Some Interesting Observations Regarding the Spidrons

Dániel Erdély and Walt van Ballegooijen
Spidron Team
Batthyány u. 31. 1.12.
Budapest, 1015, Hungary
E-mail: edan@spidron.hu, waltvanb@xs4all.nl

Abstract

In the following paper we are going to present some of our surprising new findings, which encourage us to continue our long-term investigation of the movement and other interesting properties of **Spidrons**[1]. We undertook here to present the peculiar tilting of some of the *spidron edges* during the continuous *spidron movement*, the simultaneous appearance of different angles, which are typical of cubic and diamond lattices, *spidronized* Penrose-tilings, the Kepler-tile shadows of certain edges of quasicrystals that are defined by the bisections of them by specific *spidron-nests*, and other curiosities.

Introduction

We have described the parts, the definition and several features of the spidrons many times, in earlier Bridges presentations and in the bibliography attached as reference, but since then we have created new variations. One only needs basic knowledge from earlier articles and the present paper to understand these special curiosities. So instead of giving a long description we are demonstrating the spidron (fig.1), the spidron-arm (fig.2), the spidron-ring (fig.3) and the spidron-nest (fig.4) through images.
One of the most intriguing properties of spidron-nests is the (continuous) foldability. Figure 4 shows a spidron-nest seen from the top in the flat position, and in figure 5 it is shown in a folded position. Also some edges and their midpoints are indicated. These midpoints remain in the base-plane during the folding of the nest. This can be seen easier from the side of the nest, see figure 6.

Figure 1: *Spidron*

Figure 2*: Spidron-arm*

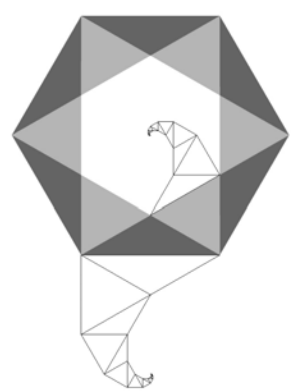

Figure 3*: Spidron-ring*

[1] Spidron has been a Registered Trademark since 2007

Every ring of a spidron-nest is surrounded at the outside by a regular skew polygon, which can be described by defining two kinds of angles:

- The *f*-angle is the angle between an edge of the skew polygon and the base-plane.
- The *g*-angle is the angle between two adjacent edges of the skew polygon.

The properties of a spidron-nest during the folding can be described by these *f* and *g*-angles.
The relation between these two angles can be found by the equation:

$$\text{Cos}[g] == 1 - 2\,\text{Cos}[f]^2\,\text{Sin}\left[\frac{\pi\,(n-2)}{2\,n}\right]^2$$

where *n* is the number of vertices of the regular skew polygon.

