From Folding and Cutting to Geometry and Algorithms: Integrating Islamic Art into the Mathematics Curriculum

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Abstract

Drawing upon visual forms of expression prevalent in Islamic arts and architecture, this workshop offers hands-on experience for understanding basic concepts in geometry, with reference to algorithms in processes of pattern-making. Art teachers and math educators may learn a variety of strategies for classroom teaching, adaptable for K-12, to acquaint students with principles underlying patterns in Islamic art. Such patterns relate to the history of mathematics at a time when Baghdad was an intellectually vibrant center of patronage, and al-Khwarezmi was engaged in the development of what we now call algebra and algorithms. Students may explore these ideas through their own experimentation, and relate their experiences to means of transmission of newly emergent mathematical ideas in the 9^{th} and 10^{th} centuries of our era.

Introduction

By the year 1000 Arab and Muslim mathematicians had advanced the understanding of geometry they inherited from the Greeks, and they had adopted Hindu numerals and methods of calculation for mathematical applications in daily life. Ongoing work led to new theoretical developments in the understanding of mathematics that continue to have an impact on the study of mathematics today. Al-Khwarezmi introduced algebra and articulated an emergent understanding of what came to be called algorithms in the West. This presentation explores the visual expression of these principles in Islamic art and architectural ornament, offering a program of activities that may be used to engage students in understanding mathematics and mathematical aspects of Islamic art, as well as to develop an appreciation for the history of mathematics and the history of art.

My goal for this workshop is to promote the incorporation of explorations of Islamic works of art into the K-12 school curriculum to link the study of mathematics and arts with Islamic cultural traditions. The series of exercises can be adapted for use in college courses, and public programs for out-of-school adults. My work has led to several collaborations and collaborative research projects among mathematicians, scientists, historians of art, artists and designers, and educators [1,2,3,4].

First Exercise – Folding Squares

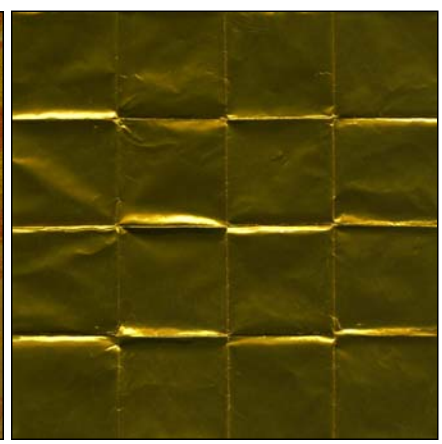

Figure 1 (left). *Two ways of folding the square in half; there are also other ways of folding it in half.*
Figure 2 (center). *Square folded into sixteen parts, each a squares 1/16 the size of the original square.*

The first exercise involves the folding of a square of paper. Students are asked to fold the square in half. Generally, some students fold the square into a rectangle; others fold it along the diagonal. This leads to a discussion of area – which is the larger half? Which has the longer perimeter? It also leads to discussions of shapes and unit lengths. What is the length of one side of the square? What is the length of the diagonal? What about the long side of the rectangle? How long is the short side of the rectangle? The students are then asked to fold the square in half again, and again, and again. What can they observe about what happens when the square is thus folded?

Together we arrive at an understanding that halving relates to establishing an equation such that ½ x ½ x ½ = 1/8, and folding it in half again results in a series of squares each 1/16 the size of the original square. What other number sequences may be observed? The unfolded square contains sixteen squares, arranged four by four. This then can be seen as a graphic model of all of the arithmetic functions (addition, subtraction, multiplication, division), halving, doubling, squares and square roots. There is also a series of odd numbers (1, 3, 5, 7, 9) by counting the squares around each successive square or cluster of adjacent squares, starting from one corner. For students (and teachers) familiar with mathematics as a symbolic language, it is often quite a surprise to work out mathematical understandings at this level of presence, without numbers (as abstractions), without symbols, without equations, and without representation! The opportunity is also present to engage the students in approaching visual proofs.

After this exercise, I ask students to identify and articulate in words what we have done. The answers may include the following ideas: we have folded paper; we have repeated a process; we have engaged in an algorithm (a one-step algorithm, and a two-step algorithm); we have made a series of squares; we have made a series of triangles, etc. I usually press for more information and ask about the shape and form of our original square – a plane. What have we done to the plane? Is what we created still two-dimensional? Well, sort of, but we have deformed the plane. But it is not exactly three-dimensional. We begin to address questions of interdimensionality, which can lead to a discussion of fractals, too.

