Pull-up Patterned Polyhedra: Platonic Solids for the Classroom

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Abstract

Drawing from many years of experience in the teaching of geometry, this workshop presents a simple method for making pull-up Platonic solids in the classroom (requiring only card nets and string). Workshop participants will have the opportunity to make pull-up Platonic solids for themselves. Further possibilities for the creation of pull-up nets for other polyhedra will also be discussed.

Introduction

It is a well-known problem that students find the visualization of three-dimensional shapes challenging, which leads to difficulties when teaching the names and characteristics of polyhedra. The traditional method for creating shapes from two-dimensional nets, where students first try to visualize how the net folds into the three-dimensional shape and then physically construct the polyhedron, can be problematic. Invariably pupils will devote much time and effort to the exercise but will often accidentally cut off the tabs and forget or misunderstand which edges must be glued together, resulting in anguish, frustration and an unsatisfactory outcome.

Geometry is an area of mathematics that appeals to many students, particularly visual thinkers. These students are often among those who either find math difficult or are disinterested in the subject. On the other hand, not all students who are good at traditional school mathematics are good at spatial work. This workshop (see [1] for further ideas) has been designed to promote the teaching of two- and three-dimensional geometry in ways that are visually stimulating for students of a wide range of abilities, between the ages of 10 and 16. The intention of this workshop is to present methods that will help engage more able students in an area of mathematics that they may find difficult and at the same time, encourage students who find traditional school mathematics difficult.

The aim of the workshop is to ensure that participants become familiar with the methods for making pull-up polyhedra and that this will act as a stimulus in their own teaching which, in turn, will allow them to introduce curriculum innovations which help to:

- increase students' motivation and enjoyment of mathematics
- explore the beauty of mathematics
- make cross-curricular connections between art and mathematics
- develop spatial awareness and visualization skills from two- to three-dimensions
- broaden students' creative outlook and develop practical skills

Under a teacher's supervision, students are fully capable of assembling the models. This visual study and physical manipulation of the nets increases a pupils understanding of space through the physical assembly and visualization required. The material presented within this workshop can be combined in the teaching of a larger cross-curricula project between art and mathematics departments, which could include subjects such as symmetry, pattern, tessellations, paper-folding and a study of Islamic geometric patterns. This workshop is currently delivered to schools alongside a current exhibition of tilings and polyhedra presented at the University of Leeds [2]. Participants in the workshop will have the opportunity to construct each of the five pull-up nets for the Platonic solids and discuss their applicability within a classroom setting.

The Development of Patterned Pull-up Nets

In 1994, mathematics educator Bob Vertes introduced Meenan to the idea of pull-up polyhedron nets. These nets could be created using only card and string (not a tab or glue stick in sight) and easily folded up into beautiful, three-dimensional shapes within the hand (see Figure 1). It was a "*mathemagical*" experience that Meenan wanted to share with others. Vertes had provided nets for the tetrahedron, cube and octahedron but Meenan was keen to develop the complete set for use as a classroom exercise. It took several years and the help of trainee teachers, Tom King and Jeff Zhao, to develop pull-up nets for the dodecahedron and icosahedron and thus to finally complete the set of pull-up nets for Platonic solids. The five pull-up nets for the Platonic solids are shown in Figure 2.

Figure 1: *A pull-up cube net showing the transition from two to three dimensions*