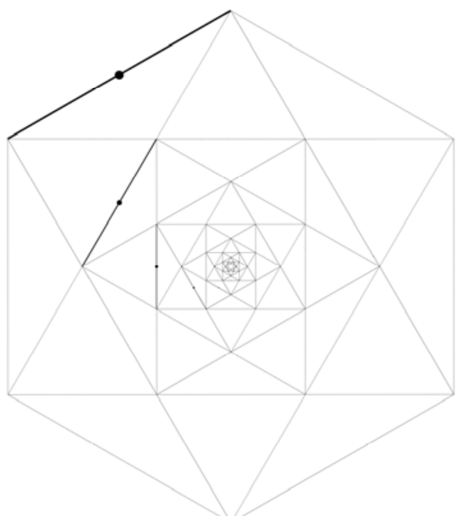

Figure 4: *Flat spidron-nest, seen from above*

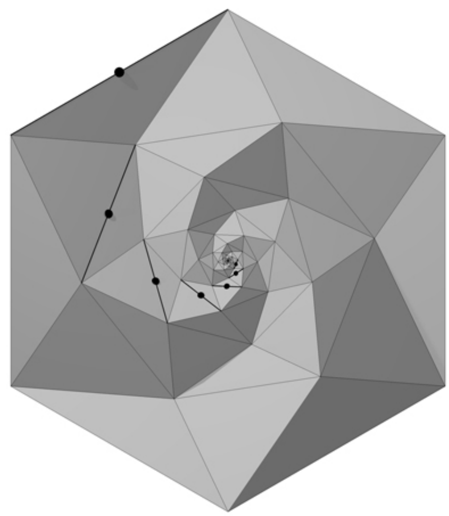

Figure 5: *Folded spidron-nest, seen from above*

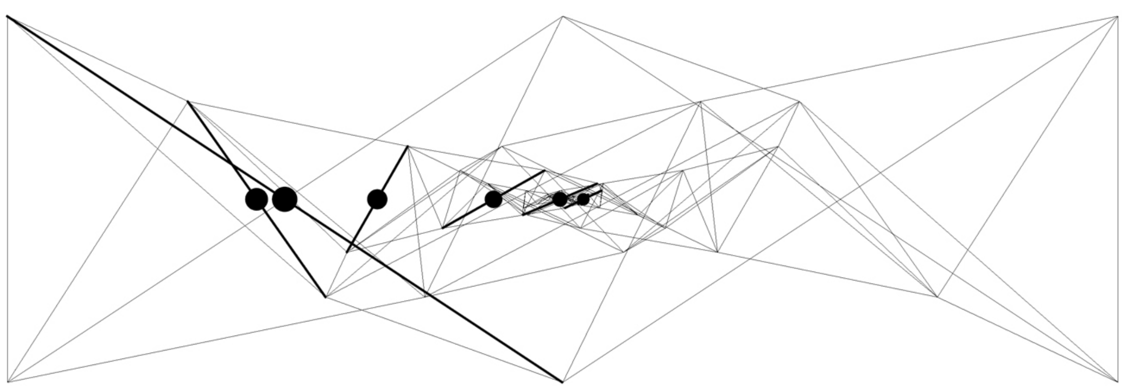

Figure 6: *Folded spidron-nest, seen from side view*
The midpoints of the appropriate edges remain in the base plane during the folding.

