Introducing the Precious Tangram Family

Stanley Spencer
The Sycamores
Queens Road
Hodthorpe
Worksop
Nottinghamshire
England
S80 4UT
pythagoras@bcs.org.uk
www.pythagoras.org.uk

Abstract

The Author of this paper has developed a family of Precious Tangrams based upon dissections of the first six regular polygons. Each set of tiles has similar properties to that of the regular tangram. In particular the property called Preciousness. It includes a discussion of some of the mathematical aspects of the dissections with examples of non periodic tessellating patterns. It continues with examples of the unique way in which they can produce an infinite number of designs. It explains the iterative nature of the process as applied to designs for mosaics, quilts and animation.

1. Introduction

This work follows from previous work on Precious Triangles and Polygons [1],[2],[3],[4], [5]. The idea has been expanded to include dissections of the n sided regular polygons for n=3 up to n=8. Each dissection results in tiles that have angles which are multiples of $\pi \div n$. Each dissection contains triangles, quadrilaterals and, as one of its tiles, a smaller version of the original regular polygon. Finally each set of tiles displays the property of preciousness.

2. What are Precious Polygons?

Precious Polygons are sets of different polygons that can be used to form other sets of similar polygons. The necessary conditions for preciousness are that a larger version of each polygon can be produced using only the original polygons, secondly, the enlargement factor in each case must be the same and finally, all the elements of the nth power of the Precious Matrix must be non zero, where n is the number of different tiles [1]. This characteristic ensures that all designs, even a single tile, are expandable to infinity. This process is similar to that of Solomon Golomb's Rep-tiles [7],[8] but involving sets of polygons rather than a single polygon. Self similarity was also a feature of 14th and 15th Century Islamic Geometry [9] as well much earlier 1st Century Celtic Art.

3. Precious Dissection of Regular Polygons.

The following diagrams show dissections of the regular polygons with the corresponding Precious scheme including the Precious Matrix. It is left to the reader to show that all the elements of the nth power of the matrix are non zero. In addition the enlargement factor is given. This must be a constant for the dissection to be Precious and is known as the Precious Ratio. Once we have a Precious set then we can take any design made from the original tangram shapes and produce a larger version using the schemes in figures 1 to 6. Since we end up with a design using only the original shapes, we can repeat the process ad infinitum. Each successive design is larger than the previous one by a factor of P, the Precious Ratio.

