Magic Stars and Their Components

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Abstract

Magic Stars is the title of a musical work based on mathematical objects of the same name. Six six-pointed magic stars provide six two-dimensional 12-tone structures, which constitute the building blocks of the work. These structures are subjected to analysis, transformations, disintegration and recombination of their components. The parts of the score, which is richly visual, look like tables rather than traditional musical pieces. While pitches ('space') are fixed, time is not, giving ultimate freedom to the performer, who may find out his own time and thereby meet quite mathematical and objective things in a very personal and intimate way.

1. Magic Stars as Mathematical Objects

The six-pointed magic star is a figure having the following appearance:

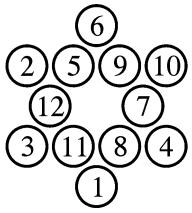

Figure 1: *An example of a Magic Star.*

Twelve non-recurrent numbers, from 1 to 12, are located on the vertices of the **main triangles** that form the star, and on the points of their intersection. The vertices of the main triangles (1, 2, 10 and 3, 6, 4 in the example) are called the **outer vertices**, and the points of intersection of the triangles (11, 12, 5, 9, 7 and 8 in the example) are called the **inner vertices**. The **side** of a star – which is also the side of one of the main triangles – is a segment, and four vertices are placed on it. The numbers are arranged so that each side can produce the same sum. For example, 1+11+12+2 = 3+12+5+6 = 2+5+9+10 = 6+9+7+4 = 10+7+8+1 = 4+8+11+3. The sum of the six outer vertices also has to be the same, for example, 1+3+2+6+10+4. There are a total of six different magic stars, disregarding variants produced by mirror images or rotation.

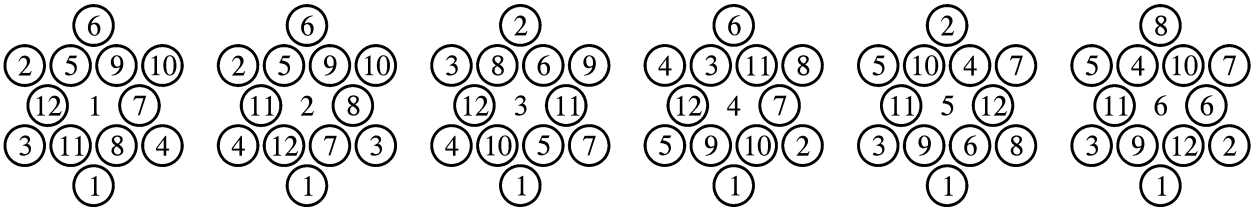

Figure 2: *All six Magic Stars in their original form.*

One can distinguish the following components in the stars: vertices (outer and inner), sides, complements, hexagons (outer and inner), triangles (outer and inner), enneagons (main triangles) and dodecagons, and also inner equilateral triangles.

There are twelve **vertices** in each star: six outer and six inner ones. Here the expression 'sum of vertices' means the sum of the numbers that lie on these vertices.

A **side** is four vertices placed on one segment. On the ends of the segment there are outer vertices and between them there are inner vertices. The sum of the vertices of any side of a six-pointed magic star is always 26.

Opposite (or **parallel**) **sides** of a star are sides that have no common vertices. Each side has exactly one opposite side. Parallel sides are formed by eight vertices, their sum being 26×2=52. A group formed by the remaining four vertices is called **complement**. The sum of the complement vertices is always 26.

A **hexagon** is six outer vertices (**outer hexagon**), or six inner vertices (**inner hexagon**). The sum of the vertices of an outer hexagon is 26. The sum of the vertices of an inner hexagon is 26×2=52.

A **triangle** is three outer or three inner vertices, which form a particular sum. An **outer triangle** is formed by three outer (corner) vertices, which are the points of the main triangle of a six-pointed star. The sum of the vertices of an outer triangle is 13. An **inner triangle** is a set of three inner vertices that make the sum of 26. For each six-pointed star, two outer triangles and two inner triangles are always clearly defined. One can add the following distinction. The **first outer triangle** is one which has the vertex '1' (vertex which contains the number 1; one of the two outer triangles always has such a vertex). The **first inner triangle** is one which has the vertex '12' (one of the two inner triangles always has such a vertex as well). The **second outer triangle** or the **second inner triangle** is one which has no vertex '1' or '12', respectively.