1. Bounding Ellipsoid

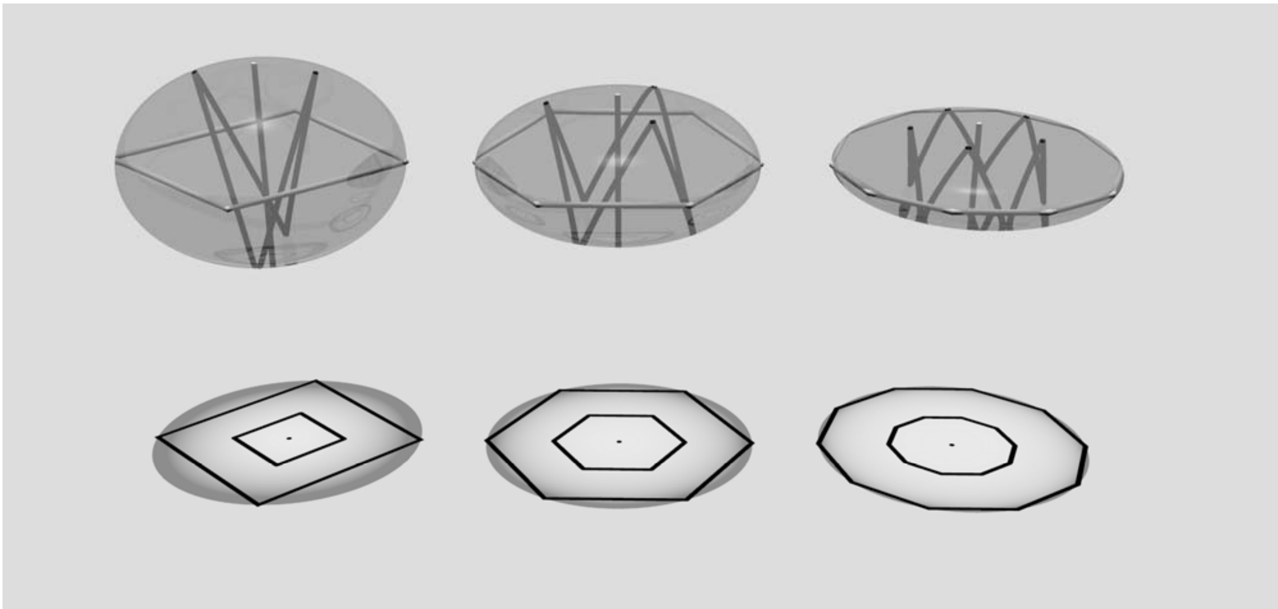

Figure 7: Bounding ellipsoids and shadows of the skew polygons

Concentric skew polygons surround the levels (i.e. rings) of the spidron-nests. The vertices of any spidron-like folding regular skew polygon move on the surface of a circular ellipsoid in 3D. The equator of this ellipsoid is the circumcircle of the flat regular *n*-gon (this is the outer polygon of the flat nest), say with radius r. The length of the vertical axis – which is the only rotational axis of the ellipsoid – in the center is equal to the edge-length of each edge in the skew polygon, say *d*. Then

$$d = 2\, r\, \mathrm{Sin}\left[\frac{\pi}{n}\right]$$

Any point (*x*, *y*, *z*) on the ellipsoid follows the equation:

$$x^2 + y^2 + z^2\, \mathrm{Sin}\left[\frac{\pi}{n}\right] = r^2$$

The ellipsoid can be seen as a sphere with radius r, vertically scaled down by a factor, which equals

$$\frac{d}{2\, r} = \mathrm{Sin}\left[\frac{\pi}{n}\right]$$

So the shape of the ellipsoid is only depending on *n*.

2. Monotonous change of one angle giving rise to local extrema elsewhere

There are two ways to introduce the spidrons:

a) The spidron-nests observed consist of joined rings, on which the inner and outer edges are n-sided regular skew polygons[2] with varying *g*-angles[3].

b) The spidron-nests observed consist of spidron-arms that are made up by placing two different sets of similar triangles in alternating sequence.

For the sake of simplicity we will demonstrate the mentioned phenomenon on the so called "*classic spidron*". The classic spidrons consist of alternating sequences of equilateral triangles and isosceles triangles with 120° vertex angles, as shown in figures 1 to 6. In case of the classical spidron-nests the f-angles of successive edges are decreasing to zero towards the center.

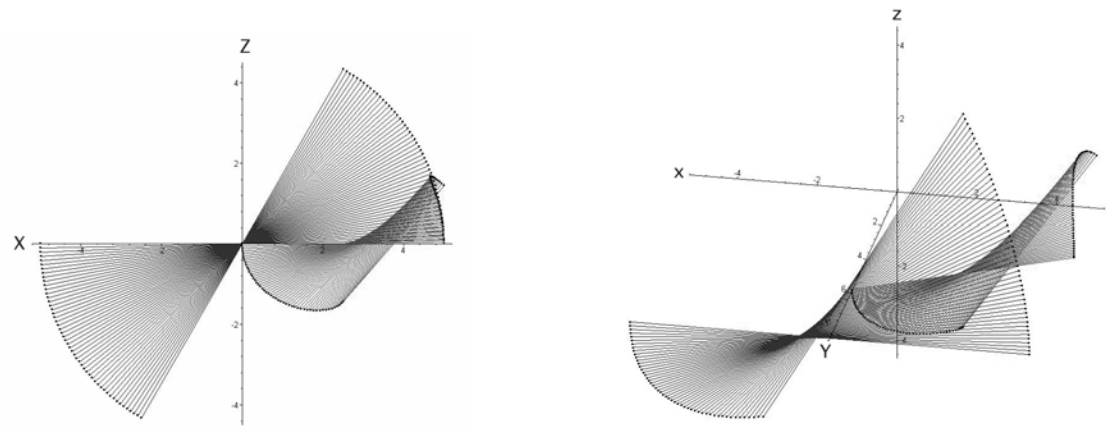

Figure 8: *The movement of two edges on the $\mathbf{L_1}$ and $\mathbf{L_2}$ levels during the folding process can be shown in an orthogonal coordinate system, seen from two different points of view (figures by Lajos Szilassi)*

