Modeling High Genus Sculptures Using Multi-Connected Handles and Holes

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Abstract

In this paper, we present the concept of a *multi-connected handle* operation to create complex high genus virtual sculptures. We have developed and implemented a simple procedure to create handles that connect a set of user-selected faces in 3D. To create multi-connected handles, we first create a convex connector mesh. We then connect each selected face to this connector surface with a simple one segment handle. If the connector mesh is completely exterior to the original mesh, we get a multi-connected handle. If it lies inside of the original mesh we get a multi-connected hole.

1 Introduction and Motivation

Sculptures with intricate networks of handles and holes are aesthetically pleasing to the human eye, particularly when the overall sculpture exhibits one or more symmetries. The complex connectivities in the sculpture exhibit a mysterious beauty which draws the attention of the viewer. We refer to sculptures with multiple holes and handles as high-genus sculptures. With the development of new mathematical and computational techniques, we have started to see more and more high genus physical sculptures. Some of the most well-known examples of such high-genus sculptures include Bathsheba Grossman's 3D metal printed sculptures [9], Brent Collins's saddle sculptures [17], Charles Perry's aluminum and bronze sculptures [14], Helaman Ferguson's bronze and marble sculptures [6, 7], Carlo Séquin's 3D printed sculptures [17], and Rinus Roelofs' [15] and George Hart's [10] puzzle like sculptures.

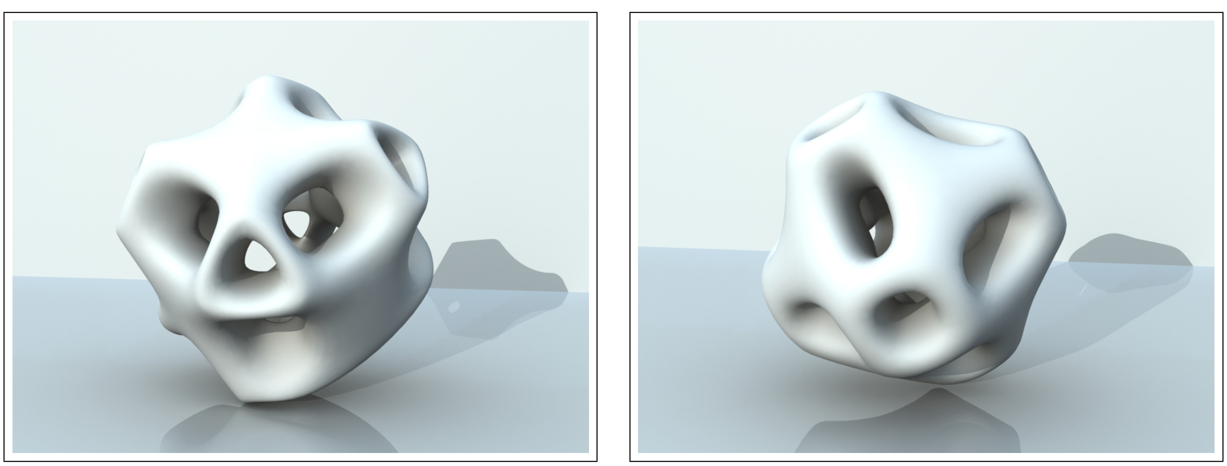

Figure 1: Hernan Molina's multi-connected handle sculptures.

Akleman and Chen recently showed that topologically there are a wide variety of high-genus surfaces and construction methodologies [4, 5]. However, their topological treatment does not take the geometry of the mesh into consideration. Akleman, Srinivasan, Mandal and Chen introduced geometrical approaches to create high-genus surfaces such as rind modeling [3, 19], multi-segment curved handles [18, 19], wire modeling [12] and column modeling [13, 20]. Such methods are based on creating handles between two faces. The simplest of all handles is a single edge that connects two different faces [1, 2]. Using multiple edge insertion operations we can obtain any type of handle. For a theoretical treatment of the variety of handles, we refer the reader to [5]. We should note that topologically there is no difference between opening holes and creating handles [2]. Holes are merely handles that go through the inside of the surface or solid shape.

