# Mathematics in Rangolee Art from India

Madhuri Bapat
Physics Department
Eastern Arizona College
615 N. Stadium Ave.
Thatcher, AZ 85552, USA
E-mail: madhuri.bapat@eac.edu

## Abstract

Rangolee is an ancient art of floor decoration from India, built on rectangular or hexagonal arrays of dots which serve as base for designs and patterns based on abstractions from the natural/cultural world. In this paper we categorize rangolee designs according to the method by which they are created and the motives that appear in them. We link these traditional and contemporary designs with patterns and models that have arisen independently in the world of mathematics. This includes a variety of symmetry groups, spirals, mirror curves, fractal self-similarity, and processes of iteration. Could daily practice of these designs transfer as kinesthetic intelligence in children? We discuss this and make a suggestion of how rangolee art might be used as an enjoyable educational practice for children.

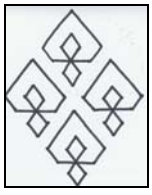
## Rangolee art from India

**Rangolee:** 'Rangolee' in the Indian language called Marathi means lines of colors: 'rang' means color and 'oalee' means lines. Rangolee is an ancient art of floor decoration, used for occasions like welcoming guests, bringing in the new seasons, weddings, birthdays, etc. The word rangolee is also used to refer to the powder that is used to draw the designs. This powder can be made by crushing flint stone or rice. In southern India rice powder is most often used.

A pinch of powder is held between the thumb and forefinger. By the rolling thumb on the finger slowly the powder is released to form a thin stream. Moving the hand freely, dots and lines are drawn. This takes significant amount of hand and eye coordination, as does the initial drawing of the equidistant rectangular or hexagonal array of dots which begins the drawing process.

**Categories**: There are three basic drawing methods, and we can use these to broadly categorize the designs.

(a) Dots are drawn as part of a rectangular or isometric lattice and are then connected in a certain order to form geometrical shapes, or patterns, or natural objects like fruit, leaves, flowers, animals or birds. Figures 1a, 1b, 1c, and 1d are of this form.



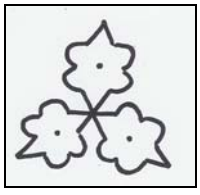
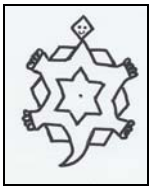
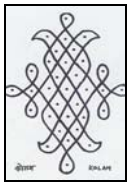
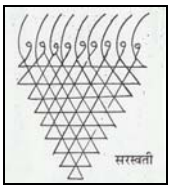




**Figure 1a**: C*oconut* **Figure 1b**: *Koyaree* **Figure 1c**: *Bilve patra* **Figure 1d**: *Turtle* **Figure 1e:***Brahma Knot*

(b) Dots are drawn as in (a) and curves are drawn around them to form geometrical patterns or natural objects. These are known as *kolam.* The Brahma knot in Figure 1e is kolam.

(c) Freehand drawings of geometrical shapes are drawn (circular forms are more common, but square and triangular forms are also found) of natural objects or scenes from history, epics or *puranas* (traditional stories). A symmetrical canopy may be drawn around a dinner plate or wooden board at which a guest, or newly weds, or a new mother are invited to sit.

One may also divide rangolee into traditional and contemporary designs.



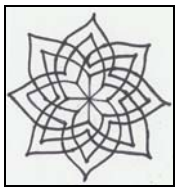




**Figure 2a**: *Saraswatee* **Figure 2b**: *Nabhi kamal* **Figure 2c**: *Dnayn kamal* **Figure 2d**: *Contemporary*

**The traditional designs:** The traditional designs in Fig. 1a-1e and 2a-2c have either special cultural significance or religious meanings. In short koyaree represents mango seed, saraswatee represents goddess of knowledge, nabhi kamal represents lotus of cosmic navel, bilve patra represents leaf of Rutaceae (favorite of Lord Shiva) and dnyan kamal represents lotus of knowledge. In addition to these freehand drawings of the sun, the moon, swastika, hoof prints of cow, and conch are also found.

**Contemporary designs:** Contemporary designs are drawn either by dots or free hand. Dots are connected to form either natural objects like flowers, birds, animals, or geometrical shapes, as in Fig. 2d.In rangolee, one more often finds flowers like the rose, hibiscus, and lotus, or birds like the swan, parakeet, and peacock, or animals like elephants, fish, or deer.

