Circle Folded Helices

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Abstract

Helices are explored as functions of circle reformation using observations that the circle functions as both Whole and parts in ways no other shape or form demonstrates. The generalization of tubes and cones, parallel surface and non-parallel surface, is fundamental to reforming the circle reveling countless variations in the helix and conical helices. The circle can generate forms that in multiples will model natural growth systems revealing a dynamic process reflecting the interrelated nature of universe order. The helix and conical helix are uniquely demonstrated in the first right angle movement of the circle to itself and fundamental to all subsequent folding of the circle.

Curving the circle

Draw a circle and cut it from the paper plane. The circle has three surfaces and two edges. The compression of the sphere shows the same properties as the cutout circle; the only form that demonstrates the concept of a singular self-referencing spherical Whole. [1].

Draw a diameter on the circle (line AB in fig.1). Curve the boundary around so the opposite points on the diameter touch (point AB fig.1). This forms a tube with a circumference formed from diameter AB, reconfiguring the flat surface to a parallel surface. Theoretically this can be carried out indefinitely as a right angle function of the self-referencing circle diameter.

The movement of line AB to point AB is perpendicular to the diameter CD on the opposite side of the curved surface. This is a right angle function of movement AB to line CD forming a triangle plane of points AB,C,D. By rolling the circumference on itself the connection point AB separates forming a tetrahedral pattern of four points A,B,C,D. The surface is now sloping away from and into itself reconfiguring the tube into a cone. The point of connection has moved away from the diameter which continues to be at right angle to line CD. An end view will show concentric openings of circles as the connection point moves away from point AB. The cone is a generalized movement towards an infinitely remote outer boundary. Infinite movement out without boundary suggests infinite movement in without center. In this regard the circle has no center point beyond the limitations of the material.



Figure 1. The sphere compressed to a circle with diameter curved end-to-end forms a tube. The tube moves into a cone as the circumference rolls. Overlapping the surface rolls the tube and cone tighter reducing diameters. The cone's open ends extend infinitely large out from, and infinitely small in to.

There are more combinations of touching points on the circumference than there are endless diameters in a circle. That suggests a preponderance of helix formations; a small part observable in natural growth forms. The potential in cone variations can be demonstrated in the numerous spiral formations that arise from folding and joining circles, in the same way spirals are observed in cosmic systems, flow forms, shells, molecular arrangements, and endless scaling universally throughout.

The four points of the open diameter AB, Fig.1 have six relationships between them forming the pattern of an irregular tetrahedron. The tetrahedron is structural pattern giving consistency necessary for reformation, transformations, generation and sustained growth. The tetrahedron/helices are observed everywhere as right angle functions of scaling patterns of movement.

Touching any two points on the circumference and creasing the circle will form two more points perpendicular and half way between the touching points. When the circle is folded in half a tetrahedron pattern is formed. Four points in space is tetrahedral. There are only six relationships between four points. The number ten describes the tetrahedron. This is observed in both curving and creasing the circle. This first movement of the circle is tetrahedral and it must follow that touching any two points on any surface and folding is a tetrahedron function.

Tetrahelix

The helix is reflected in the tetrahedron, the form is not obvious. The six edges of the tetrahedron can be divided into two sets of three, a right hand and a left hand path. There are no parallel edges showing a helix and no scale change to indicate a spiral. The tetrahedron "solid" is traditionally formed as an isolated object, an abstraction separated from any context. The context for the tetrahedron is spherical pattern, thus the importance of the circle/sphere.



Figure 2. The dual right and left hand helix individually describe all four points of the tetrahedron and show the relationship of six edges. This is observable in the first fold of the circle, principle to all folding.

The tetrahedral packing of spheres (fig.3a) shows the tetrahedron pattern to be regular and at minimum four tetrahedra containing the octahedron interval and multiple relationships of bisectors. The unity of four spheres is a 2-frequency tetrahedron fundamental to all spatial formation. (2-frequency is an edge length in 2 equal parts.)The full helix unit of six segments, 3 sets of 2 each forms parallel edges (fig.3b). The two helix forms show 4 spheres and the 6 points of connection in spherical packing.



By connecting the two ends of three edge tetrahedron through the center location of a single tetrahedron the helix becomes a loop connecting the inside and outside. Three open segments are now a five-segment path and a closed helix system connecting the center location and the outside boundary.