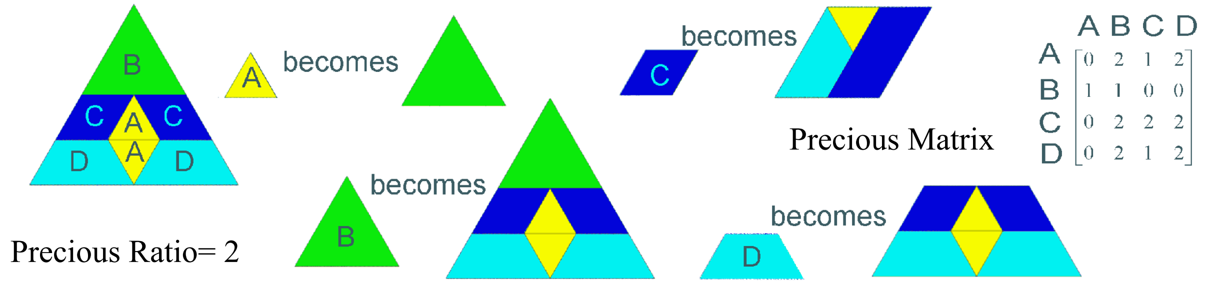


Figure 1: *A precious dissection of the regular polygon for n=3.*

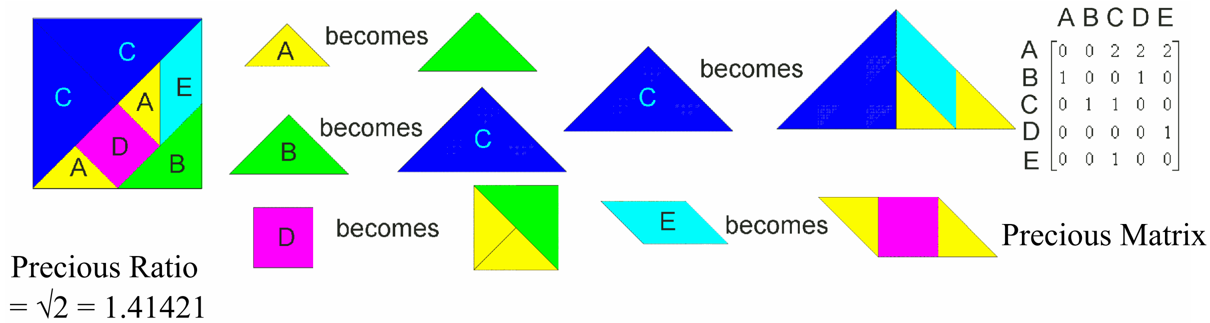


Figure 2: *A precious dissection of the regular polygon for n=4.*

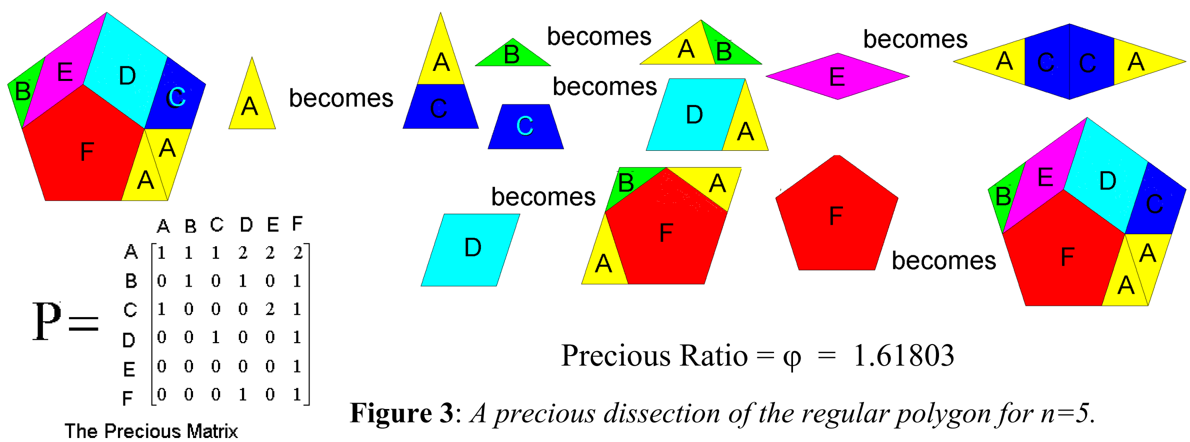


Figure 3: *A precious dissection of the regular polygon for n=5.*

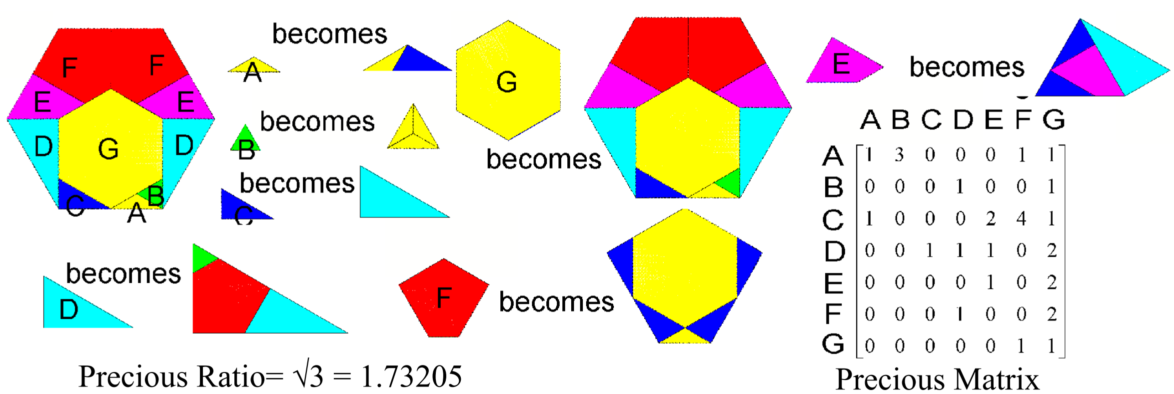
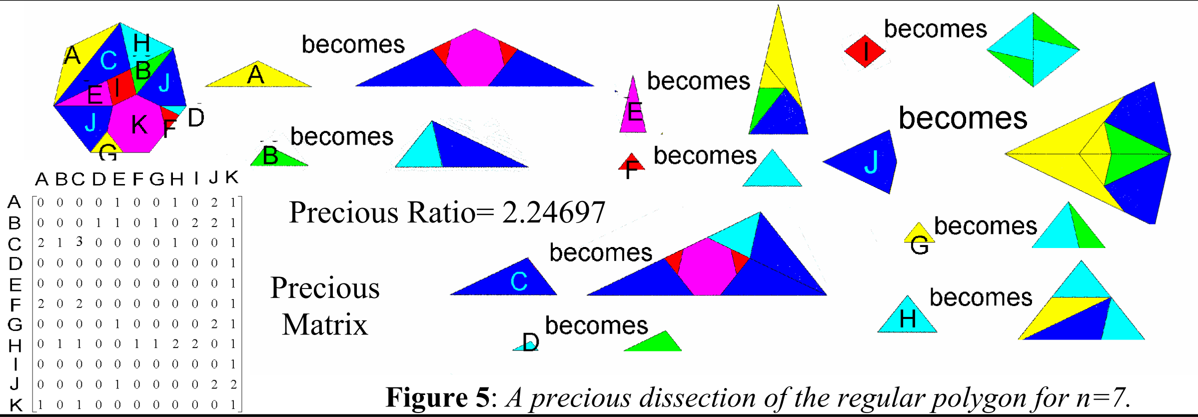