**Mathematics perceived in rangolee designs**

Since they are by their nature designs based on lattice symmetry and the abstraction of form from the natural world, it is not surprising that in rangolee designs we can find manifestations of various forms and models from the mathematical world. In this section we illustrate some of these forms that we have been able to find: symmetry, fractals, mirror curves, cyclic orders, and iteration.

**Patterns:** Drawing dots in the form of a hexagonal array, reducing the number of them in each succeeding row as one moves vertically up or down, is common in traditional rangolee designs. It is interesting to see how many possible ways one can draw a pattern from a given set of dots. A base of four horizontal equidistant dots reducing to one on both top and bottom can be connected to form closed patterns. In one case this creates the basic pattern of coconut, while in the other a small rhombus is embedded in a larger one. The coconut pattern stands out, as it is both interesting and beautiful to look at. The reason could be that all dots are somehow connected to each other.

**Symmetry:** If it is pleasing to eyes, there is often some kind of symmetry behind it. All rangolee designs show some type of symmetry. Looking over the figures one sees all sorts of types of rotational and reflective symmetries, including full dihedral symmetry and pure cyclic symmetry of Fig. 1b and 2b. Attention is paid to keeping same symmetry in their iterations as in basic patterns.

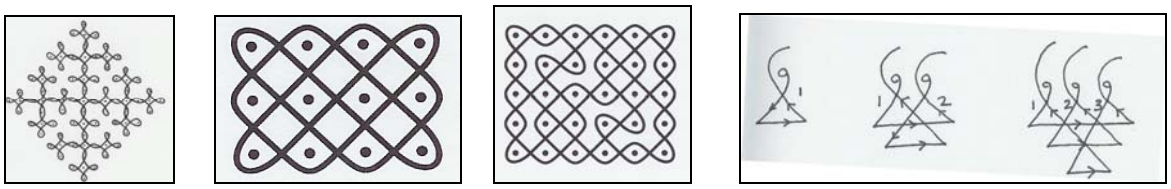


**Figure 3:(a)** *Grape vine* **(b)** *Simple figure* **(c)** *Regular figure* **(d)** *Saraswatee: first three iterations*

**Fractals:** Fractals became popular about 30 years ago, although they have always existed in nature and such famous fractals as the Sierpinski gasket date back almost a century. A common source of fractals is decreasing self-similarity. We can perceive 2-step fractal self-similarity in Figure 3a, where the basic cross is repeated on three scales.

**Mirror curves:** Gerdes [4] has described some designs from Africa as m-canonical mirror curves. These designs are very similar to kolam rangolee. The lines in these are examples of Eulerian paths that the design is completed without lifting the pencil from the surface and no part of the line is retraced. They are called mono-linear. Others are bi- or tri-linear. There are three kinds of mirror curves: simple, regular and m-canonical. See Fig. 3b, and Fig 3c.

**Cyclic order:**

A. Dnyan kamal: There are two ways of connecting dots, giving rise to opposite forms of cyclic symmetry as shown in Fig.2c.

1) 1 to 4, 2 to 5, to form two clockwise spirals; and 3 to 1, 4 to 2, and 5 to 3 to form three counterclockwise spirals. This is the most common way of connecting dots;

2) connecting 1 to 3, 2 to 4, 3 to 5 to form three clockwise spirals; and connecting 2 to 1, and 3 to 2 to produce two counterclockwise spirals.

Siromoney [2] describes the order in which to connect the dots as (1, 3, 5, 2, 4,1). After repeating this pattern for 8 times, dnyan kamal is completed. This is a mono linear drawing.

