The Tangramoid : Recent Developments

Samuel Verbiese
Plastician artist
Terholstdreef 46,
B - 3090 Overijse, BELGIUM
E-mail: verbiese@alum.mit.edu

Abstract

Tangramoids are families of 3D objects introduced in [1]. One of these objects, called here the Closed Tangramoid, or simply Tangramoid, has recently been understood as a constant volume morphing of a regular octahedron and has been proven to be a polyhedron [3] in a sense due to Buekenhout [4]. Its plastic features allowing it to be considered as a dynamic sculpture are revisited in this short communication together with technological information on new projects.

1. Introduction

I found in a US bookshop some twenty years ago probably the most complete reference on the Tangram [2], and this Chinese puzzle has ever since amazed me with the incredibly rich plastic possibilities as a shape game. Let me recall that the legend goes as follows : a Chinese boy was carrying a square stone tile, and suddenly he let it drop, so it broke in seven parts. The boy immediately went down and, grabbing the fragments, he started to recompose the square, but failed. Instead of this, he found dozens of vivid silhouettes of people, animals, objects: **the Tangram was born !**

For years I have been attracted by the recomposed square, seen as resting on the vertex joining the two large triangles, like a diamond indeed (not only the polygon but also in the sense of the familiar brilliant-cut gem, when removing the upper triangle !) Whenever I looked at it this way, I couldn't help being puzzled by the fact that it gave me a compelling symmetry feel, despite the geometrical symmetry is broken with the presence of the square and the parallelogram. Ten years ago, I suddenly “saw” why this symmetry: the figure very simply can represent the silhouette of a 3-D object. Recently I figured out *this object stems from a regular octahedron resting on a vertex and featuring four offset tetrahedral bucket-like indentations above the 'meridian', leaving four rhombs around and being exactly compensated volume-wise with four pyramidal outgrowths on the top, forming a pyramidal 'roof'*. Having indeed a rotational symmetry of order 4, this 'turbine'-like object, when seen from a distance, projects four times per revolution as kind of a Chinese lantern, and in between, the rhombs duly projecting as a square and a parallelogram, four times as a Tangram : **the Tangramoid was born !**

There are many families of objects that project into Tangrams [1], but I see this closed spinning one as particularly suited to function as a kinetic sculpture.

1. The Closed Tangramoid as a Polyhedron in the sense of Buekenhout

Francis Buekenhout, professor of Mathematics of my Alma Mater in Brussels, and a distinguished member of the Académie royale de Belgique, honored one of my exhibitions with a visit where the

Closed Tangramoid was set on display, made of Zometool [8] elements wrapped in paper "sails". He kindly suggested to extend it some mathematical legacy by proving it to be a polyhedron in a sense he developed and of which he is convinced that it is not possible to find a simpler definition without being exposed to fundamental errors [4]. This original definition of a polyhedron is not topological but combinatorial or relevant to Incidence Geometry [6], mainly based on works by Belgian-born French mathematician Jacques Tits during the period 1954-1962 when he got his deep theory of buildings (see a historical account and a geometrical context in [5, 6]). Also observe that Tits got the Abel Prize of Mathematics 2008 in this context, with American John Griggs Thomson. The detailed paper that resulted on the Tangramoid can be found on-line in [3], and I give here the final results.

A symbolic model of the object (see Fig. 1, left) has been constructed in terms of *22 vertices, 48 edges, 28 faces (which thus satisfies Euler's formula)* and incidence, which is very simply defined by inclusion since vertices are points, edges are lines, and faces are planes.
The object is shown to be a polyhedron on the basis of five steps:

(1) every edge is incident with two vertices, from the construction;
(2) every edge is incident with two faces: this required a series of checks using the list of edges;
(3) the "residue" of every face is either a triangle or a quadrangle from the construction;
(4) the "residue" of every vertex (the classical " vertex figure" due to Schläfli) is connected and is indeed a polygon. Actually, we distinguish triangles, quadrangles, pentagons and heptagons;
(5) the full set of vertices, edges and faces is connected: a given vertex can been checked to be connected to all other vertices and so to all edges and faces in view of their descriptions thanks to vertices.