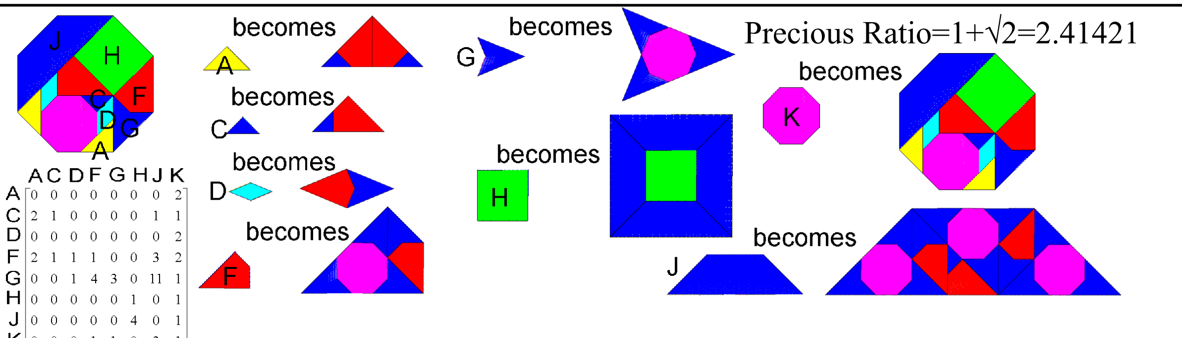


Figure 4: *A precious dissection of the regular polygon for n=6.*



	A	B	C	D	E	F	G	H	I	J	K
A	0	0	0	0	1	0	0	1	0	2	1
B	0	0	0	1	1	0	1	0	2	2	1
C	2	1	3	0	0	0	0	1	0	0	1
D	0	0	0	0	0	0	0	0	0	0	1
E	0	0	0	0	0	0	0	0	0	0	1
F	2	0	2	0	0	0	0	0	0	0	1
G	0	0	0	0	1	0	0	0	0	2	1
H	0	1	1	0	0	1	1	2	2	0	1
I	0	0	0	0	0	0	0	0	0	0	1
J	0	0	0	0	1	0	0	0	0	2	2
K	1	0	1	0	0	0	0	0	0	0	1