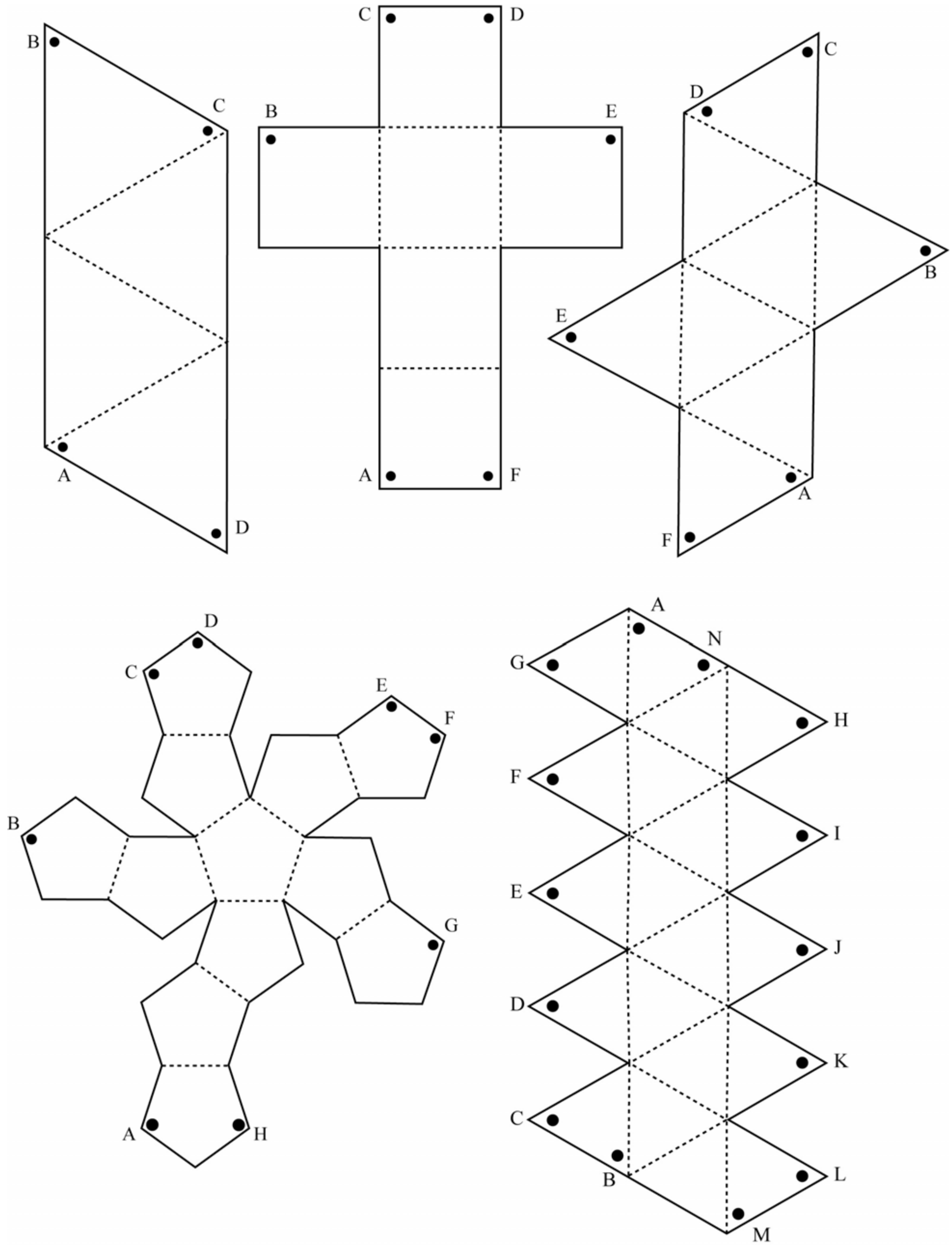

Figure 2: *Pull-up nets for a) the tetrahedron, b) the cube, c) the octahedron, d) the dodecahedron and e) the icosahedron*

In October 2007, Meenan visited *'Form, Shape and Space: An Exhibition of Tilings and Polyhedra'* at the University of Leeds International Textiles Archive (ULITA) [2]. The exhibition featured the Platonic solids regularly patterned with designs inspired by those found at the Alhambra Palace in Granada, Spain. The beauty of the solids with their colorful, intricate, symmetrical patterns struck Meenan as an ideal extension to the original pull-up nets. The research undertaken by Thomas and Hann [4, 5] identified the different pattern classes that are capable of repeating regularly across the faces of the Platonic solids. This knowledge offers immense creative potential in the designs used to pattern the faces of the polyhedra.

The collaboration between Meenan and Thomas led to the successful development of a series of workshops based around the *'Form, Shape and Space'* exhibition. Participating school groups were able to view the tiling panels and patterned polyhedra on display and create their own polyhedra, through construction of the patterned pull-up nets.

The pull-up nets have been adapted to allow them to be threaded through a base card. When a suitable pattern is applied to the base and card net, the transformation of the pattern from two- to three-dimensions is clearly evident. Workshop materials have been designed so that the pattern seems to naturally flow from the plane base onto the polyhedron. An example of a pull-up icosahedron, where the base card and the solid are tiled with pattern class *p6mm*, is shown in Figure 3. Examples of pull-up patterned nets for the Platonic solids are shown in Figure 4.

