“Gödel, Escher, Bach”, in other Eras

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Abstract

The imaginary interpretations of Douglas Hofstadter are tried out in different times, through a series of summaries. Admittedly, this paper does not present an original insight and is a mere reflective exercise. Similarities between mathematics, the visual arts and music are observed from the very beginnings in prehistoric times, Africa and Greece to the Middle Ages. From the Renaissance and the later golden age of mathematics, many more (familiar) examples could be given, but they are again summarized briefly, to reach the age of relativity and the era where mathematics and the arts seemed illusions. Two post “Gödel, Escher and Bach” examples, about modern times, conclude the summary.

Introduction

Douglas Hofstadter is known for his book “Gödel, Escher, Bach, an eternal golden braid” [12]. Acclaimed by many, such as the Pulitzer jury, it was criticized by others. The author even felt obliged to write a new preface in its recent edition. Hofstadter described similarities between the works of mathematician Gödel, graphical artist Escher, and musician Bach [21]. He stressed that Gödel worked on the impossibility of the verification of mathematical theorems within the mathematical framework. Next, Hofstadter recognized this paradox in the etchings of Escher, when, for instance, the artist made a drawing of his own hand, drawing itself. Bach wrote his name, Bach, in music staff code. Hofstadter’s opus contains many more elegant examples (or “fancy amusement”, in an antagonist’s eyes), such as the examination of how music can be played in reversed sequences, from end to the beginning.

All consistent axiomatic formulations of number theory contain non-decidable propositions.

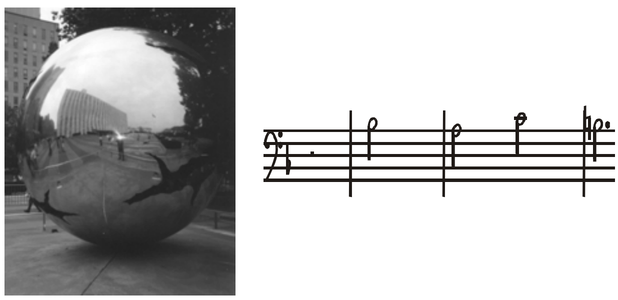


Figure 1: *Mathematical mathematics; an Escher type photo of a photographing author; Bach’s name, supposedly written on his dying bed.*

At other moments in time, and in other cultures, it is possible to observe similar relations between mathematics, the visual arts and music. Imagination and open mind are required when reading this paper, which should be considered more as an artist’s view than a scientific report. This pretext allows reducing an historical survey of mathematics, graphical art and music in the given 8 page format and allotted time slot.

The present goal is not to write an extensive summary or a philosophical essay. Others have sufficiently formulated brilliant thoughts about the subject [11]: *"The affinity [between mathematics and music] is thousands of years old. From the Pythagoreans to Aristoxenus to Boethius, philosophers have imagined music as a branch of mathematics. […] Painting, sculpture and architecture all have their mathematical components, yet there is no mystical (or statistical) connection. […] In brief, it's not that music is mathematical. It's that mathematics is musical."* On the other hand, it is hoped the conference talk will be enjoyable because of a juxtaposition of mathematical computations done on an overhead projector, combined with a projection of PowerPoint images of visual artworks and a simultaneous selection of short musical recordings.

The First Steps

It can be discussed what the very first steps were in mathematics, visual art and music. Some may object they only took of in recent times, since a few hundred years. Others may oppose the first steps are the most difficult ones and thus at least equally important. In any case, in the Lebombo Mountains on the border of South Africa, a 37000-years-old tally stick was found, with 29 clearly defined notches [2]. In art, a tiny 77000-years-old stone carved with parallel lines, found at the same site, is regarded as the oldest design object [5]. As for music, a Neanderthal cave site in northwestern Slovenia held a section of a 43000 to 82000 years old flute, suggesting it was the remnant of a "Neanderthal wind instrument" [16].