Figure 5: *A precious dissection of the regular polygon for n=7.*



	A	C	D	F	G	H	J	K
A	0	0	0	0	0	0	0	2
C	2	1	0	0	0	0	1	1
D	0	0	0	0	0	0	0	2
F	2	1	1	1	0	0	3	2
G	0	0	1	4	3	0	11	1
H	0	0	0	0	0	1	0	1
J	0	0	0	0	0	4	0	1
K	0	0	0	1	1	0	3	1

Precious Matrix

Figure 6: *A precious dissection of the regular polygon for n=8.*

4. Examples of simple Designs.

Figures 7, 8 and 9 are examples of simple designs using the dissection of the heptagon and octagon. They also show the first few of an infinite number of non periodic tessellations using the Precious properties of the tiles. Further examples for the square and pentagon can be seen at [1],[2].

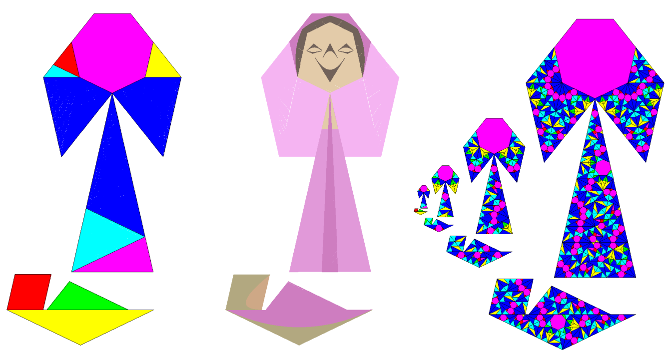


Figure 7: *Mary and Jesus: This design uses the Precious dissection of the heptagon.. The first four of an infinite number of non periodic tilings are shown on the right of the figure.*

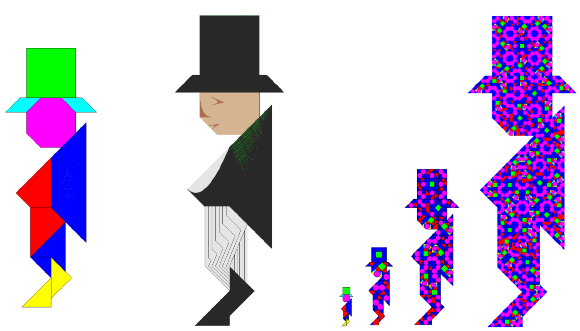


Figure 8: *The Father of the Bride: This design uses the Precious dissection of the octagon. The first four of an infinite number of non periodic tilings are shown on the right of the figure.*

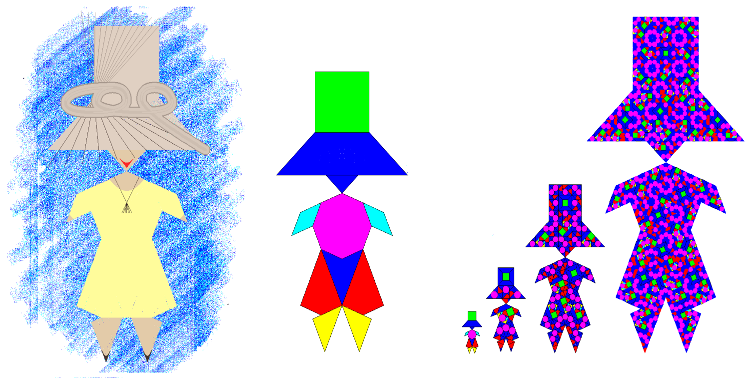


Figure 9: *The Mother of the Groom: This design uses the Precious dissection of the octagon. The first four of an infinite number of non periodic tilings are shown on the right of the figure.*