The process of inquiry to this point is then related to images of Islamic art in which a unit is repeated to cover the plane (figs. 3, 4). We have created a tessellation, a term defined as a pattern created by a single unit repeated to cover the plane with no gaps and no overlaps. The question of dimensionality is again raised, and related to interlace in Islamic art – in weaving it is actual, whereas when it is represented, as in ceramics or leatherwork, it may be illusionary. What is two-dimensional? What is three-dimensional? What lies somewhere in between?

To conclude this exercise, the students are asked why I may have chosen to use gold or silver foil, and the students begin to recognize that we have also reflected light differently with each square or triangle; we have begun to play with the effects of light on the deformed plane. There is a dynamic quality to the process as we divide the plane and examine the effects of light. Examples of Islamic art in which play with reflected light is an artistic choice are then illustrated (fig. 3) [5-15] (web resources, below).

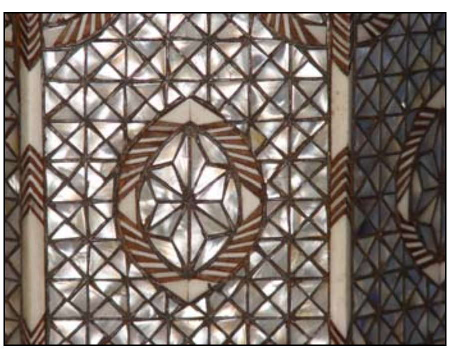

Figure 3 (left). *Tabouret set with mother-of-pearl to reflect light; collection of Doris Duke at Shangri La, Honolulu. Photograph by the author.*

Figure 4 (right). *Objects of Islamic art, author's teaching collection.*

Second Exercise – Cutting Circles

The second exercise is actually a pair of exercises, based upon the work of Jane Norman and Margit Echols. I first attended a workshop with Margit Echols at Nat Friedman's Art and Math Conference in Albany in 1996. There, she presented this exercise in relation to the practice of quilting, but she had already identified processes of making quilt patterns that she related to understandings of the repetition of forms in Islamic art.

These exercises involve the folding and cutting of circles. In order to make a circle, I have found that lids from yogurt containers, or used CDs, make excellent templates. Patty paper offers a medium that is translucent, and the fold lines are clearly visible in both reflected and transmitted light. Each student is asked to prepare two circles. Invariably, audiences no matter the age, often have difficulty following the instructions for these two exercises. Folding and cutting, as in the first exercise above, do not yield intuitive understandings of what the activities involve. Rarely do students understand the geometry in which they are engaged, nor are they familiar with means of establishing 30 degree angles, without the use of any measuring devices, nor even a straight edge. I might first ask how to establish the center of the circle. If no one suggests folding the circle in half and in half again, I might ask how we can establish a diameter. For young students, diameter, radius and circumference might be new vocabulary, requiring definitions. There are infinite possibilities for diameters of the circle, and they all meet at the center, where they intersect. Asking the students to fold a quadrant might also introduce new vocabulary. Again, folding in half and in half again yields ¼. To fold a quadrant in thirds, one can "guess and check," but this does not establish a construction. To hold one point fixed and another moveable, as the drawn instructions suggest, effects a geometric construction.

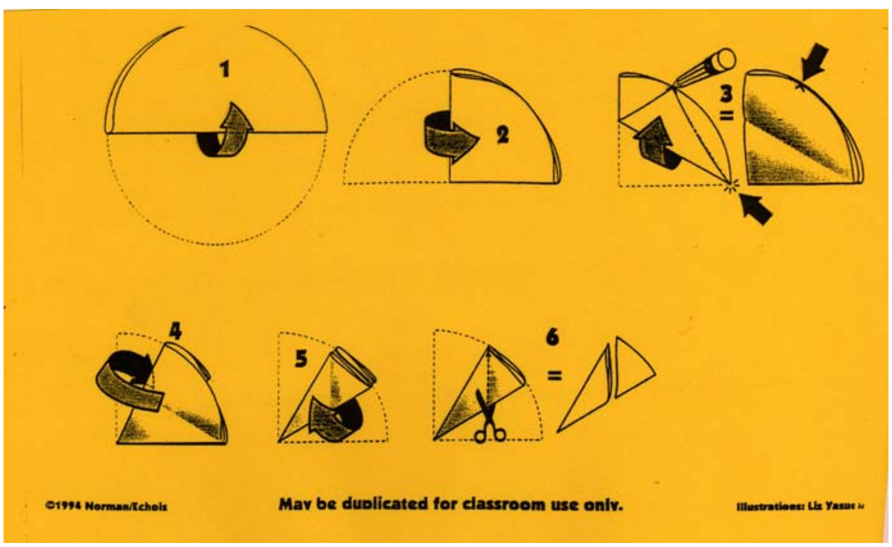

Figure 4. *Drawn instructions for cutting and folding a circle to form a six-pointed star. (c) Norman/Echols. May be duplicated for classroom use only.*

Once the folding is complete, a single cut along a fold line results in a six-pointed star. This is usually quite a surprise to the students, whether children or adults, even for adults with considerable mathematical training! The teacher may ask for explanations as to why and how a six-pointed star resulted from the steps in this exercise, and may emphasize that this achievement required no formal means of measuring or dividing, nor the use of any numbers.

