Math/Art Projects

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Abstract

In this hands-on workshop, participants will use mathematical concepts as a framework for creating their own math/art projects. In one project, participants will use modular arithmetic addition and multiplication tables for the underlying structure of project. Another project uses translations and reflections on a square grid and circular grid to create an anamorphic art picture. Handouts for the projects will be provided so that participants can use the ideas in their own classrooms.

In this hands-on workshop, participants will make two math/art projects, which they can use in their own classes. The first project uses modular arithmetic or clock arithmetic. Modular arithmetic is a system of arithmetic for integers, where numbers wrap around after they reach a certain value, which is called the modulus. Carl Friedrich Gauss wrote about modular arithmetic in his book, *Disquisitiones Arithmeticae*, published in 1801. Modular arithmetic is used in number theory, cryptography, computer science, music, and in the visual arts.

In modular or clock arithmetic, we work with a finite set of numbers determined by the modulo. In modulo three (or mod3), we use the numbers 0, 1, and 2. The basic operations of addition, subtraction, multiplication and division can be illustrated on a clock, as in figure1.

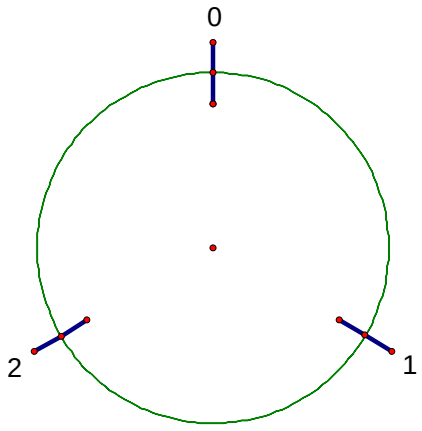

Figure 1: *Mod 3 clock*

The hand starts at 0,1,or 2 and moves around the face of the clock. The direction the hand moves is determined by the basic operation and the number of spaces is determined by the numbers used. The number on which the hand stops is the answer. To do the computation of 1 + 2 in mod 3, the hand would start at 1 and move two spaces in a clockwise direction and stop on 0. Thus 1 + 2 = 0 in mod 3.

Another way to think about modular arithmetic is by using different number bases. In the base ten-system number 123, the 1 means one group of 100 or 10^2, the 2 means two groups of ten and the 3means three groups of 1. However, the base four numbers 123, the 1 means one group of 16 or 4^2, the 2 means two groups of 4 and the 3 means three groups of 1. Other number bases can also be used for this type of art project. The first step in creating the math/art work is to create a mod 4 addition table or a base four addition table, for example. In the addition table only the last digit of the sum is written so that the sum of 2 plus 2 is written as just 0 instead of 10_4. In the artwork, only part of the addition table will be used (see [1]). A four by four square section of the table will be used that shows how to add 0,1,2,3 to 0,1,2,3. This section of the addition table will then be copied onto graph paper and through either translations or reflections; a larger pattern (8 by 8) will emerge. Colors or patterns will be assigned to each number and then put in the appropriate square. A variety of designs can be created in a similar manner by using different number bases, multiplication tables and grids to create the framework for the design.

The next project uses anamorphosis. The word anamorphosis is from the Greek ana (again) and morphe (form) (see [2]). An anamorphosis is a deformed image that appears in its true shape when viewed in some unconventional way. In other words, the images are so distorted that it is unclear what the image is unless viewed from a certain point or reflected from a curved mirror, or viewed through a polyhedron. The type of anamorphic image which we will create in this workshop can be seen clearly when viewed in a shiny curved cylinder sometimes called an anamorphoscope which "forms again" the image so it is recognizable. European painters of the early Renaissance were interested in anamorphic images. In one of Leonardo da Vinci's notebooks is one of the first known examples. Hans Holbein's *The Ambassadors* (1533) and an old newspaper puzzle by Sam Lloyd of George Washington contain anamorphic art pictures. Anamorphic paintings were popular in the 17^{th} and 18^{th} century in Europe and the Orient. Sometimes these pictures carried secret messages of political protest or erotic imagery. Salvador Dali created a set of erotic anamorphic art paintings. Anamorphic art was also used as a picture/puzzle for children's toys in the nineteenth century. The hidden picture could only be seen when viewed through the magic mirror (a shiny cylindrical tube), which transformed the distorted image into a colorful picture.

The anamorphic art picture, which we will create, is drawn on a semi-circular grid, which makes the real image look distorted (see [3]). In order to make an anamorphic picture, one first has to understand how to reflect a geometric shape on to a square grid. In this workshop, a right triangle in the first quadrant of the Cartesian coordinate plane will be reflected over the X-axis and then the Y-axis. The next step is to practice drawing the right triangle on some distorted grids and then draw a cartoon character on a distorted grids. The last step of this project is to draw your own picture on a square grid. Then translate your picture onto the semi-circular grid to form an anamorphic art image which can be seen clearly when viewed in the shiny cylinder.

References

[1] Bezuska, Kenney and Silvey, *Designs from Mathematical Patterns*, Dale Seymour Publications, 1990, pg 30 – 46

[2] Mathartfun.com

[3] Boast, Steve, *The Mathematics Teacher*, January, 1998

[4] McLaughlin Brothers, *The Magic Mirror: an antique optical toy*, Dover Publications, 1979

[5] Gardner, Martin, *Anamorphic Art,* Scientific America, vol 232, no 1, pp110-116, January, 1975

[6] J.L.Hunt, B.G.Nickel, and C. Gigault, *Anamorphic Images,* American Journal of Physics, vol 68, part 3, pp 232-237, March, 2000