5 The Creation of Mosaics from the Precious Tangram

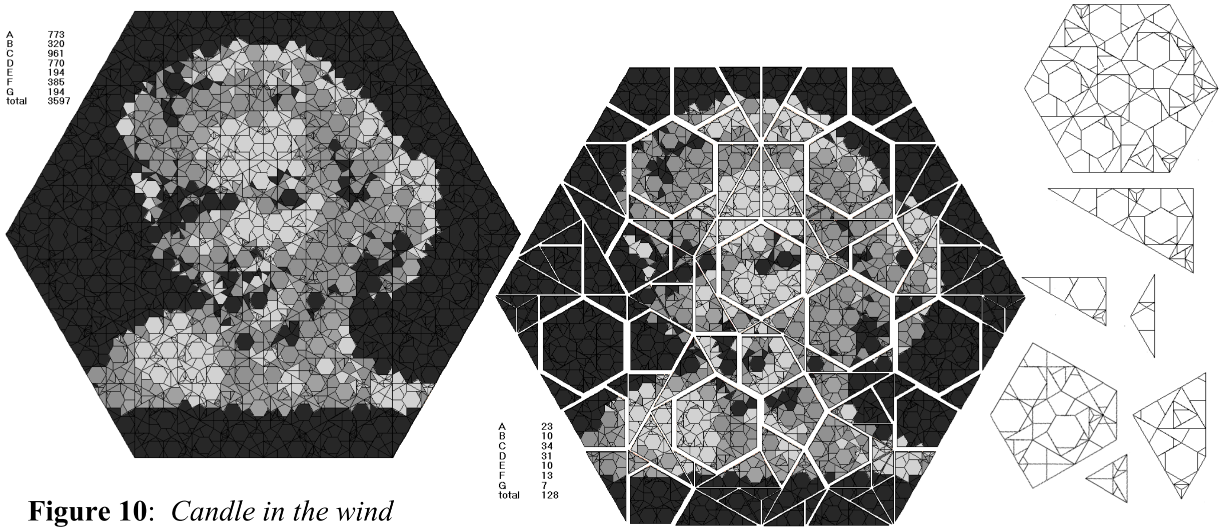


Figure 10: *Candle in the wind*
The details of a ceramic Mosaic using different generations of tile.

Conventional mosaics use small tesserae to form a picture. The picture at the left in Figure 10 is constructed from several thousand tesserae, each being one of the six Precious Shapes. The picture on the right shows how the picture can be created from 162 higher order tiles. Each one being one of the same original 6 shapes, but larger. To maintain the detail, the large tiles would need to be embossed with the original smaller tiles. The outlines of the 6 moulds can be seen in the top, right of the picture. The software, written by the Author, used to create the pictures in this paper also produces a disc that controls a CNC milling machine that makes the moulds from which the high order ceramic tiles can be accurately produced.

6 The Creation of Patchwork from the Precious Tangram

Many of the Precious schemes lend themselves to patchwork. That is, the schemes are quiltable. This means that they can be assembled by sewing in straight lines. Some preplanning of the sewing sequence is necessary to avoid sewing around a corner. To date such schemes have been developed for the equilateral triangle, the square, a subset of the pentagon [3] and a subset of the heptagon. Schemes are being developed for a subset of the hexagon and octagon. The designs in Figures 11 and 12 are based upon the equilateral triangle dissection. It is easy to see that the Precious scheme is quiltable in these cases. From a practical point of view it is necessary to be accurate with the sewing if the geometry is to work. This attention to accuracy starts with the cutting of the material. To this end a series of templates were made each with an extra border of 0.25 inches. A rotary cutter was used to cut out the various shapes. These were then sewn together with a hem of exactly 0.25 inches.