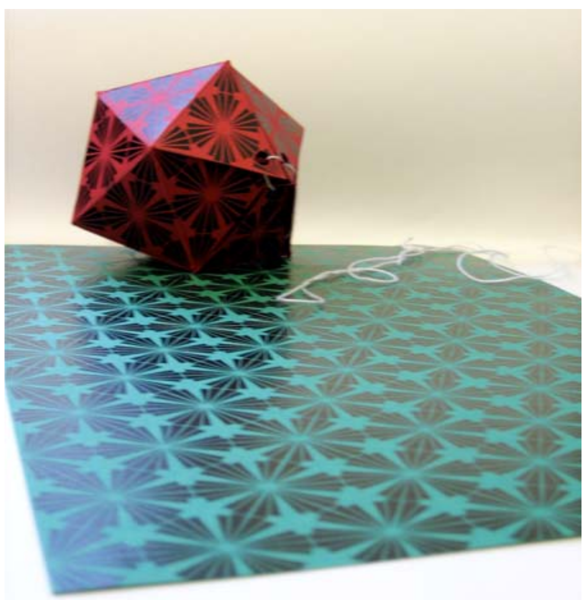

Figure 3: *A pull-up patterned net for the icosahedron on a similarly patterned p6mm base card*

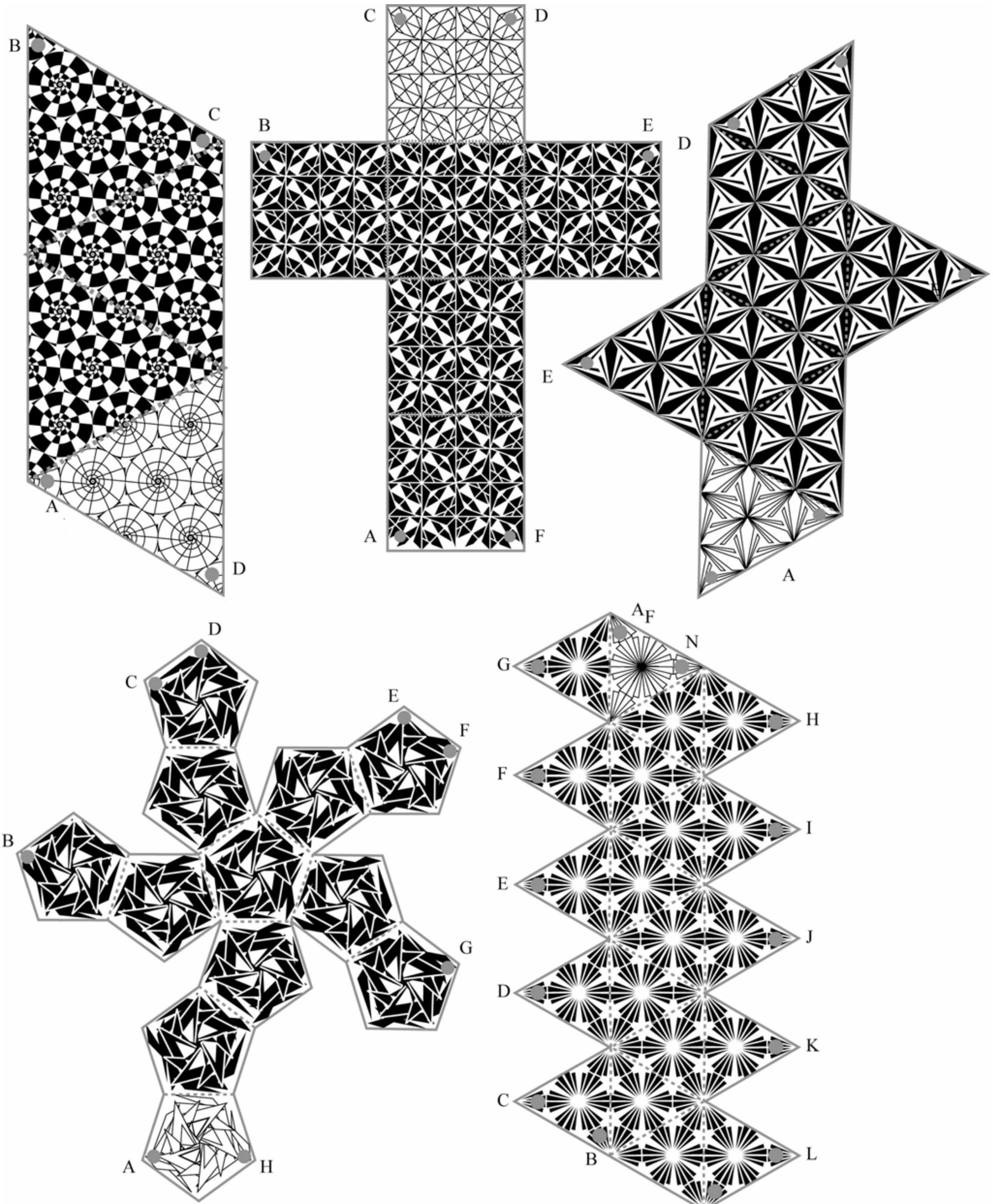

Figure 4: *Pull-up patterned nets for a) the tetrahedron, b) the cube, c) the octahedron, d) the dodecahedron and e) the icosahedron*

Making the Pull-up Polyhedra

Workshop participants will have the opportunity to construct all five Platonic solid pull-up nets. Participants may choose which nets to make and in which order. Previous experience suggests that the easiest to begin with is the tetrahedron, followed by the cube and the octahedron, and lastly the dodecahedron and the icosahedron. Sharp scissors and accurate cutting and scoring of the nets are crucial for well-made polyhedra.

Basic instructions:

1. Carefully cut out the net for your pull-up polyhedron.
2. Use a ruler and sharp point to score lightly along the remaining black lines.
3. Make holes at the points A, B, C, D, etc.
4. Thread and weave thin string or strong thread through the holes A, B, C, D, etc, to link the faces together.