We observe we may blow on the air-tight object from the "inside" and see it on a sphere (see related Bridges document [7] kindly hinted to by Jim Hausman).
The simple structure chosen so far for the object which *has now been proven to be a polyhedron in the sense of Buekenhout and* which also appears to be *an 'Embedded Polyhedron in the Euclidean space of dimension 3'*, does not entirely satisfy my original view on the Tangramoid, especially the relation to the classical Tangram puzzle of seven pieces [2] in its square shape : *for this polyhedron to merit the name of Tangramoid, the four rhombic faces must be opaque* because the sought-after projection allows for only one square and one parallelogram (see Fig. 1 left).
We also observe that *the Tangramoid is a chiral figure in 3D space.*

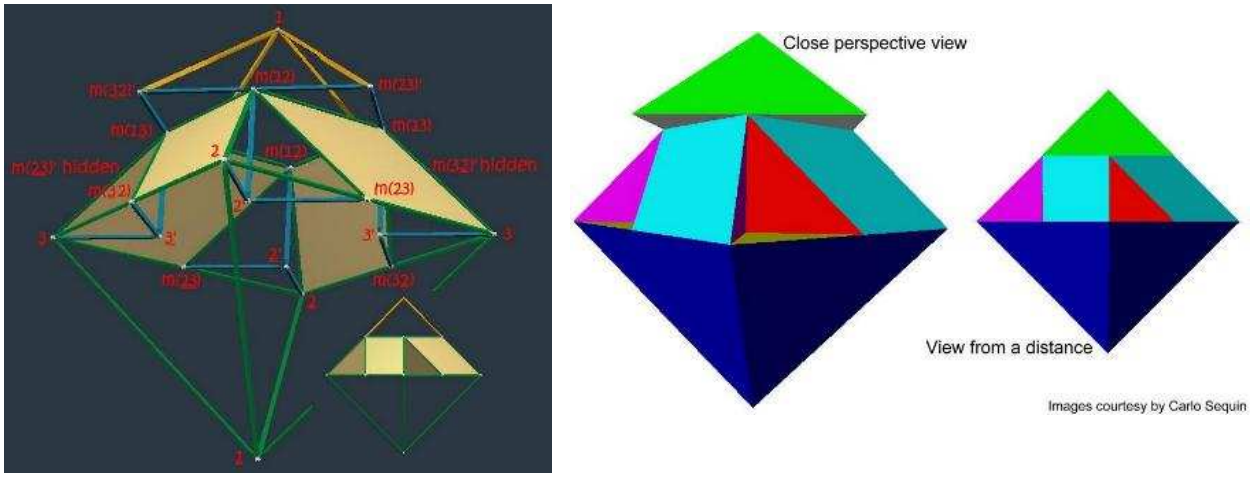

Figure 1. *Left: the vertices, edges and opaque rhombs of the Tangramoid generated from the regular octahedron, rendered after Zometool parts [8] in Scott Vorthmann's vZome modeler [9] with tiny connectors and with opaque rhombs, annotated with the notations used in [3], and, in insert, one of the four "Tangram projections". Right: images courtesy by Carlo Sequin, where each kind of face is represented by a different color.*

2. The Closed Tangramoid as a kinetic sculpture

The rotor shape suggests the Tangramoid to become a spinning sculpture actuated with the wind. Despite not being tuned as an efficient windmill, and thanks to the presence of its 'roof' where solar panels can be installed, some additional power so being generated too, it is hoped that the Tangramoid, installed on a favorable open space, could give back to the electric network on average more energy than it consumes when illuminated at night and when it needs spinning electrically in absence of wind or sun (Fig. 2).
For a total height of 10m the rotating structure would be built of steel pipes about 12 cm in diameter and 3.5m long, half the length of the sides of the generating regular octahedron resting on a vertex. In addition to this extra triangulation, an internal pipe octahedron attached to the innermost vertices of the pyramidal "buckets" could elegantly help strengthen the structure, if needed (Fig. 3). In any case, it would rest trough two large roller bearings installed 2.5m apart on an internal pivot duly bolted on a concrete slab : it has a cross-section of about 50 square meters that must resist to winds and gusts of 150km/h. Not a windmill by itself, the Tangramoid rotor needs exterior statoric sails to direct the wind into the buckets.