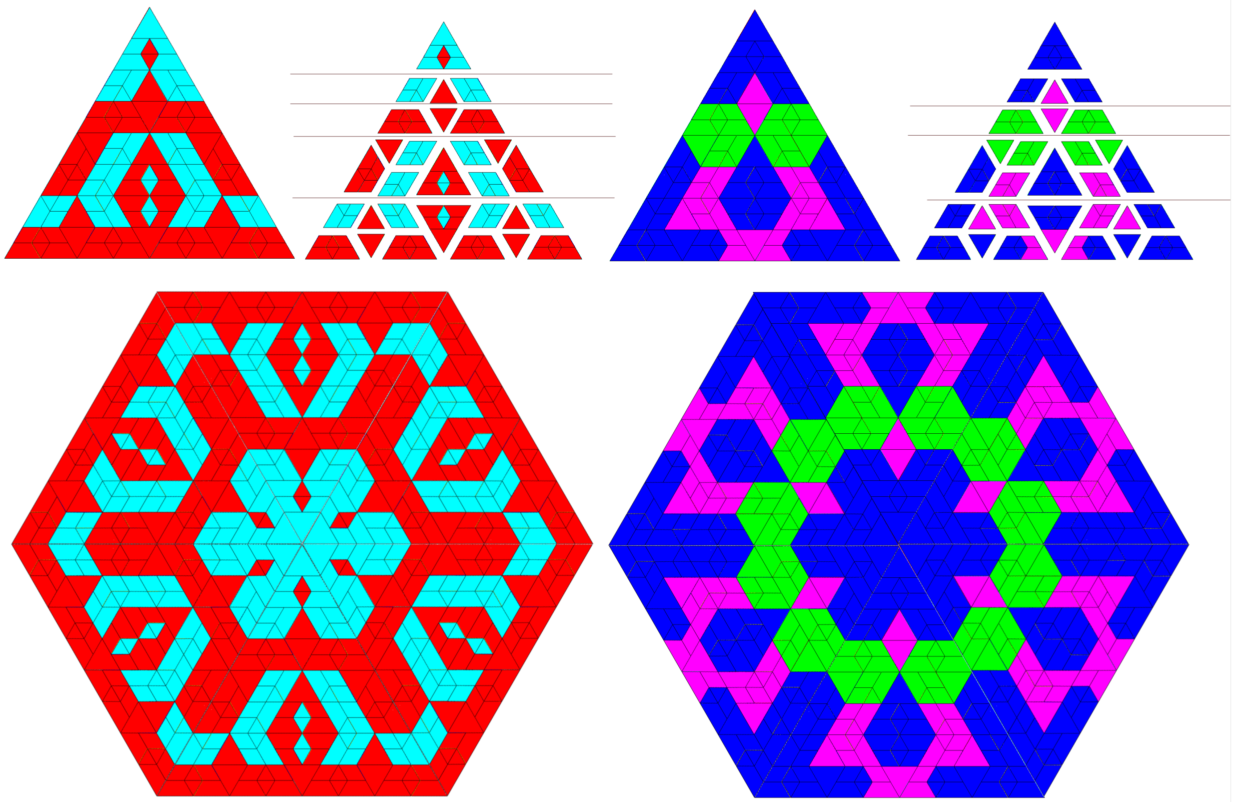


Figure 11: *The details of patchwork designs using different generations of tile.*

