement by contractions of the regular structure is visualised in an artwork by the second author, Figure 2c. The elements of this work are resistances. In equilibrium, they form a regular square pattern, Due to forces (different lengths of the resistances) the squares are deformed. In this way, the warping is captured both physically and visually. Singularities occur when resistances start to overlap each other. The resistances are typically up to 5cm long, the complete artwork varies from 9m x 1.20m to 60cm x 60cm. A similar example is visible in the Art Exhibition.

References

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