Anticlastic Form – Manifesting Fields of Tension

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Abstract

My creative practice as a silversmith and sculptor has been inspired by diagrams drawn by a device recording the interaction of two pendulums, a Harmonograph. This led to an inquiry into mathematical methods that could be useful in visualizing the complex twisting surfaces suggested by these diagrams. The various mathematical approaches drawn upon are discussed in relation to the aims and an attempt is made of finding a common language.

Inspiration

My search for form was triggered by a series of diagrams that are based on the principles embodied by a system of coupled pendulums, in a device called a 'Harmonograph' (see Cundy & Rollet [1]). The diagrams traced out by this device (called 'harmonograms') give the illusion of ribbon-like surfaces following a relatively unconstrained three-dimensional course, yet in relation to a sense of symmetry and spatial alignment (Fig.1). These loops seemed to express some fundamental principles of dynamic forms, like standing wave forms, or rhythmical orbits arising from the interplay of simple forces of attraction. Despite the evasive, pre- or para-material nature of these principles, my background as a silversmith led to the attempt of realising the 'forms' as a sheet metal surface.

Anticlastic forming

'Anticlastic forming' is a technique capable of creating surfaces of negative Gaussian curvature, by stretching the peripheral areas and compressing the central area of a sheet metal template. This is carried out with a variety of hammers and saddle-shaped 'stakes'. Fig. 2 shows stretching (green) and compression (orange) in the forming of a saddle shape, one that is similar to a hyperbolic paraboloid (Fig. 3). Anticlastic forming employs the opposite principle to the traditional techniques used for shaping a vessel or domical form, where synclastic curvature is achieved by compressing the peripheral areas and stretching the central area of the sheet metal template.

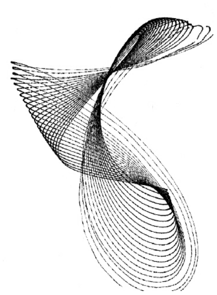


Figure 1: *Harmonogram*

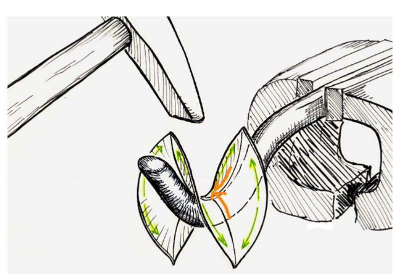


Figure 2: *Anticlastic raising*

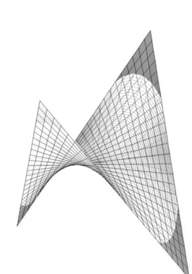


Figure 3: *Hyperbolic paraboloid*

Anticlastic raising creates a simple saddle-shaped curvature locally, yet allows for the creation of a great variety of forms where the parameters of the process are varied over an elongated or more complex template. In terms of my own initial inspiration, the negative curvature enables the ‘fluid‘, helical twisting of a surface around its own axis as well as around an external center, thereby promising to allow the creation of the complex surfaces suggested by the ‘harmonograms‘.

It was during the process of creating these desired forms that I encountered difficulties with visualising the surfaces in their three-dimensional complexity. Despite having worked out the three-dimensional continuity of the curves, the suggested curvature had to be fully understood locally, in terms of the different parameters of the anticlastic geometry. Another problem was posed by the design of the templates: could there be a means of better predicting the transformation of their two-dimensional shape into the three-dimensional form? It was at this point that I wondered how the application of mathematical methods could enhance my understanding of the qualities and manifestation of these forms.

The Role of Mathematics

The similarities to certain geometric and mathematical principles suggested that a study of these principles could be potentially beneficial, for several reasons:

- It could aid in the conceptual understanding of the technical parameters, i.e. the parameters of the surface curvature of the tools and the desired outcomes.
- It could improve methods of visualising the complex forms, both to be able to better resolve existing ideas as well as for future creative developments.
- The mathematical principles promised to inform the philosophical enquiry into ‘generative form principles‘.