In this paper we present a method to create handles between multiple faces. We refer to such handles as *multi-connected handles* or *n-connected handles*, where n refers to the number of faces that the handle connects. When the handle passes through the interior of the object, we get a *multi-connected hole*.

Differentiating between holes and handles is useful for visualizing the geometric impact of the operation in our minds. For instance, the rind modeling tool [3] allows the user to *punch holes* in a surface. On the other hand, the multi-segment curved handle tool [18, 19] allows the user to connect two faces of a mesh with a smooth looking *handle*. The column modeling tool, converts each vertex of a manifold mesh into a convex hull and connects these convex hulls with straight *handles* [13, 20].

2 Methodology

Our method consists of three steps and works on a user-selected set of faces, F. A connector mesh M_c is created using geometrical information from the selected faces. Handles are then inserted between the selected faces and matching faces in the connector mesh.

The user specifies a displacement parameter d (which can be positive or negative) which determines where the connector mesh will be created, and a scale factor s which determines the size of the connector mesh.

1. We first collect the set of points P that will be used to create the connector mesh.
   (a) For each face f in F
      i. Compute a displacement vector v_d of length d along the normal to f.
      ii. For each vertex v in f, compute a new point $p = v + v_d$.
      iii. Add p to P.
   (b) We scale all points in P around their centroid by the scale factor s.
   (c) We go through the points in P and remove duplicate points to ensure that we do not have multiple points at the same location.
2. We create the connector mesh M_c from the points in P.
   (a) Compute the convex hull of all points in P. This will give us a set of triangles.
   (b) Clean-up the convex hull by removing edges which separate co-planar faces. The resulting mesh elements will constitute M_c.
   (c) Analyze the location of M_c in relation to the faces in F. If the faces in F point in the same direction as the faces in M_c, we reverse the orientation of all faces in M_c. This step is necessary to ensure that combining M_c and the original surface, will produce an orientable surface.
3. We insert handles between faces in F and the connector mesh M_c.
   (a) For each face f in F we find a matching face f_c in M_c. We consider f_c to be a matching face for f if it is parallel to f and their normals point in opposite directions. In other words, the dot product of the normals of f and f_c should be -1.
   (b) We insert a single-segment handle between f and f_c.

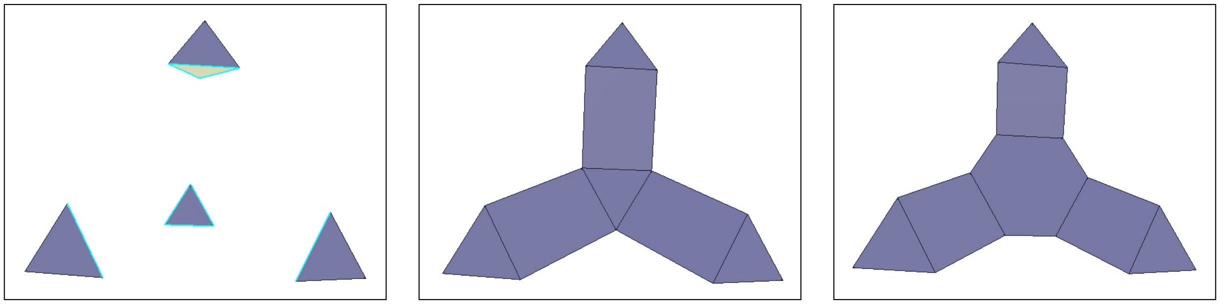

Figure 2: Connecting 4 tetrahedra with a 4-connected handle. The image on the left shows the initial 4 tetrahedra. The image in the middle shows a 4-connected handle with a large value for d. The image on the right shows a 4-connected handle with a smaller value for d.