An **equilateral triangle** (unlike a triangle) is three outer or three inner vertices which form a particular geometrical figure, namely that of an equilateral triangle. An **outer equilateral triangle** always coincides with an outer triangle. An **inner equilateral triangle** never coincides with an inner triangle. The sum of the vertices of an inner equilateral triangle, while being different in different cases, is never 26.

An **enneagon** (or **main triangle**) is three outer vertices of one of the two outer triangles and all six inner vertices. In other words, it is a set of all the vertices of a main triangle of a star (of all the vertices of three adjacent sides of a star, which form such a triangle). The sum of the vertices of an enneagon is 13+52=65.

A **dodecagon** is a figure formed by all the vertices of a star. The sum of the vertices of a dodecagon is 26+52=78.

In all cases (with the partial exception of complements), fundamental importance attaches not only to which elements (vertices) form this or that component of a star, but the order of these elements as well. Thus, for example, we distinguish the side '1, 11, 12, 2' from the side '2, 12, 11, 2' (reverse order) or from '1, 12, 11, 2'. For the same reason, one can say that for the first star there exists the dodecagon '1, 11, 3, 12, 2, 5, 6, 9, 10, 7, 4, 8' or '8, 4, 7, 10, 9, 6, 5, 2, 12, 3, 11, 1', but not '1, 3, 11, 12, 2, 5, 6, 9, 10, 7, 4, 8'.

One can transform magic star also by 'inside-out/outside-in' operation. As the result, outer vertices of an original star take now inner positions and vice versa. A star transformed in such a way is called an **Antistar**. In antistars, hexagons of equal type will still have an equal sum (now, 52 for outer hexagons and 26 for inner), while sides lose their quantitative sameness.

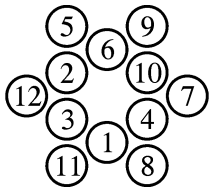

Figure 3: *The first Antistar (compare with Figure 1).*

How can stars be turned into music? For example, like this: let 1 be E, 2 be F and so on, up the chromatic scale... Now one can examine stars and their components, in different combinations and correlations, by means of music.

2. Magic Stars as Musical Work

The full title of the work is **Magic Stars // Tables for Piano or Other Appropriate Instruments**. There are a number of tables – more than two hundred and the work is still in progress. The tables are grouped in a following way:

Stars, this section includes Vertical Stars, Vertical Stars Expanded, Vertical Stars Compressed, Horizontal Stars, Horizontal Stars Sloped, Horizontal Stars Extra Sloped, Horizontal Stars Compressed, Horizontal Stars Degenerated (8 tables on 36 pages in total in this section).

Components, this section includes Sides and Complements (in different forms, sorting and combinations, 44 tables on 40 pages), Triangles (12 tables on 12 pages), Hexagons (30 tables on 20 pages), Enneagons (18 tables on 26 pages), Dodecagons (18 tables on 24 pages), Opposite Vertices (4 tables on 4 pages) (126 tables on 126 pages in total in this section).

Ordered Components, this section includes Ordered Sides (20 tables on 20 pages), Ordered Complements (19 tables on 20 pages), Ordered Triangles (16 tables on 16 pages), Ordered Hexagons (26 tables on 26 pages), Ordered Enneagons (6 tables on 12 pages), Ordered Dodecagons (6 tables on 12 pages) (93 tables on 106 pages in total in this section).

Stars in Base-n Mode, this section includes Stars represented in different numerical systems, from Base-2 to Base-13 Mode (12 tables on 12 pages in this section).

Antistars, this section includes Vertical Antistars, Horizontal Antistars, Horizontal Antistars Expanded (3 tables on 12 pages in this section).

(So, by now there are 242 tables on 292 pages in total.)