If the external edges of the outermost ring of a spidron-nest – the first level ($\mathbf{L_1}$) – are rotated out of the base plane (the angle of rotation is the *f-angle*), then the $\mathbf{L_2}$ internal edges of the same ring also rotate, but they rotate less. In the meantime, due to the folding process, the midpoints of the edges move towards the centre of the nest. The angles of rotation of those two groups of edges are shown in the table below. The first row shows the angles of rotation f_1 of the $\mathbf{L_1}$ edges, while the second row shows the angles of rotation f_2 of the $\mathbf{L_2}$ edges. The maximum possible f_1 angle is 60°, as at that point the plane triangular faces approaching each other touch. But at f_1 = 48.1897° there is an interesting phenomenon: The f_2 angle reaches a local maximum! The same turnaround also occurs on the further rings ($\mathbf{L_3}$, $\mathbf{L_4}$, $\mathbf{L_5}$, etc.), although to a lesser and lesser extent. At that point, the value of f_2 is such that the $\mathbf{L_2}$ edges are perpendicular to each other, so these edges of the spidron nest fit onto six edges of a cube (so g_2 is 90°). This subnest bounded by the $\mathbf{L_2}$ edges divides the cube into two parts of equal volume with chiral symmetry.

2 In our paper we call *regular skew polygons* each of the skew polygons which have equal f- or g-angles and equal edge-lengths and where the vertices aren't in the same plane.

3 In the flat position, when the nest is in the base plane, these g-angles are equal to π (n-2)/n.

f ANGLES TABLE

f_1	30,0000	33,5573	35,0000	40,0000	45,0000	48,1897	50,0000	55,0000	60,0000
f_2	27,5622	30,0000	30,8905	33,4422	34,9657	35,2644	35,1604	33,6679	30,0000
f_3	25,7206	27,5622	28,2021	29,9263	30,8700	31,0468	30,9856	30,0704	27,5622
f_4	24,2514	25,7206	26,2160	27,5084	28,1876	28,3126	28,2694	27,6135	25,7206
f_5	23,0368	24,2514	24,6527	25,6786	26,2049	26,3007	26,2676	25,7606	24,2514

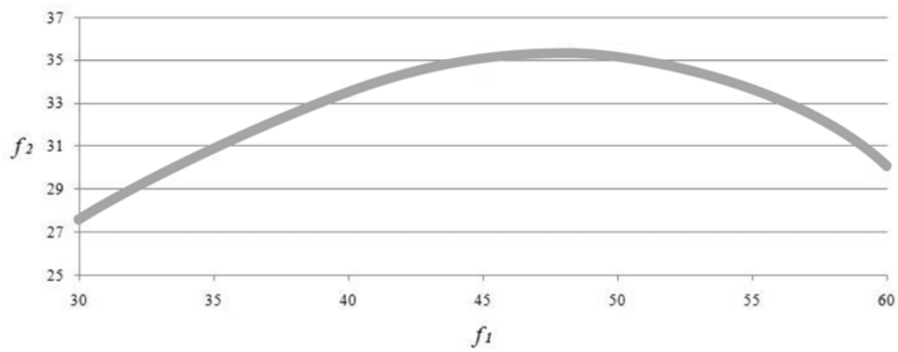

Figure 9: *Angles of* **L_2** *edges (f_2) as a function of the angles of* **L_1** *edges (f_1): at f_2=35.2644°, at the maximum, these edges coincide with the edges of a cube*

3. Simultaneous appearance of the typical angles of cubic and diamond lattices

The second interesting feature is also related to the *f* angles. When the edges of **L_2** coincide with the edges of a cube, then f_1 = 48.1897°, and in that position, g_1 is 70.5288°, which is exactly the adjacent angle of the characteristic angle of the diamond lattice, 109.47°. The interval of f_1 edges in which pairs of distinct f_1 values have identical corresponding f_2 values (with the single exception of f_1 = 48.1897°, which only has one f_2 value, 35.2644°) is demonstrated in the table above. It is for instance quite clear that the f_2 = 30° value corresponding to the f_1 = 60° extreme value reappears where f_1 = 33.5573.

