Using Binary Numbers in Music

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Abstract

Composing using binary numbers as rhythms can create music that is both interesting and accessible. I shall demonstrate, using whimsical examples, that binary numbers and music can indeed live in harmony.

Why use Binary Numbers?   Play Figures

Binary numbers used as rhythms (not to be confused with binary rhythms) can be used in music to create a mathematical aspect to a piece which can be heard and appreciated by mathematicians, yet still be pleasant to the ears of the uninitiated. Even someone who has never heard of binary numbers can still understand the patterns on an intuitive level. Binary numbers, especially those affiliated with computer science, have a structure which goes very well with the standard forms of music.

Mathematics and constraints. Artists love to come up with limitations which they must figure out how to work around. Working with tight constrictions requires creativity, giving the artist opportunity to stretch her limits, show off, and have fun. Bach's fugues and canons are still known and admired because of the incredible things he managed to do with such strict forms. Escher did similar things with art, using mathematical rules to create his work [1]. A French literary group, Oulipo (Ouvroir de Littérature Potentielle, or Workshop of Potential Literature), also enjoy writing using rules, for example: writing a novel without the letter E, or writing "snowball sentences," where the first word has one letter, the second has two letters, then three and so on [2].

Many composers find ways to work meaningful numbers into their pieces. Alban Berg found importance in the number 23, and also gave significance to other numbers, which he then reflected in his pieces. The score to George Crumb's *Black Angels* is saturated with the numbers 7 and 13, from the number of movements to the number of notes in phrases, and different combinations of instruments are used methodically [3].

Schoenberg developed serialism, organizing pitches into a row and using them systematically. The possible transpositions and inversions of the row can be organized into matrices, a technique used by Pierre Boulez and Peter Maxwell Davies. John Cage used chance, rolling dice to determine the pitch and length of notes. Iannis Xenakis, inspired by architecture, created musical curves out of straight glissandi [4]. Indeed, throughout the 20th century, music relied increasingly on mathematics. However, most melodies composed from such rules are appreciable only by academics. Often the math isn't recognizable to the listener, and only an unpleasant sounding string of notes remains.

It should be noted that very few techniques of this type directly generate music; most are organizational tools which still leave many musical choices to the composer. The technique which is the topic of this paper constrains only the rhythm of one or more voices, leaving the choice of notes up to the composer. In the following examples, I compose using not just the rhythm derived from binary numbers, but also using other compositional choices based on the character of the numbers. The examples can be heard online at [5].

Binary Numbers as Rhythm. Let 1 be a note and 0 be a rest. Thus a number becomes a rhythm! The numbers one through eight, in binary, are shown in Figure 0. Below each number is the corresponding rhythm. In computer science, binary numbers are usually seen in groups of eight bits (a byte), which happens to fit perfectly into a 4/4 measure if we assign bits to eighth notes. Powers of two are involved in the usual form of western music, where four or eight bar phrases might make up a 16 measure section of a 64 measure song. This creates a constraint on the rhythm, but leaves the composer free to choose the melody.
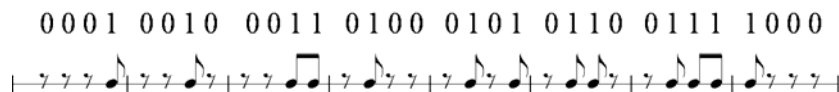

Figure 0: *Binary numbers 1-8 above corresponding rhythms.*

Ways to use Binary Numbers as Rhythms

ASCII Code. There is a computer standard, ASCII, for encoding data into binary. Every letter of the alphabet (as well as other characters) has a corresponding 8 bit ASCII code. While mathematically arbitrary, it is a natural encoding for people familiar with Computer Science. This means that one measure can encode one letter (if you're using eighth notes). Figure 1 shows a self-referential string quartet where each instrument plays the letters of its name as the rhythm. The code for each lowercase letter used begins with 011, and the following five digits are the position of the letter in the alphabet. Lowercase "a," then, is 01100001. The melody is unconstrained.

If one heard the segment without knowing there was any math behind the rhythms, one would still hear how each measure starts the same way and how the violin and viola play the same rhythm for the first four measures. Even when not consciously noted, patterns make a piece cohesive. A random string of ones and zeroes would not have nearly as nice an effect.