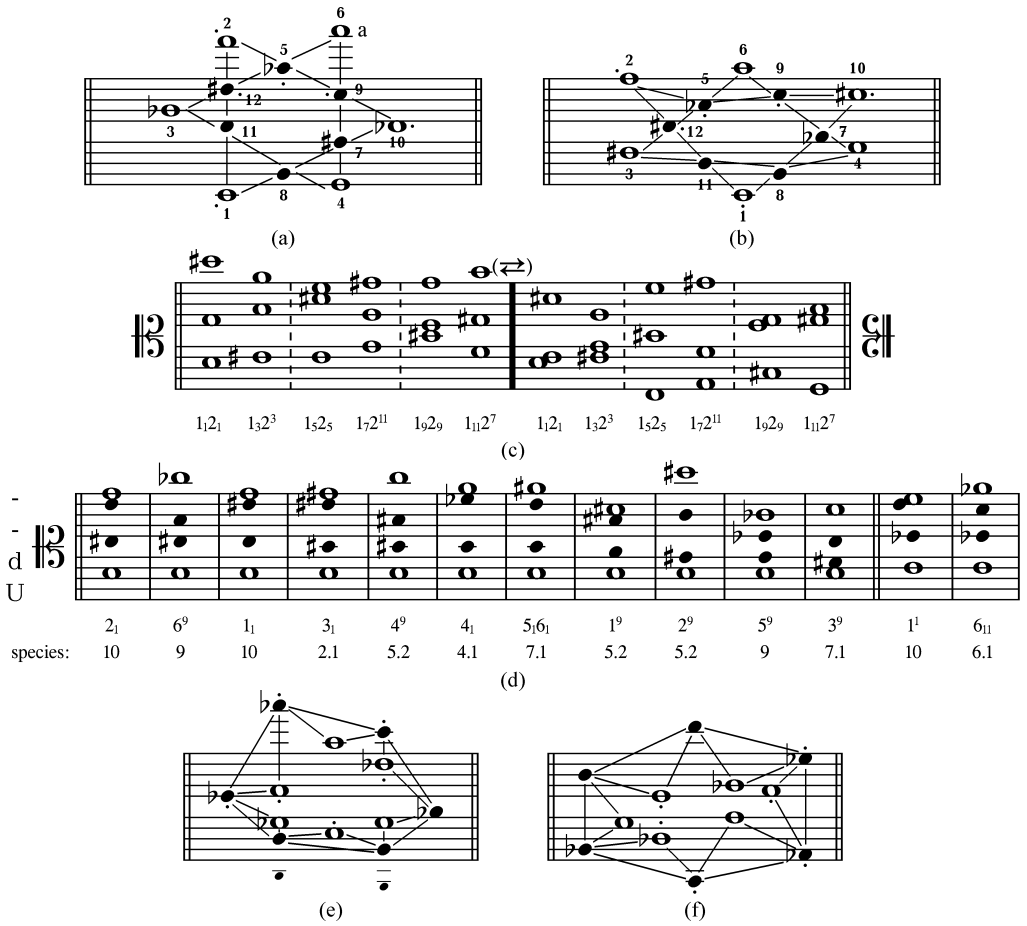

Figure 4: *The fragments from the score: from* Vertical Stars (a); *from* Horizontal Stars (b); *from* Outer and Inner Triangles 1 (c); *from* Ordered Sides 1 (d); *from* Vertical Antistars (e); *from* Horizontal Antistars (f).

Stars and their components are sorted, classified and distributed in the score in a systematic way (often followed by index marks of different types). In terms of comparison, the score is a dictionary rather than a novel. Any traces of 'subjectivity' and 'expression' have, as far as possible, been eliminated. Conversely, the performer's subjectivity is entirely permissible: any sort of expression may be generated by him and 'added' to the music. While note pitches are fixed in the score, note durations and their time positions are not. Each table or its fragment may be played from left to right, from right to left, from top to bottom, at any angle 'diagonally', etc. If desired, the notes may follow one after another in any other order, with reiterations and omissions. (For example, instead of playing a three- or four-note chord progression, the performer can play only the *soprano* or the *alto* part.) This principle also applies generally: for any particular performance any number of tables can be selected, which may follow one another in any order, with reiterations, etc. When playing, one can follow and emphasise – or hide and break – a logical structure in any desired way.

The score in its entirety and any of its components can also be compared to a landscape or a park. While the spatial structure is well defined, the routes are not, but are determined by the walker's moods, wishes and decisions, either spontaneous or planned ahead, which may be different each time. One can experience the things of the outer world in a deeply personal and intimate way. The music in itself is 'emotionally void'. However, when communicated with, it may be richly 'coloured' and 'filled up' with human emotions and feelings, like a sunset, a river or a starry sky...