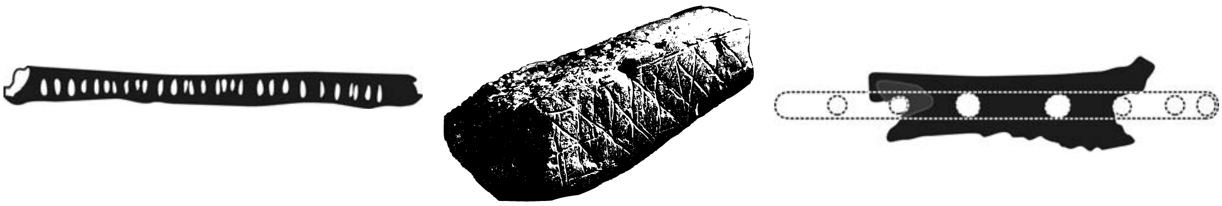


Figure 2: *29 notches on a bone; is it mathematics? Parallel lines on a tiny stone; is it design? 2 holes in a bone; a music instrument?*

The oldest artifacts with an undisputable mathematical significance are the Ishango rods (plural: a second bone was revealed on the dying bed of the discoverer). They date from about 22000 years ago [7, 18]. In art, the oldest example of a veritable so-called "key-pattern" is 25000 years old, and was found in Mezin, Ukraine [14]. Ancient aerophones are the six 9,000 years old flutes, found in an excavation at Jiahu in the Chinese Henan Province (Zhu Zhi, 1962) [7].

Figure 3: *The mathematical Ishango rods; the Mezin key-pattern; old Chinese music instruments.*

Mathematics, patterns and music developed in many African, Egyptian, Arab, Chinese and Indian cultures before they came to the West. In Rwanda for instance, counting methods went all the way up to words for 2 billion minus 1 [15]. The geometric drawings along the guidelines of King Kakira ka

Kimenyi were inspired by a desire for geometric purity [13]. Traditional percussion was not a horrendous banging (as some colonial observers wrote), but followed structured yet asymmetric patterns [17].

1 999 999 999
= urukwavu na impyisi cyenda na uduhumbagiza cyenda na uduhumbi cyenda na utuyovu cyenda na inzovu cyenda na ibihumbi cyenda na magana urwenda na mirongo urwenda na icyenda

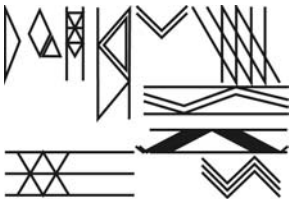
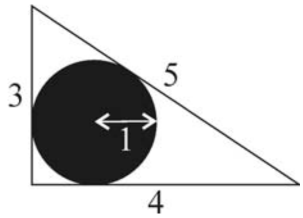
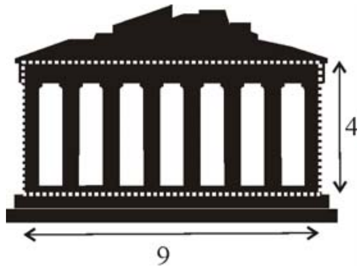


Figure 4: *Rwandan numbering, geometric patterns and drum configurations.*

From Greece to Gödel, Escher and Bach

No doubt, “mathematica” started in Greece and the Greek contributions are well known [20]. A quote from one of the numerous publications on the subject states that [10]: *“Greek philosophers saw a relationship between music and mathematics, envisioning music as a paradigm of harmonious order reflecting the cosmos and the human soul”*. Sometimes, Greek mathematical insights were even overestimated, for instance, when the golden ratio myth identified this proportion as the crucial key to comprehend Greek sculptures and architecture.





do	**1:1**
re	**9:8**
mi	**81:64**
fa	**4:3**
sol	**3:2**
la	**27:16**
si	**243:128**
do	**2:1**

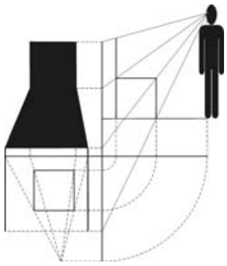
Figure 5: *The Pythagorean triangle; the Parthenon and the 9:4 ratio [3]; Pythagorean ratios.*

Scholars in the Middle Ages collected (and neglected) what was left from Greek models through different cultural heritages. “Mathematics” again turned out to be a set of rules and guidelines to solve practical problems. In architecture, simple construction rules were used, based on multiples of a local unit measure [22]. In music, the “Kyrie” of the “Messe de Nostre Dame” (1365) by Guillaume de Machaut can be constructed using but the simple proportions 1:2 en 2:3.

do	**1:1**
e	**9:8**
f	**32:27**
fis	**81:64**
g	**4:3**
a	**3:2**
b	**27:16**
c	**16:9**
cis	**243:128**
d’	**2:1**

Figure 6: *Fibonacci, author of Liber abaci (The Book of the Abacus – 1202); proportions in medieval buildings were based on the “local foot”; medieval tone system.*

Renaissance scholars returned to a more scientific approach. Rules for the construction of the Platonic solids were (re-) discovered, whereas Brunelleschi's rules explained graphical drawing. Though later in time, simplified "rationalized" ratios found their way in music as well.