Doris Schattschneider has offered a proof for the construction of a 30 degree angle by this method:

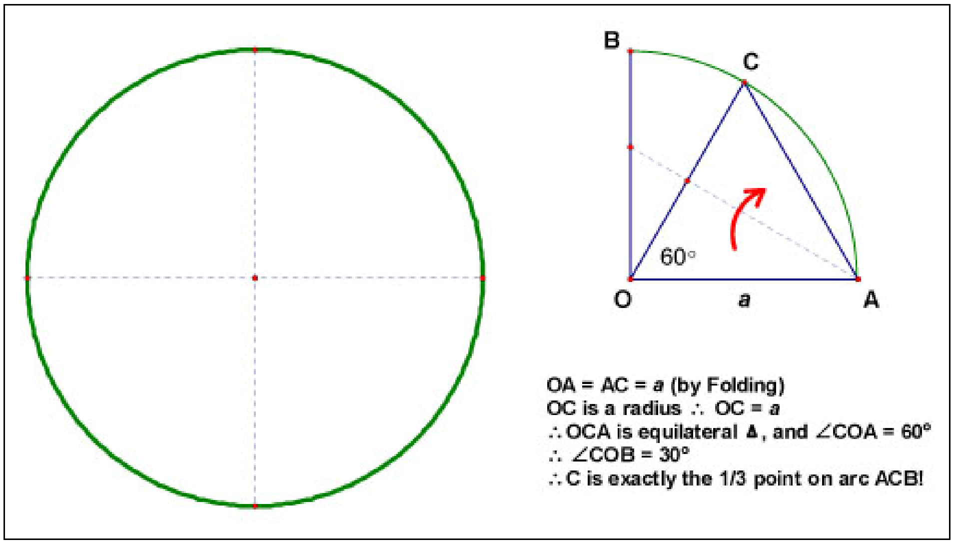

Figure 5. *Proof by Doris Schattschneider, demonstrating how this folding exercise yields a 12-fold division of the circle, resulting in a 6-pointed star.*

In a group situation, the six-pointed stars may then be gathered and tight-packed to form a variety of arrangements. The notion of symmetry may then be introduced and discussed. What are the symmetries of a six-pointed star? How many axes of reflection? What is the order of rotation? How is this affected locally and globally by the different arrangements of stars when tight-packed?

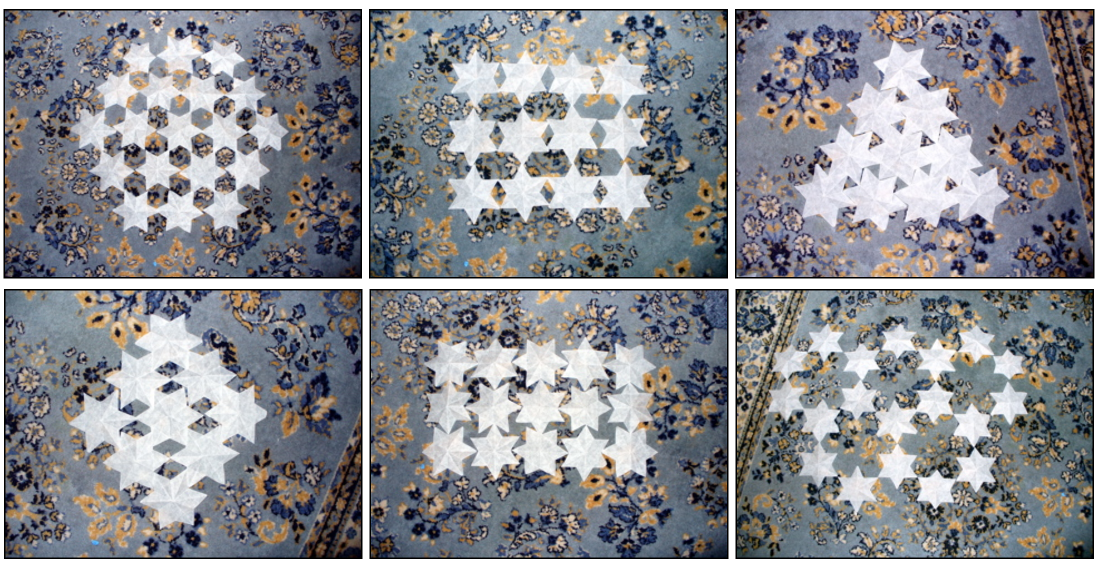

Figure 6. *Six-pointed stars tight-packed in several ways to create semiregular tessellations. Star arrangements and photographs by Aviana Edwards.*