In approaching related mathematical methods, it was found that they held some qualitative differences. Besides the fact that the mathematical definitions vary in their computational methods, they express different approaches to the generation of ‘form‘. Several questions posed themselves: which mathematical approach would best communicate the parameters of the forming process? Which mathematical approach was capable of expressing the dynamical principles initially sought for? And, being an idealist, the question was whether there was not an approach that could unify the different areas into one: could the understanding of the local curvature parameters, of the global dynamic parameters, of the tactile, kinesthetic sense of form and the conceptual aims somehow find a common level of understanding? On the one hand this posed the danger of confusing or misinterpreting the principles, while it could on the other hand lead to new insights.

It is here that the attempt of reading the principles as a ‘field of tension‘ enters.

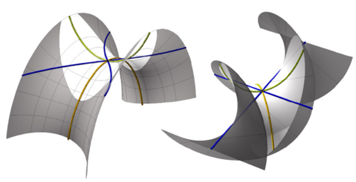
The Forming Process as a ‘Field of Tension‘

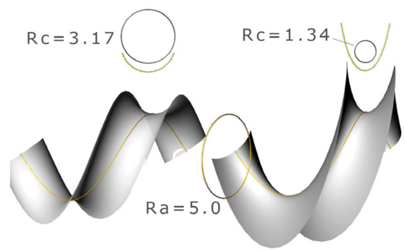
As mentioned above, the forming process exerts tensile and compressive stresses on a sheet metal template, given by the shape of the support and the placement of the hammer blows. While these stresses act locally, the entire workpiece generally extends beyond the central saddle region of the tool action.

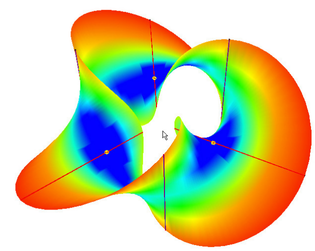
The outcome of the forming process depends on the following:

- The *orientation of the axes of symmetry* of the template in relation to the principal curvatures of the stake's saddle influences the form fundamentally. The differences in orientation relate to the basic complimentary nature of the catenoid an the helicoid (see Fig. 4, left and right, respectively).

- The local *Gaussian curvature of the surface,* i.e., the product of the two principal curvatures. The greater the negative curvature, the more the metal needs to be stretched and compressed.
- The *difference between the two principal curvatures*. Fig. 5 shows two 'catenoidal channels' of equal axial radius (*Ra*) and varying cross-sectional radii (*Rc*).
- The *focus of curvature* expresses the location of the saddle point in relation to the midpoint of the template. This can be illustrated by analyzing the curvature of a given surface (Fig. 6).







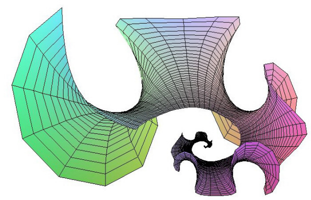
Figures 4 and 5: *Parameters in anticlastic raising* **Figure 6:** *Curvature analysis*

The interpretation of the forming process as a 'field of tension' does involve a degree of abstraction, of withdrawing from the physical difficulties of controlling the technique. Yet on another level, this 'dynamic understanding' of the form principle at work can be found intuitively once the technique is mastered, and the strenuous control is not absorbing the attention any more. The forming process could then be understood as a field, which the sheet metal template is 'fed' into. According to the above parameters, the template will slowly take form. Another approach to understanding the parameters was to see the forms as being generated by the cross-section being 'swept' along the axial curve of the form, yet this principle of axial curve and generator was limited to certain cases only.