Figure 5. a) A stick model of a two-frequency tetrahelix showing the octahedron and vector equilibrium (traditionally called a cuboctahedron) helix forms. The relationship of these forms reveal 13 individual arrangements of primary helices that run through the two-frequency tetrahelix. b) This drawing shows two tetrahedral intervals between each of two tetrahedra touching on the points. The connection of one half of the three axis of each octahedron (3 of 6 radii at right angles) runs through the center of each tetrahedron and models the DNA double helix ladder. The direction of twisting of the double helix is opposite to the tetrahelix.



Circle Reformation to Helix

Forming a tetrahedron using the circle reveals nine folds [2]. By reconfiguring these nine creases many different forms of the circle can be created. Any number of these reconfigurations when produced in multiples of the same diameter circle will when consistently joined in a line create a helix formation. The most fundamental is in the form of the tetrahedron/octahedron combination reflected in the helix function of the two-frequency tetrahedron (fig.3).

The nine tetrahedron folds are a primary subset of folds in the 24 creases of a folded equilateral triangle grid (fig.6). Three diameters are perpendicularly divided into eight equal parts forming this grid [3]. The parallel creases of this grid allow the circle to reform into the fundamental helix forms. Folding the circle into an 8-frequency diameter grid is similar to an octave in music. From that primary differentiation of fundamentals and the intervals between there is endless potential of reformation.



Figure 6. Folding the circle into an eight-frequency grid allows reforming the circle into the individual reconfigurations pictured above. The square tube (1) transforms into a triangular tube (2), into a tetrahelix (3), into an octahelix (4), then to a four-sided helix (5), and then opens to a conical helix (6). The triangle tube and helix forms are triangulated and rigid. The conical helix breaks the parallel edges and is less rigid. The square tube is without triangulation and collapses. The diameter in the triangle tube is joined to itself perpendicular to the tube length as shown in Fig. 1. The tetrahelix (3) comes from opening the diameter and connecting with the next parallel cord. The direction of opening determines right or left handedness. The diameter when opened further and keeping edges parallel forms the octahelix (4) with both right and left hand twisting edges. The square and triangle tubes are prisms in design and the twisting of parallel planes are anti-prisms.

Transformational Nature of the Tetrahelix

The tetrahelix modeled as a static object has properties but is without function (fig.6, #3). Using individual tetrahedra with hinge-joining edge connections between them rather than rigid face-to-face joining reveals some interesting changes. Fig. 7 shows the edge-to-edge joining of tetrahedra that can be considered a helix pattern since corresponding edge positions are parallel and twist around in a continuous path. There are two right hand and two left hand edge paths, four of the six edges. The top view shows a square image where each side is an edge path and diagonals are joining edges.



Figure 7. Eight tetrahedra hinge-joined on opposite edges. The square shows the tetrahedron, top view of the tetrahelix, where the diagonals are connecting edges perpendicular to each other. This right angle hinge-joining shows many variations in helix reformation. (By connecting the ends together with a tape hinge the helix will form a rotating Torus ring.)



Figure 8. a) Eight tetrahedra twisted in one direction will reform to a tetrahelix joined surface-to-surface. Twisting in the opposite direction exposes the inside surfaces, hiding the outside to the inside. The direction of the twisting planes dose not change. b) Twisting to either direction, with alternating two triangle sides out and two triangle sides in, will change the direction of the three twisting planes from a right to a left hand helix. (Notice the dark and light differentiation between triangle surfaces.)



Figure 9. The tetrahelix with tetrahedra joined surface-to-surface



Figure 11. Here the tetrahelix is hinged in sets of six and hinge-joined together.



Figure 10. Using hinge-joining at intervals between sets of two tetrahedra the helix can be reformed to a tighter twisting.



Figure 12. Another variation of the tetrahelix wound tighter through hinging.

Modular Growth of Conical Helices

Diminishing or enlarging the diameter of the circles used for a helix segments will turn it into a conical function. Much like curving the circle and separating the diameter points AB (fig.1) making multiple levels of parallel circle planes in different sizes, Each segment in a helix changes scale depending on how much the diameter for each circle folded is reduction or increased. The circle configurations are folded the same, the diameters differ corresponding to scale differential in individual stages of growth (fig.13a,b). There are many options in forming conical helices. The degree of open and closedness of the helix is correlated to the proportional difference between diameters and specific reformations being used. Generally the closer the intervals between diameters the number of units used the tighter it is. These differences can be generalized whereas the controlling development appears to be conditional, dependent on configuration interaction within the environment towards specific growth function. Natural growth forms are regulated by larger systems inherent to pattern. In modeling conical helices all variations and diversity of forms are inherent to the circle, otherwise they could not be formed. The possibilities lie between the generalizations of mathematical functions and the forming interactions of parts inherent to life systems. This seems to apply to both the circle and to growth forms in nature.