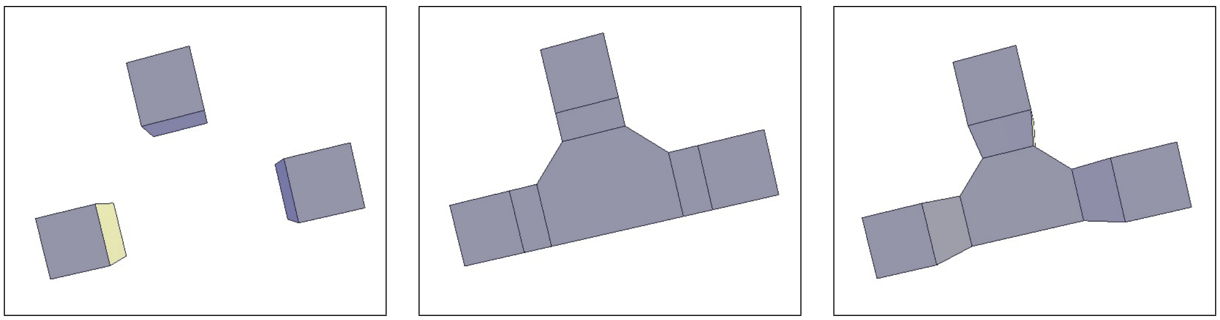

Figure 3: Connecting 3 cubes with 3-connected handles. The image on the left shows the initial 3 cubes. The image in the middle shows a 3-connected handle with $s = 1$ (no scaling). The image on the right shows a 3-connected handle with a $s < 1$.

Figure 2 shows the effect of the displacement parameter d and Figure 3 shows the effect of the scaling parameter s.

3 Implementation and Results

We have implemented our method in an interactive modeling system developed using C++, OpenGL and FLTK[8]. Figure 4 shows the creation of 3-, 4- and 5-connected holes with a cube as the base mesh. Figure 5 shows the creation of multi-connected holes with an octahedron as the base mesh.

Our method produces the best results when the base mesh is convex and the set of selected faces is symmetrically distributed on the surface of the original mesh. This creates a connector mesh that is very similar in shape and topology to the original surface, thereby allowing for clean connections between the two. Additional high-genus operations such as rind-modeling can then be performed on the resulting surface. Figure 6 shows two examples of sculptures created using the multi-connected handle tool and the rind-modeling tool.

A similar process was used to create the sculpture shown in Figure 7. Two 3-connected holes were used for this sculpture. The final model was obtained using the “Inout” method [11].