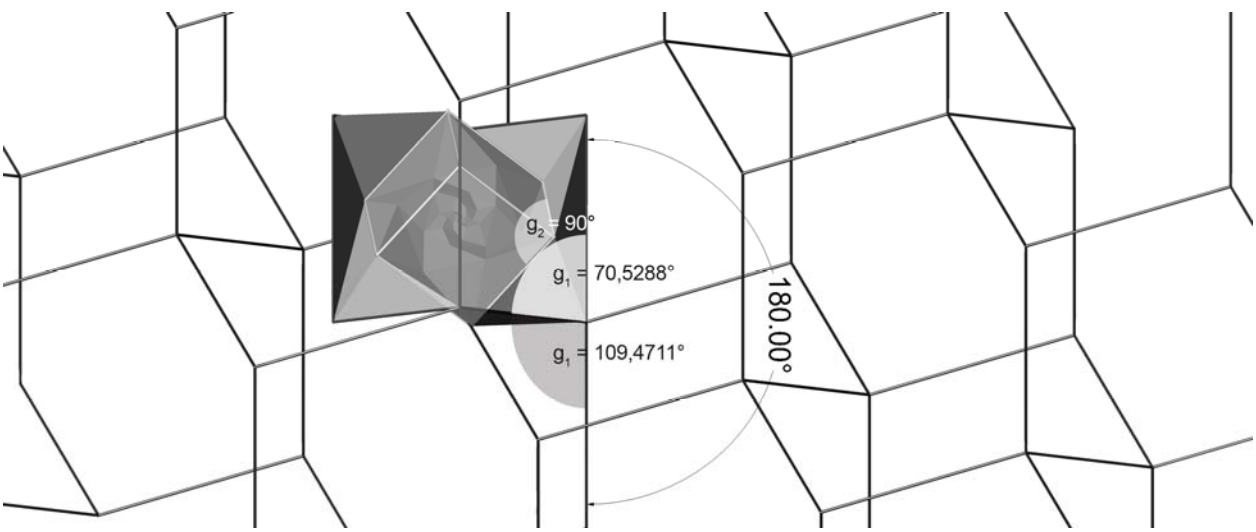

Figure 10: *Simultaneous appearance of cubic and diamond angles*

4. Kepler's shadows

Based on an idea of Marc Pelletier and Amina Bühler-Allen we constructed regular 10-sided skew polygons oriented in planes perpendicular to the edges of the acute and obtuse (*A6,O6*) golden rhomboids of quasi-crystals at their midpoints. Then we noticed various Kepler tilings[4] from certain views. Once the regular skew polygons were filled with spidron nests, we obtained "dodeca spidro-balls" and aperiodic spatial labyrinths.

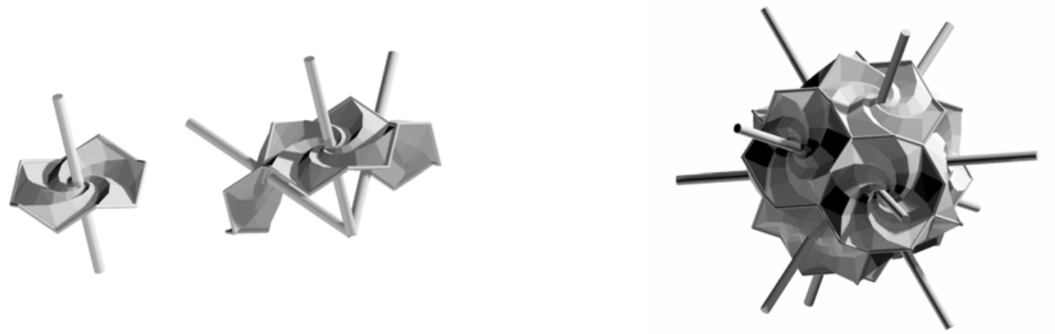

Figure 11: *Joined 10-sided spidron nests in planes perpendicular to edges at their midpoints*