Figure 13 Two conical helices folded from different reforming of circles. a) 24 circles with a printed design have been folded, each fitting into the other. b) 14 circles folded into tetrahedra forming an open pentagon.





b)



Figure 14. 18 paper plate circles. The first unit shows the form of the reconfigured, triangle folded circle. The folded segment is consistently the same with diminishing diameters.

held together with hair pins.

Figure 15. 12 reformed paper plates



Variations in Conical Helices

There are endless variations in forming conical helices. This is obvious in nature and in folding and joining circles. The circumference can be folded in, out, and in combinations; only the circle can do that. The changing diameters, the number of units, sequencing of segmented sets, and open/closed forming of units are all variables that can be creased, curved, or both. When working with triangular units there are three primary positions that can change direction of orientation. The sequencing of set numbers and the rotational positions of each unit will change the angle and direction of the movement in relationship to the possible ways of joining. This is similar to the articulation between musical notes.



Figure 16. Double end conical helix using paper with printed image in gradation. Angles of direction changed by rotation of units.





Figure 17. Variation of printed units used in fig.16.



Figure 18. a) View of 14 circles in tight twist. b) Opposite view shows how the circles have been formed where each circle is folded into an equilateral triangle and two ends twisted into a mobius surface.







Figure 20. a) This model is made using 162 circles all folded the same. It is a variation from the unit used in Fig. 14. These circles are folded tighter and joined closer together making a more compact growth path. Each revolution touches itself. The units decrease in sets of eight until they get to the end where the numbers of units in a set diminishes. b) The end view shows the axial hole open all the way through both ends that is unobservable from the side.



Figure 21. One rotation using 15 reconfigured circles.



Figure 22. The units used here are a variation to those in fig.15. They are arranged differently showing $1\frac{1}{2}$ turns.

Figure 23. This is another view of Fig. 14 with more units added to it. Each added unit is sequentially opened creating a tessellation type change. By making small alterations to the folded grid in the reconfiguration of individual segments a system can tessellate through many form changes.



There are great advantages for using circles to model the geometry in nature. The greatest advantage is that it is far more comprehensive than any other form of modeling and will generate forms

and designs not possible in other ways. The circle is not restricted by limited number of sides or scale. It can form all traditional planer configurations as well as those things that can not be done by other means.

Nothing is added or taken away. There is simply the movement of the circle to itself demonstrating the process of transformation and change which these pictures of models indicate. With the circle everything becomes informational, nothing is ambiguous or arbitrary. The circle reveals a principled process of movement in the context of endless forming systems through folded reformations. Generalized functions, discrete information, and beautiful forms are modeled that reflect patterns of growth in the natural world. These models show but a few forms of the endless possibilities of the helix growth pattern.





Figure 25. Detail of a helix branching system of tightly folded circles in conical form showing a fractal like growth.

Figure 24. The units for this system are the same used in Fig.16,17, and 20. In each case the fit of one segment into another has been modified changing the angulation. Different diameter intervals change the angle of rotation making each model a uniquely different form.

Conclusion

Folding circles expands the information of traditional geometry and generalized mathematical functions, revealing beautiful and dynamic forms in a rather poetic way, that allows exploration of the richness in diverse forms observed in nature, particularly evident with helical growth. The forms tend to show process and structural design rather than reproducing outward appearances. This is a small part of what is an enormous benefit in folding circles.

References

[1] Bradford Hansen-Smith, *The Geometry of Wholemovement: folding the circle for information*. Wholemovement publications, 2000

[2] Demonstrated on website; www.wholemovement.com

[3]Bridges Mathematical Connections in Art, Music, and Science. Conference Proceedings, 2004. Reza Sarhangi & Carlo Sequin, Editors. *Folding the circle, both Whole and Part*, B.Hansen-Smith, pp.87-94 The images in **Figures**, **1**, **2**, **4**, **5b**, **7 8a**, **b**, **9-12**, **13b**, **15**, **19b**, **21**, **24**, are from the author's book *Folding Circle Tetrahedra: truth in the geometry of wholemovement*. Wholemovement Publications, 2005