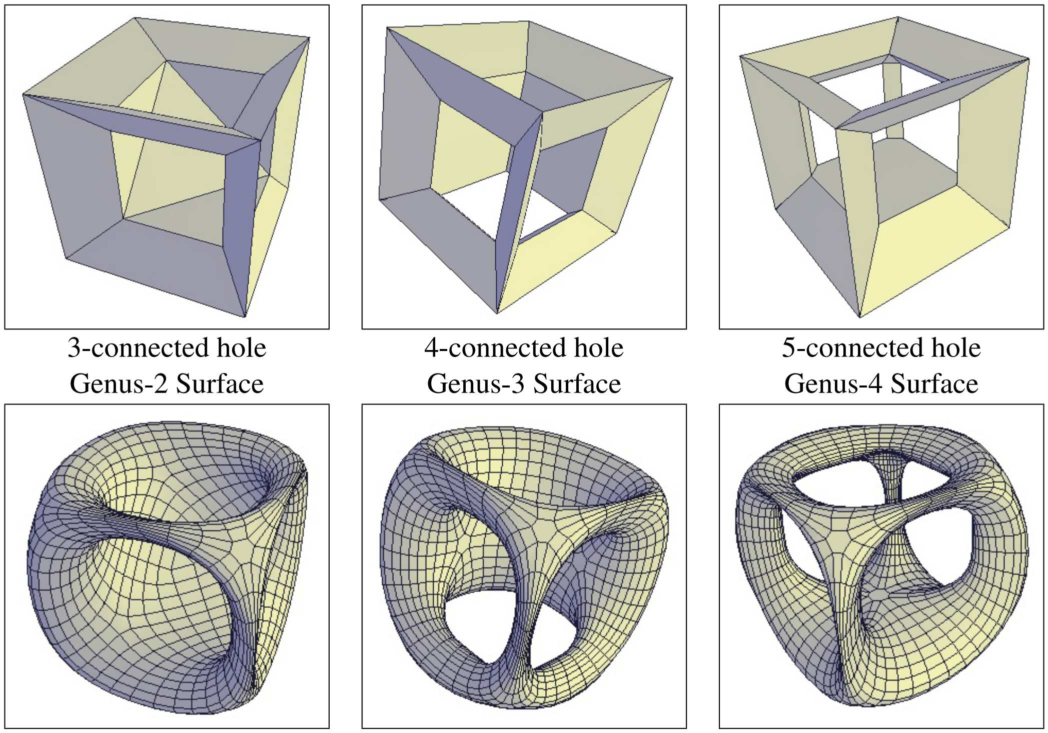

Figure 4: The top row shows three examples that illustrate the creation of multi-connected holes in a cube. The images in the second row are Doo-Sabin smoothed versions of the shapes in the top row.

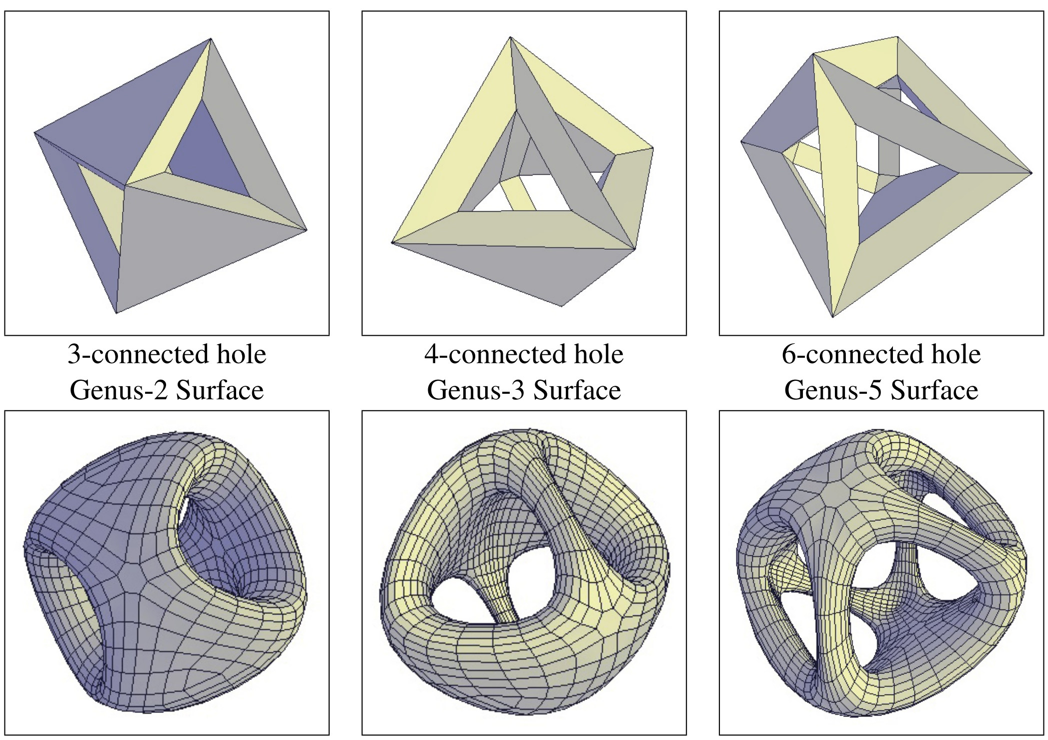

Figure 5: Multi-connected holes in an octahedron. The images in the second row are Doo-Sabin smoothed versions of the shapes in the top row.