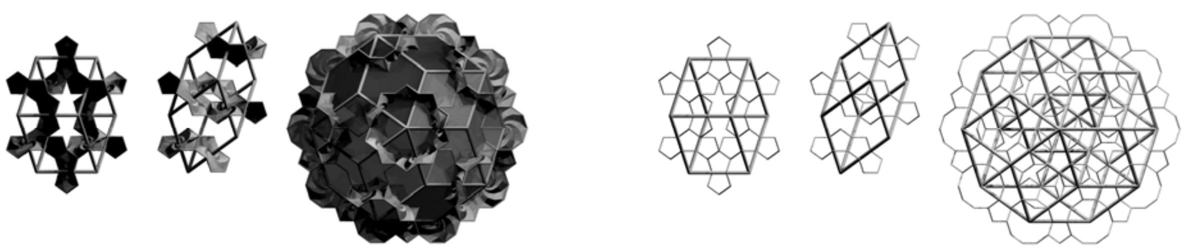

Figure 12: *The two golden rhomboids (A6 and O6) and the rhombic triacontahedron, with 10-sided spidron nests bisecting their edges. The resulting Kepler tiling is clearly visible from this point of view.*

Figure 13: *Spidron nests on the edges of a rhombic icosahedron also project to a Kepler tiling*

[4] Kepler tilings are pentagonal tiling systems, first described by Kepler in his book "*Harmonia Mundi"* in 1619.

5. “Spidrose” tiles

The next interesting result is that we were able to transform Penrose tiles into plane figures delimited by special spidron edge sequences. In this way we got “Spidronised Penrose Tiles”, and so we called them “Spidrose”-tiles. They eliminate the need for the markers enforcing the matching rules described by Conway, because now the rules are encoded in the shapes themselves.

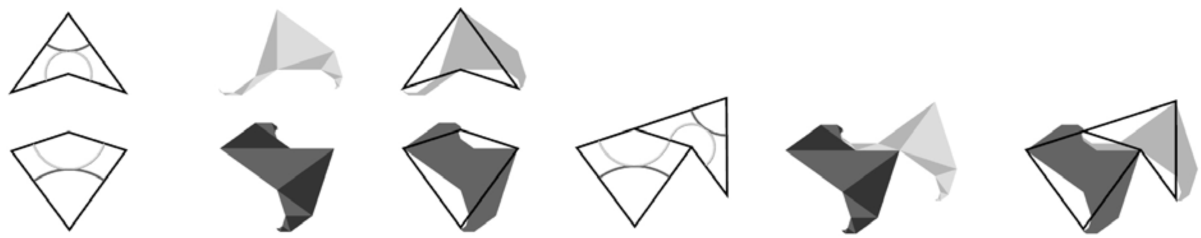

Figure 14: *With a clever idea, Marc Pelletier replaced the edges of the darts and kites of the Penrose tilings with special spidron edge sequences. This change makes the matching markers unnecessary.*