Figure 2: *artist impression of monument*

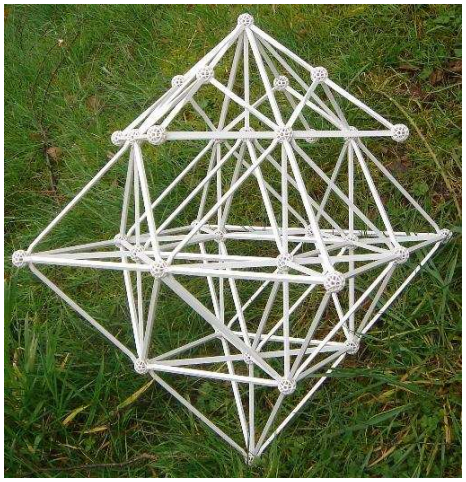

Figure 3: *draft structure of monument*

Beyond this, for several years now, I nurture a still unfulfilled dream : I want to "give back" this idea to the Chinese People as a friendly interpretation of a Chinese cultural icon, and I envision the Tangramoid spinning in the Pudong skyline on top of what I suggested to call a "Tangram Tower" to be readied for the Shanghai 2010 Expo. The idea was to convince a party already decided to erect another tower, to consider designing one from scratch with an embedded Tangramoid on top. At that time I imagined that the sculpture to be build with aerospace technology could be accessible to the public as a restaurant revolving around a central elevator shaft. Emergency exits could have been provided at rest when a couple among the four "equatorial" corners of the Tangramoid would have "docked" with the specially to be designed glasshouses. A principle model of the top of such a tower was built in Zometool covered with paper to show the asymmetric "statoric" glasshouses (Fig. 4, where gaps should be reduced). Combined teams of architecture and engineering schools from Shanghai and Belgium could have studied the early project.
But 'selling' such ideas evidently isn't easy and considerable time and efforts couldn't help me getting there so the time left over before the Expo became too short. At the end of that period, I learned about the construction of a World Financial Center that would feature a circular opening on top, but this design looked so plastically pure to me that it never could be harmed by any addition, so I didn't even bother to try to squat it. Then recently, in a fortunate circumstance perhaps, I discovered that the WFC Tower is now being completed with a rectangular opening instead, which may look a bit unfinished : couldn't it be graced by a shiny spinning Tangramoid (Fig. 5) ? Considering the levels of financing required for huge tower projects, funding should be available for such a visible but relatively inexpensive stunt : a mere spinning aluminum/steel pipe structure with sails, 40 meters tall, with huge bearings on its base and top.

Figure 4: Zometool model of initial concept of top of Tangram Tower

Figure 5: *Possible WFC project for Shanghai 2010*

Conclusion

While this paper brings a new insight to view the Tangramoid as a regular octahedron slightly transformed with constant volume, its major purpose is to reflect a recent mathematical result that answers, for the case of the Closed Tangramoid, the hope expressed in [1] that thorough mathematical developments could be made to establish that Tangramoids created in the 'sphere of art' also truly belong to the 'sphere of mathematics'. Finally, technological considerations are shared about the ongoing efforts to bring the Tangramoid to life as an eye and mind pleasing and environmentally sound kinetic monument with strong playful content.

Acknowledgments

Warm gratitude is extended to Francis Buekenhout for the interest and work he wanted to devote to the Tangramoid. Carlo Sequin also deserves our appreciation for having been willing to delve in this subject.

References

[1] Samuel Verbiese, *Tangramoids*, Proceedings of ISAMA-Bridges 2003 Alhambra Meeting, Granada, Spain, (Eds. Javier Barrallo et al), pp. 245-252, 2003.
[2] Joost Elffers, *Tangram, the ancient Chinese shapes game*, Penguin Books, 1976.
[3] Francis Buekenhout and Samuel Verbiese, *The Tangramoid as a Mathematical Polyhedron*, UREM-ULB, Unité de Recherche sur l'Enseignement des Mathématiques, Université libre de Bruxelles, 8p, 2008, http://dev.ulb.ac.be/urem/The-Tangramoid-as-a-Mathematical.
[4] Francis Buekenhout, *Une définition de polyèdre*, CeDoP Université libre de Bruxelles, 10p, 1995, http://www.ulb.ac.be/docs/cedop/index_12.html ; Translated in English as *A Definition of Polyhedron*, UREM-ULB, Unité de Recherche sur l'Enseignement des Mathématiques, Université libre de Bruxelles, 9p, 2008, http://dev.ulb.ac.be/urem/A-Definition-of-Polyhedron-par.
[5] Francis Buekenhout, *The rise of incidence geometry and buildings in the 20th century*, in Charlemagne and his heritage, 1200 Years of Civilization and Science in Europe, Eds. P. L. Butzer e.a., Brepols, Turnhout, pp. 235-256, 1998.
[6] Francis Buekenhout, *Handbook of Incidence Geometry*, Elsevier, Amsterdam, 1995.
[7] Robert J. MacG. Dawson, *Some New Tilings of the Sphere with Congruent Triangles*, Renaissance Banff Proceedings 2005 (Eds. Sarhangi, Moody), Canada, http://www.mi.sanu.ac.yu/vismath/bridges2005/dawson/index.html.
[8] http://www.zometool.com.
[9] Scott Vorthmann, http://vorthmann.org/zome/jnlp/v21latest/vZome-v21latest.jnlp.