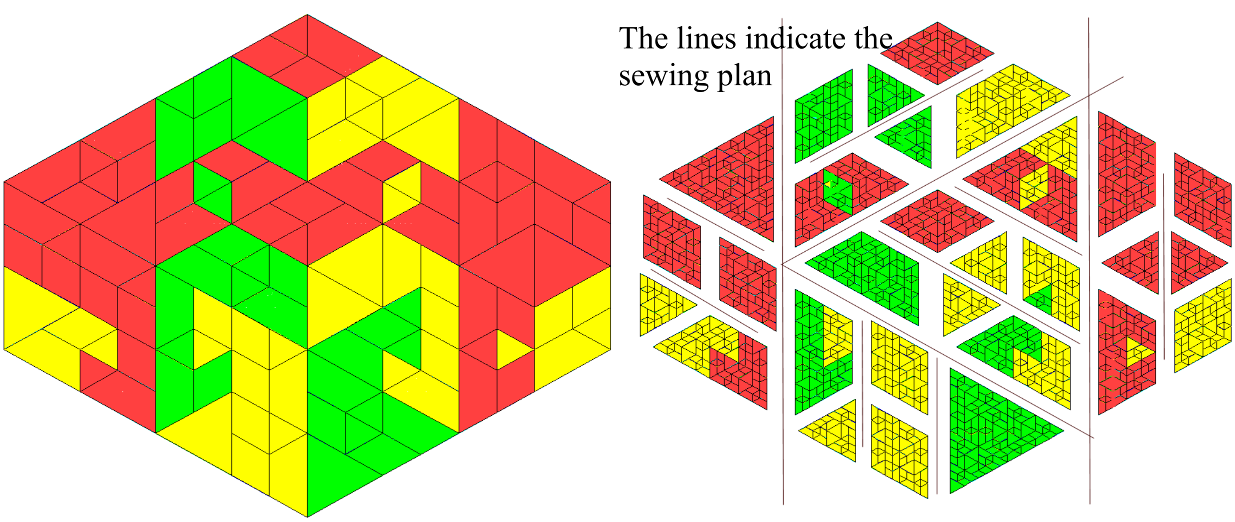


Figure 12: *The details of a patchwork designs using different generations of tile.*

7 The creation of Animation from the Precious Tangram

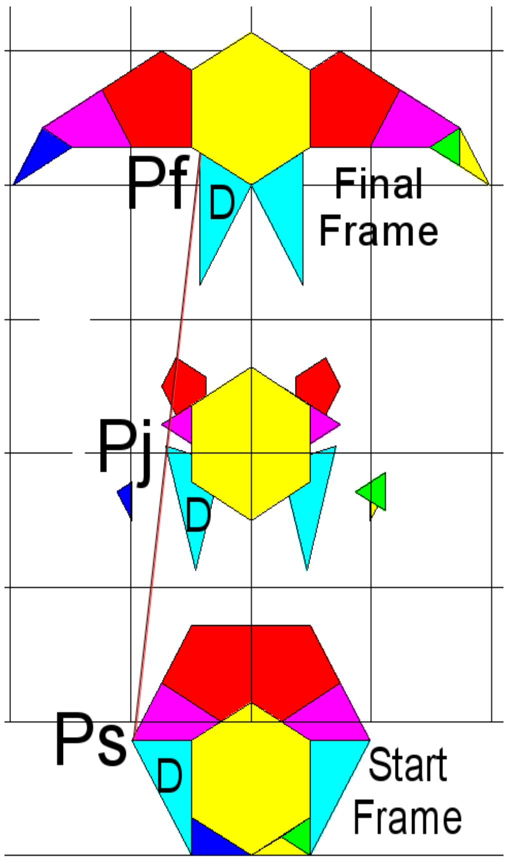


Figure 13: *The hatching of the swallow is an example of an animated Tangram design.*

To produce animated clips it is necessary to create a number of frames, each being a snapshot of the required movement. The method adopted by the Author was to produce two pictures from a set of tangram shapes. These represent the starting and finishing positions of the animated clip. The example in Figure 13 shows a hexagon and a swallow as the first and last frames of the sequence. It is important to note that the two designs use the same set of tiles. The intermediate frames can be produced by considering a line joining corresponding points on the start and final frames. If these points are called Ps and Pf, the number of frames n and the number of the frame being considered j then the distance PsPj is PsPf x j÷n.

If this calculation is repeated for each corner of each tile then an intermediate frame can be produced. Figure 13 has the start, finish and jth frames. Normally these are separate frames but have been assembled together for illustration.

The frames were printed and assembled into a booklet that can be flicked through and given the illusion of movement. An alternative was to assemble the frames into an animated .gif file. The software used to create the individual frames was written by the Author and the JASC Animation Shop2 was used to assemble the individual frames into an Animated gif. The size of the individual frames and the number of frames both have an effect on the quality and speed of the animated clip. A few examples of animated gifs will be presented at the conference.

8 References

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