do	**1:1**
re	**9:8**
mi	**5:4**
fa	**4:3**
sol	**3:2**
la	**5:3**
si	**15:8**
do	**2:1**

Figure 7: *Luca Pacioli, De Divina Proportione (1495); rules of perspective; simplified ratios.*

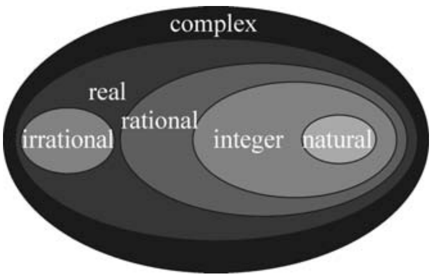
In the golden age of Newton, Leibnitz and their followers, the universe became a mechanical device obeying mathematical rules [4]. Visual arts achieved a faultless representation of reality, finalizing in styles such as realism and naturalism. In the 18th century, musicians discovered the twelfth root of 2, or 1.05946…, could represent the music scale: its square, 1.1224…, corresponds to re, its fourth power to mi, etc. By the end of the 19th century, mathematics, the visual arts and music accomplished an image of a universe supposedly fully understood by man.

$$-1 = e^{i\pi}$$

do	**1**
re	**1.122…=$2^{2/12}$**
mi	**1.259…=$2^{4/12}$**
fa	**1.334…=$2^{5/12}$**
sol	**1.498…=$2^{7/12}$**
la	**1.681…=$2^{9/12}$**
ti (si)	**1.887…=$2^{11/12}$**
do'	**2**

Figure 8: *Euler related famous mathematical constants; Jean-François Millet ("The Sowers" - 1857) applied the painting skills known before photography; the relation between tones turned out to be $2^{1/12}$.*

Now that the branches of the mechanical universe flourished, the three fields turned to the roots of their well-established view of the world. Cantor made mathematicians consider their numbers as "elements of sets" in an attempt to discover a definition for what numbers are. Pointillist painters thought about what a painting is, using small distinct points of primary colors. Music seemed to be nothing more than a game with physical phenomenon, i. e. sound waves with a given frequency.



do	**261.63 Hz**
re	**293.66 Hz**
mi	**329.63 Hz**
fa	**349.23 Hz**
sol	**392 Hz**
la	**440 Hz**
ti (si)	**493.88 Hz**
do'	**523.25 Hz**

Figure 9: *Set theory: numbers are but "elements"; a personal Seurat-type "drawing": paintings are but dots; a unit, "Hertz", for sound waves: music is but frequency.*

At the turn of the 19th and 20th centuries, several flaws began to disintegrate the established view. Cubism illustrated the painter himself influences the representation of “his” reality and some even dare to pretend it preceded relativity. However, the latter came to that similar conclusion through more involved reasoning, such as transformation formulas for coordinate systems, in movement with respect to each other. In music, Hermann Helmholtz (1821-1894) studied what “consonance” is based on. Jean-Philippe Rameau (“Traité de l’harmonie réduite à ses principes naturels", 1722) and Georg Andreas Sorge ("Vorgemach der musikalischen Composition", 1745) had already understood that the notion was related to the relative distance of the “partials” in the overall sound. However, Helmholtz was the first to investigate in physiological aspects of the ear ("Die Lehre von den Tonemfindungen", 1862). He showed dissonance has nothing to do with the interference in the air, but with what happens in the ear of the observer. Consonance became a consequence of overlapping overtones, in the ear of the observer. Composer Gérard Grisey, who used this theory, became the founder of “spectralism”.

$$\mathbf{v} = \frac{\mathbf{v}_1 + \mathbf{v}_2}{\sqrt{1 - (\mathbf{v}_1 \mathbf{v}_2)^2 / \mathbf{c}^2}}$$

Figure 10: *Adding velocities following relativity; cubism (Picasso); Gérard Grisey.*

Perception of sound and vision became a special topic in music and art. Sound effects and optical illusions competed in creativity. Admittedly, it takes more imagination to associate a mathematical equivalent to this framework. We opt for theories about “perception” of reasoning, such as Russell’s work. His formalism gave the impression of being completely correct, though it turned out differently, just like the correct paintings or music, which seem to turn into non-existing yet perceived observations.

$$\mathbf{M} = \{\mathbf{A} | \mathbf{A} \notin \mathbf{A}\}$$

Figure 11: *Russell’s set was an illusion of a set; optical illusion by Vasarelly (1910); a sound illusion: (1) would be perceived as (2) [9].*