The stars may also be affixed to a window so that the light comes through the negative spaces between the arts, creating interesting visual effects. Again, the relationships to Islamic arts and architecture may be explored.

The same exercise may be conducted to produce eight-pointed stars, adjusting the folding and cutting, according to these drawn instructions.

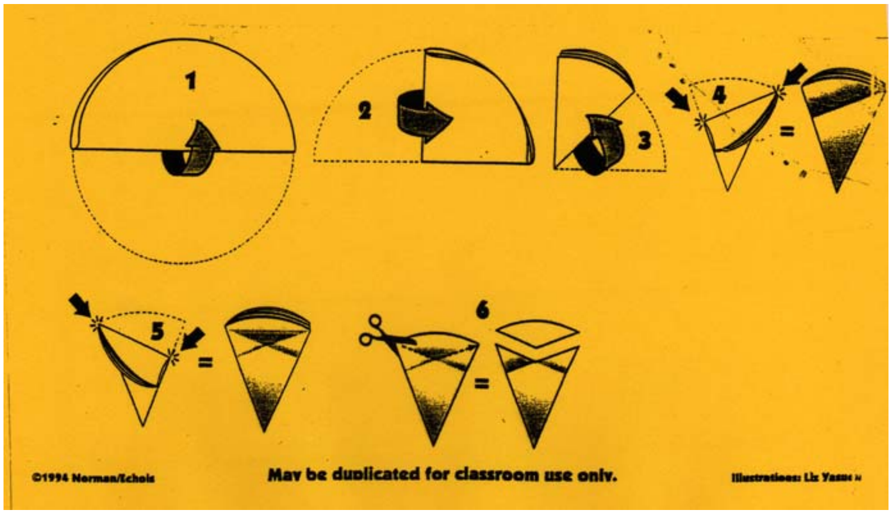

Figure 6. *Drawn instructions for cutting and folding a circle to form an eight-pointed star. (c) Norman/Echols. May be duplicated for classroom use only.*

Islamic Art in Museums – Collections and Exhibitions On-Line (not a comprehensive list)

http://www.ee.bilkent.edu.tr/~history/Ext/palace.html Topkapi Palace, Istanbul

http://www.shangrilahawaii.org/ Doris Duke's Shangri La, Honolulu

http://www.lacma.org/islamic_art/islamic.htm Los Angeles County Museum of Art

http://www.asia.si.edu/collections/islamicHome.htm Freer Gallery of Art/Arthur M. Sackler Gallery

http://www.lacma.org/khan/index_flash.htm "Courtly Art and Culture in Western Asia"

http://www.metmuseum.org/explore/Flowers/HTM/INDEX.HTM Indian Carpets of the Mughal Era

http://www.metmuseum.org/toah/hd/orna/hd_orna.htm Timeline of Art History: Nature of Islamic Art

http://www.dia.org/collections/ancient/islamicart/islamicart.html The Detroit Institute of Arts

http://www.nga.gov/exhibitions/islamicinfo.shtm Islamic Art from the Victoria & Albert Museum

http://www.textilemuseum.org/fsg/ Flowers of Silk and Gold: Four Centuries of Ottoman Embroidery

http://www.metmuseum.org/Works_Of_Art/department.asp?dep=14 The Metropolitan Museum of Art

http://www.hermitagemuseum.org/html_En/03/hm3_5_5_4.html Hermitage, Petersburg, Russia

http://vam.ac.uk/collections/asia/islamic Victoria & Albert Museum – Islamic Collections

Other On-Line Resources for Teaching about Geometry and Islamic Art

ArchNet http://archnet.org is an international online community for architects, planners, urban designers, landscape architects, and scholars, with a focus on Muslim cultures and civilizations.

Carol Bier is an historian of Islamic art who served as Curator for Eastern Hemisphere Collections at The Textile Museum in Washington, DC (1984-2001); she has taught "Pattern in Islamic Art," "Plato, Euclid and the Arabs," "Plato, Geometry and Islamic Art," "Islamic Ornament: Forms and Meanings," and "Sufism, Spirituality, and Science."