Figure 1: *A string quartet where each instrument plays its own name in ASCII.*

This is a great way to hide secret messages and suggests various types of musical steganography. Famous examples are the motives which Bach and Shostakovich used to sign their name, Bach with the notes BACH (Bb A C B), and Shostakovich with his initials: DSCH (D Eb C B). The German system of note names uses B rather than Bb, and H instead of B. Using Eb to represent S is a precedent created by Shostakovich, which I follow in Figure 2. Brahms, Schumann, Elgar, Berg, and others have also used the letter names of pitches to sign or dedicate their work. The technique of encoding binary into music improves upon this, allowing messages of any length and with any letter. Of course, both techniques could be used at once. As seen in Figure 2, which is a section of a longer piece, I hide a name within the notes of a lyrical melody, while the rhythm of the chords spell it in ASCII. Also note that instead of 1 meaning play and 0 meaning rest, a 0 means hold the previous note, to allow a more lyrical melody.

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Figure 2: *"sasha" is hidden in both note names and rhythm.*

Equations. One can 'add' numbers by making an equation where a number played by one instrument, plus a number played by a second instrument, equal a number played by both together, as seen in Figure 3. It shows the creation of the Fibonacci sequence by adding previous terms to get new ones. Each eighth beat represents a 0 or 1. A 1 must start a note but it may be held over any number of 0s, and all other 0s are rests. Because previous terms are used again in addition, there is a structure to the form. The numbers grow from beginning to end of the piece, and so the trend is an increasing number of ones with earlier entries, making the piece gain momentum and excitement as it progresses. The beginning of the Fibonacci sequence can also be found hidden in the piano part, where middle C is 1, D is 2, E is 3, G is 5, and so on.

Figure 3: *The Fibonacci sequence.*

One could write music to a whole variety of sequences, and many sequences have a number of unintended side-effects. By simultaneously using two different but related sequences in different voices, connections can be heard between them (for example, if you multiply a binary number by two, the aural effect is a metric shift one note to the left). The Online Encyclopedia of Integer Sequences [6] is an excellent resource for this, and one I have used in the composition of examples not shown in this paper.

Fractional Binary Numbers. Binary numbers with fractional parts have an additional problem: how to indicate the binary point. One could use bar lines or a musical cue, such as having all the bits played in a different range or by a different instrument. In the following example (Figure 4), π has been turned into music. The first two bits which make up three (binary 11) are a pickup, and from the first measure onwards the violin and flute play the non-repeating bits for, in theory, an infinitely long piece (unfortunately this example only shows four measures of it). Also new in this segment is that both instruments play the same number, with the flute sounding on ones and the violin on zeroes.

While π is functionally a random number generator, the compositional choices made using this 'random' number are very specific to this number, and a different number (such as *e*) could lead the composer to an altogether different piece. When notes skip back and forth between two voices, one must be careful to give shape to the groups of notes which are together (as in the end of the first measure in the flute) and keep the music from sounding too jumpy when the parts are very disjoint (such as in the second measure). It is not merely a matter of writing a single melody and then separating it out into two voices.

Figure 4: *A segment using π to 64 binary places.*

Other Possibilities. There are many other encodings possible from binary strings to rhythms, which I have used in examples not shown here. Instead of making an eighth of a beat correspond with a 0 or a 1, more control could be had over the rhythm by making a 1 a note of any chosen length and 0 a beat of rest. Also, the roles of the digits could be switched to create a similar piece, where zeroes sound and ones do not. The two important things are to know where the zeroes and ones are, and to know where numbers begin and end. There are always more numbers and patterns to explore, more ways of encoding information, and more ways to turn ones and zeroes into notes.

Composers are always looking for inspiration, challenges, and tools. Creating music using binary numbers is a lot of fun, and I have enjoyed trying to discover different ways of composing using them. The results have often been unexpected, with new side-effects constantly popping up. There is a challenge in making an arbitrary rhythm into coherent music, and even greater challenge comes from adding additional constraints: new and related rules that govern the relationships between numbers or measures. It helps one to look at composing in a new way, breaking old patterns by forcing a start from a different direction than usual. It is almost addictive, and I know I won't be able to resist sneaking secret messages into future compositions.

References

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