B. Saraswatee*:* Fig.3d shows saraswatee drawn with one, two and three canes. The numbers on the canes should help in understanding the sequence of connecting them as described below.

| | |
|---|---|
| Saraswatee drawn with one cane connecting | order 1 to 1 |
| Two canes | order 1 to 2, 2 to 1 |
| Three canes | order 1 to 3, 2 to 2, 3 to 1 |
| --- | --- |
| N canes | order 1 to N, 2 to N-1, ...N-1 to 2, N to 1 |

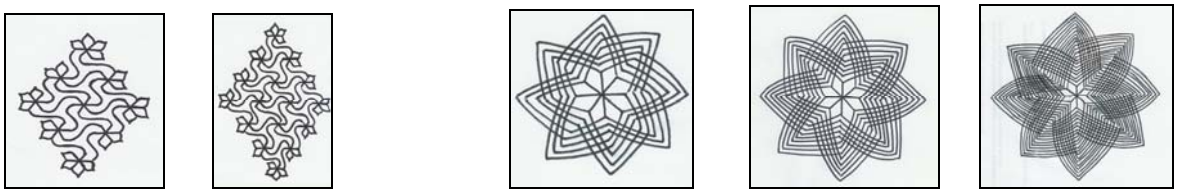


**Figure 4a,b:** *Koyaree: First two iterations* **Figure:4c,d,e** *Lotus of knowledge, first three iterations*

**Iterations:** Making larger patterns out of a basic pattern is an interesting and challenging activity. For example, we saw above how to draw the basic pattern of the coconut. We can iterate the construction, based on eight dots from the hexagonal lattice. A further iteration is based upon twelve dots. One can write this in the form of a recursion, where the number of dots is given by $n(i+1) = n( i ) + 4$.

The coconut patterns in complex iterations are not connected to each other in any way. Whereas bilve patra has either one or two points connected to the next pattern in first and successive iterations. All others show overlapping of one or more lines or shapes in successive iterations. One also finds $(n+1)^2$ basic patterns in nth iteration of the coconut, turtle, nabhi kamal , and koyaree designs, based on hexagonal arrays of dots. See following description of example in Fig. 4a and 4b.

**Koyaree iterations:** The basic pattern of koyaree is drawn by first drawing 7 horizontal equidistant dots and then decreasing rows from a hexagonal array until we reach one at each of the top and bottom ends. The next iteration starts with 11 dots as in Fig 4a. Fig.4b shows a further iteration of koyaree that starts with 15 dots. You find four basic patterns in first iteration and nine in the second iteration.

While drawing iterations of dnyan kamal, we want to keep the mono linearity in the drawings as it allows writing efficient computer program to create the designs with the aid of computers. Fig. 4c, 4d and 4e show drawings with 8, 13 and 21 points respectively. Although beautiful to look at Fig. 4c with 8 dots is not mono linear. Fig. 4c will be completed by following number pattern (1,4,7,2,5,8,3,6,1) 8 times showing 5 clockwise and 3 counterclockwise spirals. Fig. 4d will be completed by following number pattern (1,6,11,3,8,13,5,10,2,7,12,4,9,1) 8 times showing 8 clockwise and 5 counterclockwise spirals. Iterations are also drawn with different number of arms. E.g. instead of 8 there can be 4, 6,12,16 ..petals.

**Computer science:** Siromoney [2] from Christian College, Chennai, India and Ascher [3] from Ithaca College in United States have studied extensively the logic and recursive language hiding in dnyan kamal, grape vine, koyaree and other kolam designs. Many more are have been revealed recently in analysis of kolam designs [4].

## Rangolee in education?

After investigating the mathematics and science involved in rangolee designs, it seems possible that its daily practice may bring the mind to a greater awareness and understanding of logic, patterns and symmetry through transfer of a sort of kinesthetic intelligence. Although minimal the use of contemporary rangolee designs in schools in United States is on the rise. It is merely used as an art and social studies activity rather than a mathematical tool. This author is currently engaged in developing mathematics curriculum according to NCTM standards for grades K-8 based on traditional rangolee designs. She gives power point presentations and workshops in local schools to educate teachers about mathematical nature of rangolee designs and their use in classroom. She is teaming up with Dr. Ron Eglash on DR-K-12 exploratory research grant proposal to National Science Foundation to test her hypotheses.

## References:


[1] P. Gerdes, *Geometry from Africa: Mathematical and Educational Explorations.* Washington, D.C.: Mathematical Association of America. 1999.

[2] G. Siromoney, Kolam, Kavyashastra Quarterly, April 1978. http://www.cmi.ac.in/gift/Kolam.htm

[3] M. Ascher, The Kolam Tradition; American Scientist, 2003.

[4] Y. Kawai, K. Takhashi, S. Nagata, PsyKolo3D-Interactive Computer Graphical Content of "Kolam" Design Blocks. Forma, Vol. 22, pp.113-118, 2007