Symmetry and Pattern: Art of the Oriental Carpet http://mathforum.org/geometry/rugs/ (The Math Forum @ Drexel University, Philadelphia, ©1996-2005)

Student Practicums in Islamic Art: Maryland Institute College of Art
http://mathforum.org/geometry/rugs/resources/practicums/ (The Math Froum @ Drexel University, Philadelphia © 2005)

Activities for Students http://mathforum.org/geometry/rugs/resources/act/

Fourth Grade at Park School: Student Gallery
http://mathforum.org/geometry/rugs/students/parkschool/

Eric Broug http://www.broug.com/courses.html Broug Ateliers, London, is an artist practitioner who bases his work on traditional Islamic geometric design. His website offers sample lesson plans for classroom use.

Jay Bonner http://www.bonner-design.com/ is an internationally recognized designer who studied with Keith Critchlow in London, UK. His work focuses on arabesques and geometric patterns; some of his designs were prepared for the restoration of the Prophet's tomb in Madina.

Craig S. Kaplan http://www.cgl.uwaterloo.ca/~csk/washington/taprats/ teaches mathematics and computer science at the University of Waterloo in Ontario. He has developed Taprats, a Java applet program that enables computerized generation of Islamic star patterns and offers the source code with Java applet for use.

Mamoun Sakkal www.sakkal.com is a contemporary artist whose work expresses many Islamic themes through geometry and calligraphy, combing the content of writing with its form. He teaches about computational geometry in Islamic architecture at the University of Washington in Seattle.

Mohamed Zakariya www.zakariya.net is an Islamic calligrapher whose works are displayed in museums around the world, including Asian Civilizations Museum in Singapore. With an *ijzazet* (diploma) in calligraphy from Istanbul, Turkey, Zakariya is an American-born Muslim, who has achieved international recognition for his work. He has received numerous commissions for his work in Arab countries.

References

[1] Bier, C. 2007. Patterns in Time and Space: Technologies of Transfer and the Cultural Transmission of Mathematical Knowledge across the Indian Ocean, *Ars Orientalis*, v. 34, pp. 174-96.

[2] Bier, C. 2006. Number, Shape and the Nature of Space: An Inquiry into the Meaning of Geometry in Islamic Art, *How to Talk about Religion*, ed. J. B. White, pp. 246-77. Erasmus Institute Publications, University of Notre Dame Press, South Bend, IN.

[3] Bier, C.2006. Islamic Art: An Exploration of Pattern, *Bridges: Mathematical Connections in Art, Music, and Science*, ed. R. Sarhangi and J. Sharp, pp. 525 – 32. Tarquin Publications, London.

[4] Bier, C. 2004. Islamic Art at Doris Duke's Shangri La: Playing with Form and Pattern (with D. Masunaga), *Bridges: Mathematical Connections in Art, Music, and Science*, ed. R. Sarhangi and C. Sequin, pp. 251-58. Winfield, KS.

[5] Blair, S. and J. Bloom 1994. *Art and Architecture of Islam, 1250-1800*. Yale University Press, New Haven.

[6] Bourgoin, J. 1973. *Arabic Geometrical Pattern and Design*. Dover, New York.

[7] Castéra, J.-M. 1999. *Arabesques: Decorative Art in Morocco*. ACR Éd. Internationale, Paris.

[8] Critchlow, K. 1976. *Islamic Patterns*. Thames & Hudson, London.

[9] Ettinghausen, R. O. Grabar and M. Jenkins-Madina 2001. *Islamic Art and Architecture, 650-1250*, 2nd ed. (Yale University, New Haven).

[10] Frishman, M. and H. Khan, eds. 1994. *The Mosque: History, Architectural Development & Regional Diversity*. Thames and Hudson, London.

[11] Gerdes, P. 1999. *Geometry from Africa*. Math Association of America.

[12] Jones, O. 1868. *The Grammar of Ornament*. Bernard Quaritch, London [1856].

[13] Necipoglu, G. 1995. *The Topkapi Scroll --Geometry and Ornament in Islamic Architecture*. Getty Trust Publications, Santa Monica, CA.

[14] el-Said, I. 1993. *Islamic Art and Architecture: The System of Geometric Design*. Ed. T. El-Bouri. Garnet Publishing.

[15] el-Said, I. and A. Parman 1988. *Geometric Concepts in Islamic Art*. Repr.[1976]. Scorpion.