Surface Tension as a 'Field of Tension'

The similarity of the anticlastic surfaces to soap films and mathematically generated minimal surfaces suggested that an understanding of the latter could be very useful. This was suggested by several articles in the book *The Visual Mind* [2]. Fig. 7 shows a minimal surface spanning of a trefoil knot. Conceptually and physically, the minimal surface is defined by its boundary or boundaries and the principle of surface-tension acting on the intermediary membrane. Here, the principle that generates the boundary(s) can be completely independent from the principle of surface-tension.

To better understand the mathematical generation of minimal surfaces, algorithms were 'borrowed' from different sources and experimented with (for the main source, see Oprea [3]). Here it was found that most equations generated surface grids with u and v coordinates, which somewhat relates to the principle of building a surface from a series of generator curves along an axial curve. This is a very different approach to the one described above, where the form is dictated by its boundaries and the principle of surface tension. The two methods of understanding and employing the 'generative principle' are not necessarily mutually exclusive. It is rather the case that thinking about the forms according to either principle will influence the conceivable outcomes. In my perception, the mathematical methods were trying to mimic the physical principle, but struggled to represent it. Yet these are limited observations: since carrying out my experiments I have been told that Ken Brakke's *Surface Evolver* software [4] does permit a good control of the boundary curves.



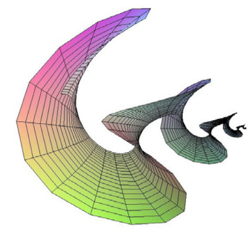
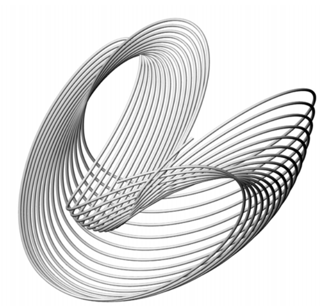


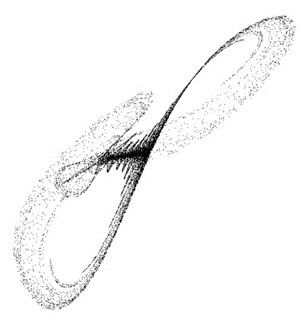
Figure 7: *Surface-spanning of a Trefoil knot*

Figures 8 and 9: *Minimal surfaces generated with Maple code written by John Oprea*

The Dynamical System as a 'Field of Tension'

The system of coupled pendulums represented by the Harmonograph and its 'recordings' are fairly closely represented mathematically by Lissajous' curves. The latter can also be generated in three dimensions and served as a source of inspiration (Fig. 10). Yet the principles can also be linked to other models of dynamical systems: the curve could then be interpreted as tracing out an orbit circling around the centers of gravity of the two pendulums. There are similarities to the computations of the three-body problem in astronomy, where the aim is to understand the interaction of three celestial bodies, generally where one moon orbits around two planets. Further similar principles were given by the strange or chaotic attractors that have been arising in the visual modeling of dynamical systems (Fig. 11, modeled with a program written by J. C. Sprott [5]). The literature states clearly that the path of the trajectory is not to be confused with any actual motion the system could be associated with, but this I did not accept so readily. While this approach was conceptually very closely related to my aims, the occurrence of anticlastic/minimal curvature in the suggested surfaces was much more arbitrary.