5. Gently pull up the net to make your polyhedron.

Further Exploration

Is there only one pull-up net for each Platonic solid? A good starting point to explore this question is to consider the eleven distinct nets of a cube [3], illustrated in Figure 5.

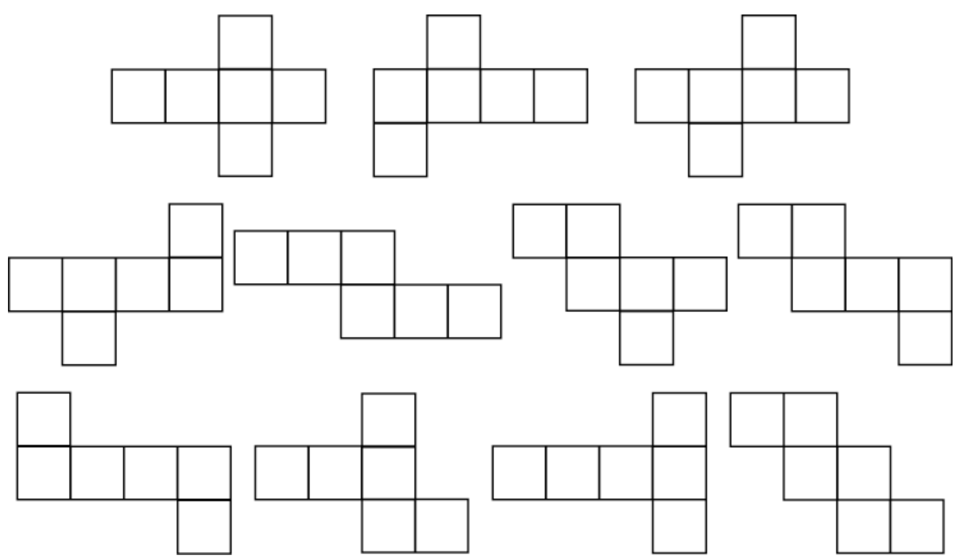

Figure 5: *The eleven distinct nets of a cube*

Further problems to consider include:

- Can any of the above nets be used as pull-up nets for a cube apart from the cross-type one which has already been used?
- How could the pull-up net for a cube be modified to make a pull-up net for a cuboid?
- What about other nets for a tetrahedron, octahedron etc?

- What about other polyhedra, do they have pull-up nets? Try making a pull-up net for a square-based pyramid and you'll see that other pyramids can be made by changing the number of triangles and the shape in the middle.
- What about prisms other than a cube and cuboid? See what changes have to be made to construct a pull-up net for a triangular prism. What about a pentagonal prism?