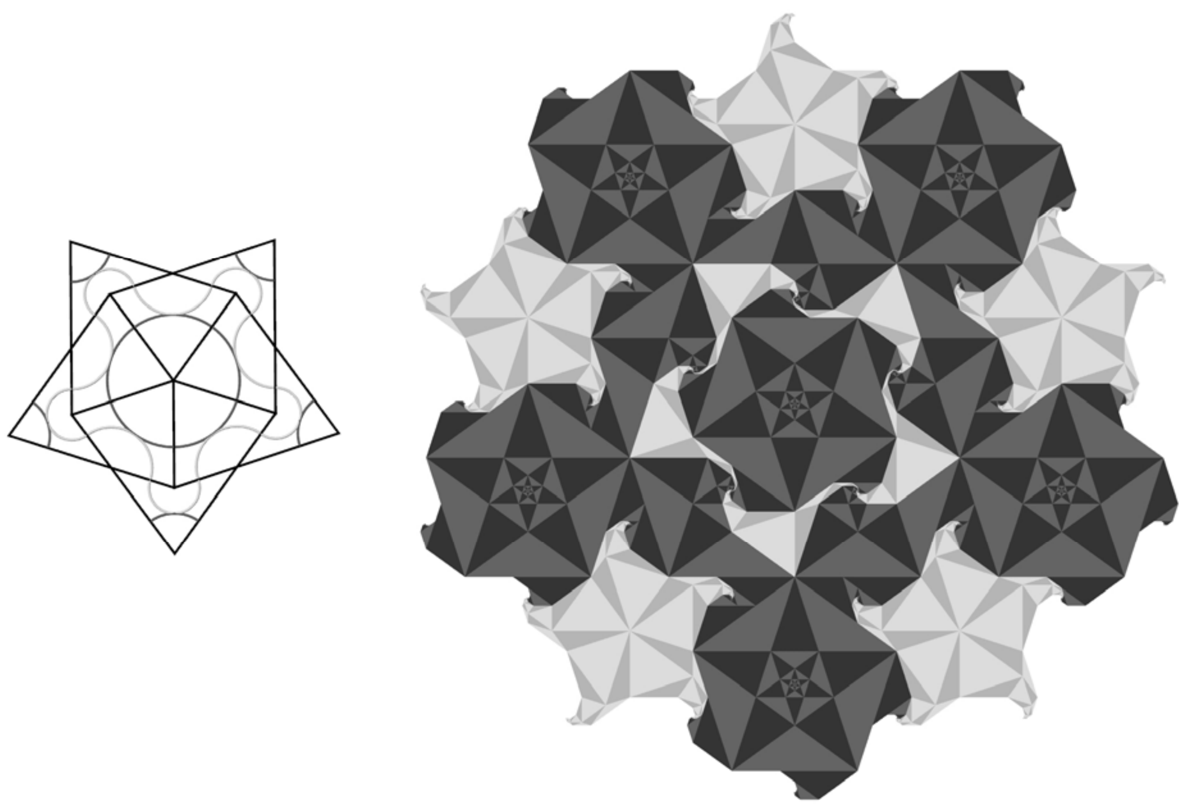

Figure 15: *Nicely shaped aperiodic tessellation corresponding to a Penrose tiling*

Aknowledgements

The family of geometric shapes has undergone a number of developments and received international recognition thanks primarily to Dr. Lajos Szilassi, Rinus Roelofs, Walt van Ballegooijen, Marc Pelletier, Amina Bühler-Allen, Craig S. Kaplan, Cristiana Grigorescu and Paul Gailiunas. In addition to those already mentioned above, we would like to thank the following people for their steadfast and committed work: Emil Molnár, Gergő Kiss, Péter Kőszegi, István Sági, Zsófia Végvári, György Falk, Miklós Laczkovich, Andrea Szekeres, Adéle Eisenstein, Balázs Földvári, Regina Márkus, Ildikó Szigeti, János Saxon-Szász, Zsuzsa Dárdai, John Hiigli, István Tenke, Wenninger Magnus, György Darvas, László Beke, Szaniszló Bérczi and my family, Ági, Matyi, Simon, Jakab, Janka and Mara for their patience.

References

[1] Branko Grünbaum and G. C. Shepard: *Tilings and Patterns*, W. H. Freeman and Company, 1989

[2] P. Gailiunas: *Some Unusual Space-filling Solids*, 2003

[3] Dániel Erdély: *Spidron System*: *A flexible Space-Filling Structure*, POLYHEDRA- *Symmetry: Culture and Science,* International Symmetry Foundation, 2004

[4] Dániel Erdély: *Spidron rendszer*, Régi-Új Magyar Építőművészet / Post Scriptum melléklet (In Hungarian) January, 2004

[5] Lajos Szilassi: *The right for doubting – and the necessity of doubt*; *Thoughts with regard to the analysis of the Erdely's Spidron-System,* 2004

[6] Dániel Erdély: *Some Surprising New Properties of Spidrons*, Renaissance Bridges proceedings, Banff, Canada – Page 179-186, 2005

[7] Dániel Erdély – Marc Pelletier: *Spidron Domain* – Expanding Spidron Universe; Bridges proceedings, London, Great Britain – Page 549-550, 2006

[8] Ivars Peterson: *Swirling Seas, Crystal Balls – Spirals of triangles crinkle into intricate structures* Science News, 21 October, 2006

Web pages

[1] http://www.spidron.hu

[2] http://www.rinusroelofs.nl/pr-spidron-00.html