# Building Simple and Not So Simple Stick Models

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### Abstract

Physical models are invaluable for conveying concepts in geometry. In this paper, I explain how to build stick models based on the Platonic polyhedra. Supplies for these models were thin bamboo shish kebab sticks from a grocery store, and vinyl tubing from a hardware store; both supplies are inexpensive and readily available. The tools used were a ruler for measuring the length of sticks, a clipper to cut the sticks, a scissor to cut the tubing, and a punch to make holes in the tubing. These tools are also reasonably inexpensive and readily available. Grade school, high school, undergraduate and graduate level students have made models with these supplies and tools and all of them have taken away something meaningful related to their existing level of knowledge.

## 1. Introduction

Inspiration to produce these simple stick models came from Ron Resch, my Ph.D. thesis advisor, with his intriguing film entitled “The Paper and Stick Thing Film” [1]. My investigation into building stick models includes an interest in using them in the class room. With some preparation of materials before the class, these models can be built in class. The supplies are widely available and inexpensive, and the tools exist in most home tool boxes.

It is important to think of a physical model of a regular polyhedron as a “sketch” of the concept for that polyhedron. There is perfection associated with a regular polyhedron which makes it impossible to achieve with any physical materials. Failure to meet perfection is true no matter how precise the physical pieces are for building the model. Consequently, the less exact the pieces become, the more of a “sketch” the model becomes. However, these “sketches” support the concept and are useful examples for discussing many geometric relationships.

My models are all based on the Platonic solids. They serve as a significant foundation for the study of three-dimensional space [2]. The better a student understands these models, the better that student is able to understand and imagine more sophisticated concepts about three-dimensional space. Additional understanding of these models is achieved by adding the sense of touch to the sense of sight. A student who is able to handle these models, reinforces their knowledge gained through sight. The size of the models is also important. It has been studied by J.J. Gibson, [3] that a model which is held in the hands provides insight. A model that is approximately a handful, is better than one that is considerably smaller or considerably larger [3]. A model the size of a students hand can be quickly rotated providing many views. A geometric relationship that exists in many places in a model can be seen nearly simultaneously. This multiplicity of views reinforces both the local and global nature of geometric relationships.

Regular polyhedron models have a very simple definition. A regular model consists entirely of a single-sized regular polygon, whose edges are all the same length. All vertices of a regular solid have the same number of regular polygons or edges meeting at that vertex. The number of edges meeting at a vertex is the ***degree*** of that vertex. Each vertex looks the same as every other vertex of a regular solid. Therefore, once a single vertex is made for a model the remaining vertices of that model are all the same. This condition implies that such vertices have equal solid angles. In Coxeter's Regular Polytopes [4], page 15, it states that solids with regular faces and regular solid angles are regular solids.

The remainder of this paper provides a list of supplies, tools, and directions for constructing stick models related to Platonic solids. The models serve to highlight the Platonic solids, their mid-edges, their mid-faces and their mid-cell, as well as, provide views of their dual polyhedron.

## 2. Supplies and Tools

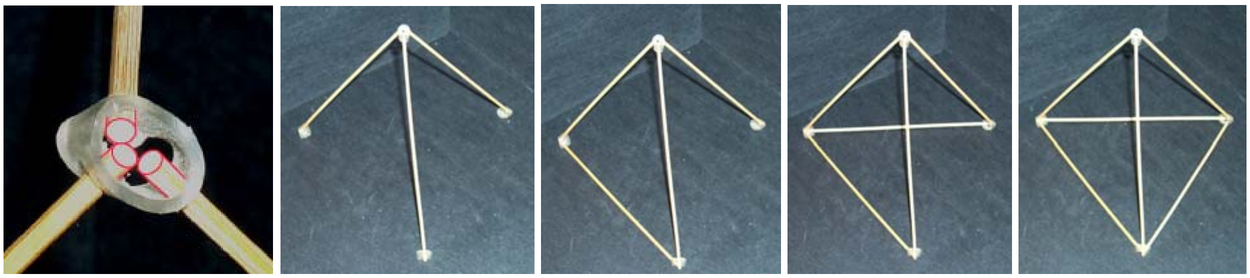
Supplies for these models consist of thin bamboo shish kebab skewers and three sizes of vinyl tubing. Sticks were purchased from a variety of supermarkets, and come in a standard length of approximately 10 inches. Any length of stick in this vicinity can serve to build these models. The vinyl tubing can be purchased from a variety of hardware stores. The size of the tubing used relates to the ***degree*** of the vertices for the model. Tubing measuring ¼" is used for ***degree*** three vertices, with ½" tubing used for ***degree*** four vertices and ¾" tubing used for ***degree*** five and ***degree*** six vertices.