Summary

This paper presents a novel approach to the teaching of three-dimensional solids. Attention has been focused on the Platonic solids and the development of this cross-curricula, artistic and mathematical investigation of polyhedra is reviewed. The material presented in this paper forms a component of a larger workshop, which has been designed to promote the teaching of two- and three-dimensional geometry in a visually stimulating way. The material has been found to engage students of a wide range of abilities, and is aimed particularly at those between the ages of 10 and 16. The pull-up patterned nets are closely associated with the exhibition *'Form, Shape and Space'*, which has provided an important visual stimulus to pupils involved in the workshops. Figure 6 illustrates a completed pull-up net together with the associated patterned polyhedron from the exhibition. The octahedron patterned regularly with a class p31m-derived pattern is shown alongside the pull-up net and base card patterned with the same design. Pattern appears to flow from the plane to the faces of the polyhedron. Geometric connections are thus made, and this allows the student to cross the visual bridge between two- and three-dimensional space.

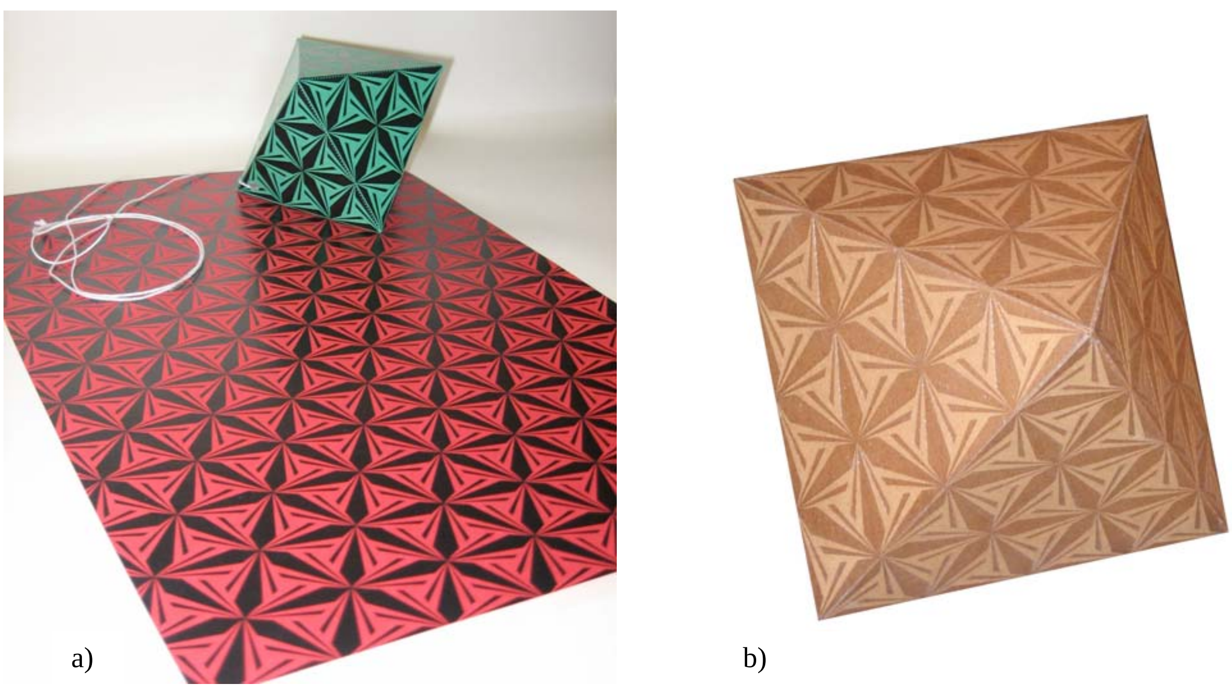

Figure 6: *The octahedron regularly patterned with class p31m presented as a) a pull-up net on patterned base card, and b) a model from the 'Form, Shape and Space' exhibition [2].*

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Acknowledgments

The authors wish to thank the NCETM, particularly Tony Shepherd, for their support and encouragement throughout this project. The authors also gratefully acknowledge Bob Vertes for initiating the investigation into pull-up nets for the classroom, a project that has developed over many years. The assistance of Tom King and Jeff Zhao is also recognized in the development of nets for the dodecahedron and icosahedron. Thanks are also due to Professor Michael Hann, Margaret Chalmers and Tony Smith at the University of Leeds International Textiles Archive, for their support and assistance with the exhibition and workshops.