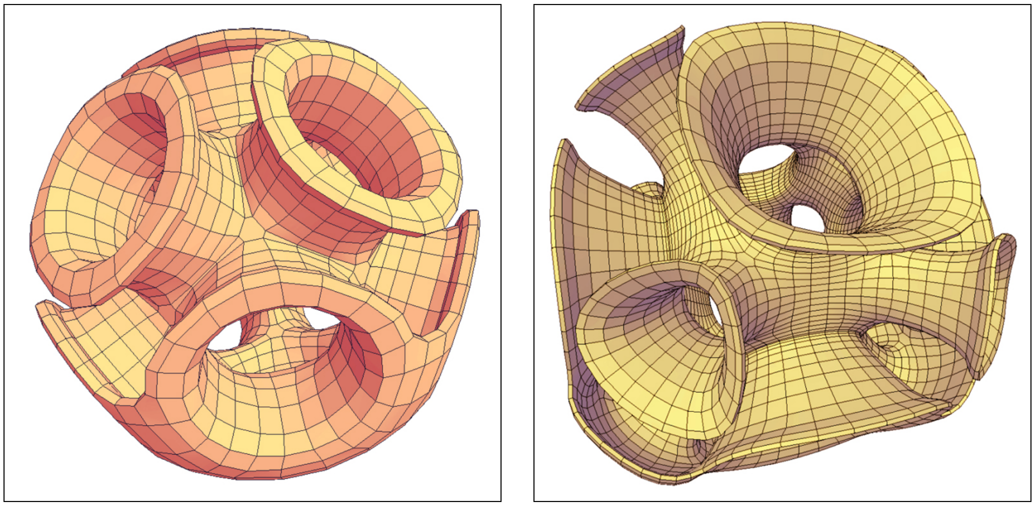

Figure 6: Two sculptures created using multi-connected holes and rind-modeling. The shape on the left was created from a soccerball mesh with a 12-connected hole between all the pentagons. The shape on the right was created from a cube after Doo-Sabin subdivision. A 14-connected hole was created between all the triangles and squares in the mesh. Both meshes were smoothed using Doo-Sabin subdivision.

4 Conclusion and Future Work

In this paper we have presented a new high-genus modeling operation that can be used to create complex high-genus sculptures. Our method builds on previously introduced topological operations that allowed to user to insert a handle between two faces. Our method is based on the concept of a *multi-connected handle* which is a handle that connects multiple faces. Using our method, users can create an intricate network of handles and holes, producing visually appealing sculptures. We have constructed both virtual and 3D printed physical sculptures as shown in Figures 1 and 7.

There are some situations where the connector mesh may not have a matching face for each of the selected faces. This occurs when the length and scaling parameters specified cause some vertices to be excluded from the convex hull. The user has to undo the operation and repeat it with different parameters.

Currently, our method does not include a way of interactively changing the size or position of the multi-connected handle (or hole) after it has been created. We would like to include this capability in future versions. We would also like to add an option to automatically select faces with similar properties and create the multi-connected handle. This will allow the user to create very high-genus models very quickly.

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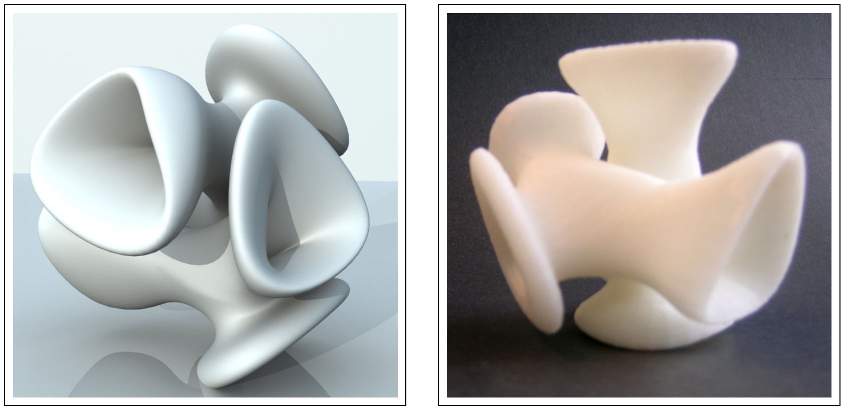

Figure 7: Hernan Molina's "Inout sculpture" created using multi-connected handle operations. The left image shows the virtual sculpture rendered in 3D Studio Max. The right image is a photograph of a physical sculpture made from ABS plastic and printed with a Fused Deposition Modeling (FDM) machine.

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