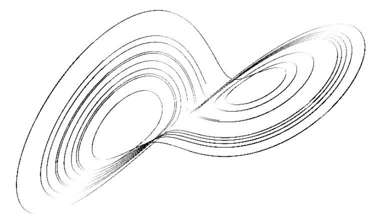


Figure 10: *3D Lissajous plot*

Figure 11: *Strange attractor*

Figure 12: *Lorenz attractor*

The principles rather expressed the overall dynamic continuity of the form, whereas the local anticlastic regions arise through offset iterations of the recurring 'orbit'. Yet one source [6] revealed a surprising connection between the disparate approaches: illustrations were found where a transverse manifold arises from the Lorenz attractor, a 'side-effect' that is rarely illustrated. A simplified band of this manifold is shown in Fig. 13, as illustrated by B. Krauskopf & H. M. Osinga [7]. While I was lacking an understanding of the generation of this manifold and its connection to the trajectory of the attractor itself, it illustrated how the attractor creates a field that generates 'form': the orbit spirals around two points in approximately two orthogonal planes. In my interpretation, these planes in turn can be seen as the cross sections of two intersecting tori (Fig. 14). The helicoidal manifold arises in-between these tori in a similar way to the helicoid being shaped in the anticlastic forming process. Here the saddle of the stake can be seen as presenting the inner half of one of the tori, whereas the other torus has to be imagined to be arising from the action of the hammer blows (Fig. 15). The concept of form thus returns from the global dynamic principle to the local principle of the saddle surface, which in this case creates the separatrix between the two attractors.

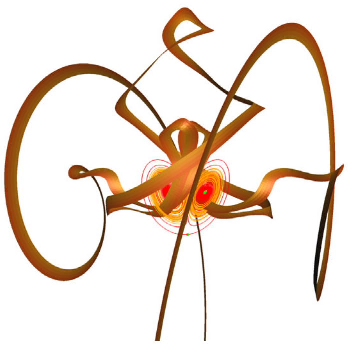


Figure 13: *Lorenz manifold*

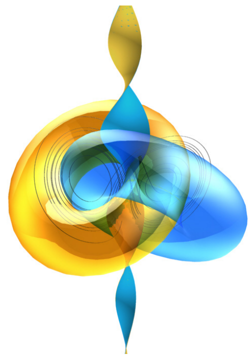


Figure 14: *Lorenz attractor as 2 tori*

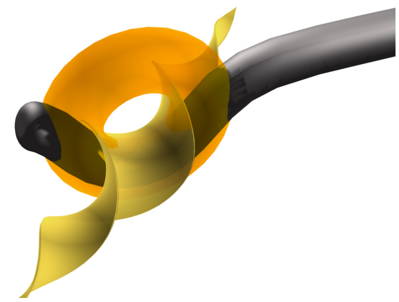


Figure 15: *Helicoidal forming*

Summary

The questions posed above, about the benefit of the mathematical approaches for my own practice, are difficult to answer. Each approach revealed different ways of understanding complex forms, and of defining complex forms. And the question was whether one naturally has to involve the other, whether the understanding depends on a definition. The understanding of the geometric parameters gained from the mathematical description of minimal surfaces was useful to better define the parameters experienced in the forming process. Initially it was thought that the engagement in the mathematical modeling of forms via the equations could be a way of arriving at new developments that could not have been visualised without this help. In retrospect, only very few forms were arrived at via mathematical modeling that were seen as successful models for my own practice. Yet most of the experimentation had been carried out independently, and it is certain that a collaboration with a mathematician could have been a lot more fruitful. The same held true for the exploration of three-dimensional Lissajous curves and strange attractors. Many of the resulting plots showed unsuitable transitions when interpreted as a surface. Others were more shallow in the third dimension than initially thought. Nevertheless, the computational visualisation did allow a much better three-dimensional perspective of the overall continuity as well as local transitions. Aspects of the plotted 'forms' could still serve as inspiration for the design process, without the expectation of being offered finished 'designs' by the mathematically generated models.

The other question addressed the possibility of finding a language that is capable of communicating between the conceptual approaches and the practical, possibly haptic, non-verbal understanding of form and process. Several of the connections implied by the research of dynamical systems seemed to confirm that they 'speak a language' one could readily empathise with. Although the dynamic principles described above are rather abstract, acting on an invisible level, I believe that their language of basic attractive and repulsive forces is accessible to our kinesthetic, bodily senses. A dancer would readily relate to these forces, though maybe he or she could not easily describe them in abstract terms. Likewise it might be difficult for a sculptor to verbally express the tension or the dynamics of a form, and any mathematical methods used to approach the principles might not do justice to the non-verbal understanding of form.