Beyond Gödel, Escher and Bach

Quantum physics introduced probability in the (physicist’s) mechanical world of atoms. Jackson Pollock’s art was seen, by some, as fractal based art. Others, such as art historian Tsion Avital [1], could prove a probabilistically justified random process applied on The Dripper’s paintings does not change much. As for the musical equivalent, we turn to Iannis Xenakis, the close collaborator of Le Corbusier’s who, in fact, built the pavilion of Philips for the world exposition of 1958 in Brussels. Later, Xenakis used more mathematical inspiration for his composition “Metastasis”, but his piece “Achorripsis", as well as other later works are composed stochastically. He used an analogy with statistical models for the behavior of gas. In “Herma” he even created abstract “stochastic clouds of sound". The tones get an arbitrary intensity and duration, though they still are distinguishable.

$$i\hbar \frac{d}{dt}\psi(t) = H(\psi(t))$$

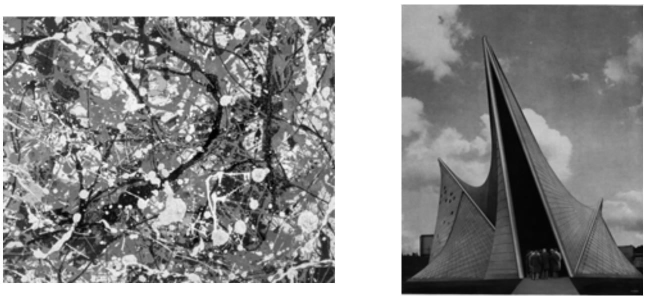


Figure 12: *Schrödinger's equation for the state of a system at time; a personal Pollock-type "artwork"; Xenakis's musical pavilion.*

Mathematics, (visual) arts and music later all developed in various disperse directions, though many subfields again seemed to meet. Finite geometry, about planes and spaces with only a finite number of points and lines is an important mathematical subfield. The minimal art movement explored similar "lower bounds" in art, and it is no surprise scientists became particularly fascinated by this style. Musically, minimal music seems the straightforward equivalent, but we prefer to turn to an example from the componist Jan van Vlijmen, used in his work "Monumentum" (1998). The example is more graphical and thus easier to explain in a printed text: tone series in dodecaphonic music can be represented by clock calculation modulo 12 and thus provide a better insight in the structure of the music (see figure 13).

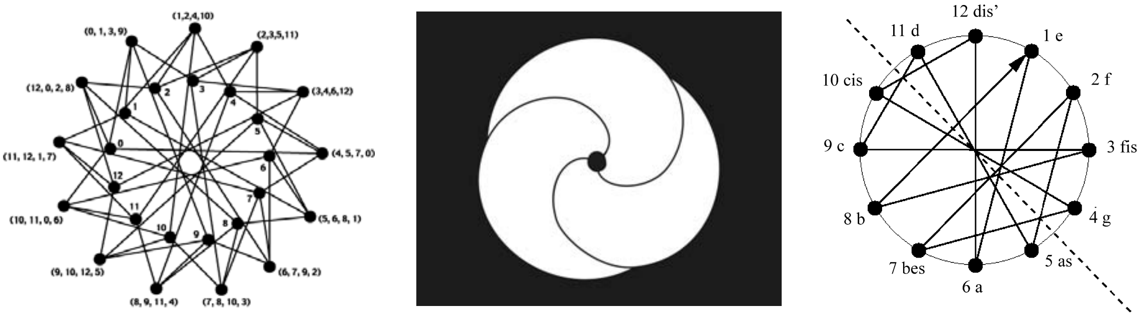


Figure 13: *An illustration for a finite geometry course (Noelle Evans [6]); "Minimal Saddle Trefoil" (Carlo Sequin [19]); clock representation of a twelve-tone system: in this example, the series e - a - dis - cis - g - bes - f - as - d - c - fis - b has a symmetry, meaning the fragments e - a - dis - cis - g – bes and f - as - d - c - fis - b are mirrored.*

Of course, Gödel, Escher and Bach did not really fit in between Russell and Schrödinger, as Bach's music is from a much earlier epoch. As for summarizing similarities in mathematics, visual art and music of the most modern times, we hope to elaborate this exercise in a future paper.

References

Note: images are personal "creative" interpretations. They were an attempt to streamline the format. Another reason is that it exempts the paper from copyright problems. When such a "creative" interpretation was too challenging, images on stamps were used, as they involve no special copyright authorizations.

Information about the history of music was obtained in collaboration with Derk Pik, Leiden University.

The list of references is available on http://etopia.sintlucas.be/3.14/ .