Despite an uneasiness about the abstract language mathematics employs, I cannot help but think that on some hidden level 'life' does speak the mathematical language, and that our minds have an affinity to it, without the very strenuous, intellectual effort of abstraction. Maybe the plea for a more 'embracing' language is impossible to answer, as the principles are inherently complex and not readily simplified to serve everybody. Nevertheless, I believe that there is hope, and the trend towards improved methods of visualisation via the computer is positive. Yet here it is also easy to overlook the complexity of the specialised programming language, based on mathematics in itself. I cannot say much about the trend towards virtual reality, but intuitively I would much rather suggest that we remain rooted in the real. The search for forms discussed here expresses an attempt to defy gravity, but in no way is this experienced as a desire to leave the physical behind. If anything, the forms are trying to relate to something deeply physical, possibly so deep that it is invisible.

The last three Figures show some of my recent work. Figure 16 is based on a space curve passing through a central circle, both acting as the two edge curves of the surface. Figure 17 is based on a minimal surface-spanning of a trefoil knot. Figure 18 shows a vortex-like trajectory, based on illustrations of air currents.

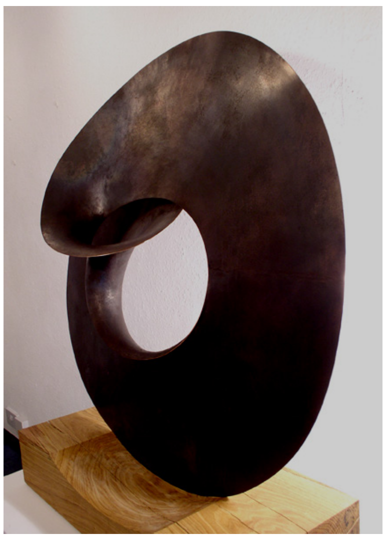


Figure 16: Through the Centre

Figure 17: Motion III 2006

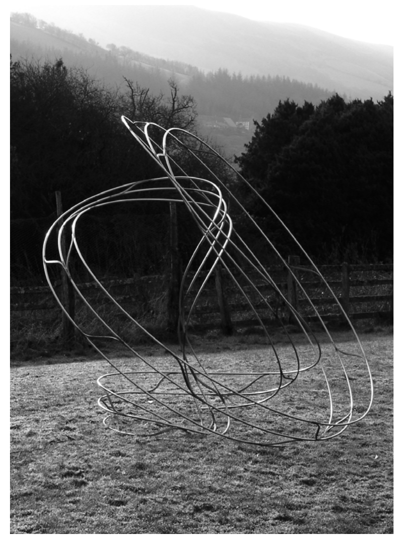


Figure 18: Flux

Acknowledgements

I owe thanks to Heikki Seppä and Michael Good for having shared their discoveries in anticlastic forming, enabling me to explore the field much more quickly. I am also grateful to the mathematicians and scientists who readily made their algorithms and illustrations available to the public, allowing me to explore mathematics without a thorough understanding of the equations.

References

[1] H.M. Cundy & A.P. Rollet (1961), *Mathematical Models*. Oxford: Oxford University Press.

[2] M. Emmer (ed.)(1993), *The Visual Mind*. London: M.I.T. Press.

[3] J. Oprea (2000), *The Mathematics of Soap Films: Explorations with Maple*. Rhode Island: American Mathematical Society.

[4] See K. Brakke's webpage at http://www.susqu.edu/brakke/ .

[5] J. C. Sprott (1993), *Strange Attractors: Creating Patterns in Chaos*. New York: M&T Books.

[6] Argiris , G. Faust. & M. Haase et al. (1994), *Texts on Computational Mathematics Vol. VII: An Exploration of Chaos*. Plates XI-XIII. London: North Holland.

[7] B. Krauskopf and H.M. Osinga. Computing geodesic level sets on global (un)stable manifolds of vector fields. *SIAM J. Appl. Dyn. Sys.,* **2**(4):546-569, 2003.