A ruler is used for measuring the sticks. A clipper is used to cut the sticks. A scissor is use to cut the tubing and a punch is used to make holes in the tubing. Once a stick is cut to length it becomes an ***edge*** in a model and is referred to as an ***edge*** for the remainder of this paper. The strength of your grip comes into play when cutting the sticks. Different people may well decide on different tools for this repetitive task. A scissor with blades of approximately 4" will successfully cut the vinyl tubing. The tubing is cut producing a ring of tubing approximately ¼" in length. For the remainder of this paper this ring of tubing will be referred to as a ***vertex***. A leather punch with a single #6 size punch can be found at a crafts store. It is used to make holes in the vinyl tubing, so that ***edges*** easily insert into the holes to create a ***vertex***. It is best if the size of the hole and the size of the stick are closely matched so that the tubing will serve to hold the sticks firmly, and do not easily slide out of the hole. Also the size matching allows the stick to be easily inserted into the ring.

## 3. Platonic Solids, High-Lighting Their Mid-Edges, Mid-Faces, Mid-Cells and their Duals.

**3.1. Tetrahedron.** I started my stick models with the most basic of regular Platonic solids, the tetrahedron. I cut 6 sticks 8 3/4" long for the six ***edges*** and cut four pieces of ¼" tubing for the 4 ***degree*** 3 ***vertices***. I used the punch to make three evenly spaced holes around the ring of tubing at approximate 120 ***degree*** separation between the holes in each of these 3 ***vertices***. With these ***edges*** and ***vertices***, I assembled a model. This model, although very simple, served to establish an approach for building more complex models. I started by inserting three ***edges*** into one ***vertex*** **Figure 1a**. On the end of each of these three ***edges*** I placed a ***vertex*** **Figure 1b**. An ***edge*** was inserted between two of these three ***vertices,*** **Figure 1c**. I inserted another ***edge*** into the remaining hole of one of those ***vertices,*** and placed the other end of that ***edge*** into a hole of the remaining ***vertex*** **Figure 1d**. Now only one ***edge*** remained and two holes remained in two ***vertices***. When this final ***edge*** was inserted into the final two holes of those ***vertices*** a tetrahedron was formed **Figure 1e**.

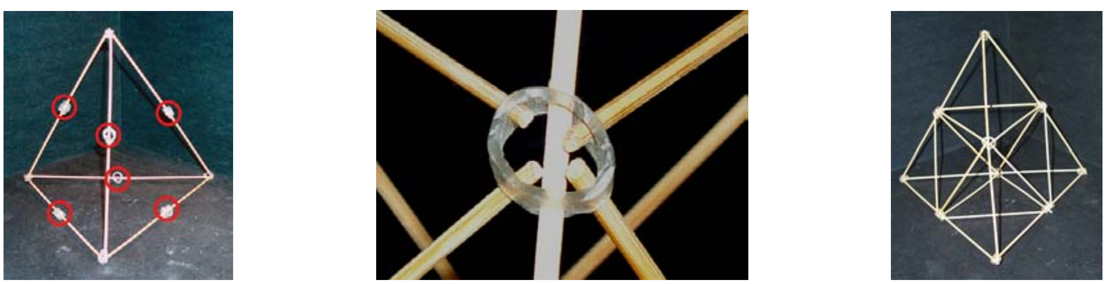
Note: A finished tetrahedron can be adjusted to more closely approximate the ideal of a regular tetrahedron. Take a close look at each of the ***vertices*** to adjust the position of the ***edges*** within the hole of the ***vertex*** so that the ends of these ***edges*** closely resemble edges that meet at a theoretical point in the center of this ***vertex*** **Figure 1a**. Repeat this observing and adjusting of the remaining ***vertices,*** and your model will have an appearance closer to the ideal.



**(a)** **(b)** **(c)** **(d)** **(e)**

**Figure 1**: 1 ***vertex*** **(a)**, 3 ***edges*** **(b)**, 4 ***edges*** **(c),** 5 ***edges*** **(d),** a tetrahedron **(e).**

**3.2. Mid-Edge Tetrahedron.** The next model built drew the viewer's attention to the mid-edge of the tetrahedron. I cut 6 11½" ***edges*** and 4 ***degree*** 3 ***vertices***. In addition, 12 5¾" ***edges*** were cut, as well 6 ***degree*** 6 ***vertices*** were cut and punched from ¾' tubing. Before assembling the tetrahedron, each ***edge*** of the tetrahedron had one of the ***degree*** 6 ***vertices*** slide onto each ***edge*** so that the ***vertex*** was positioned near the middle of the ***edge*, Figure 2a**. Holes on opposite sides of each ***vertex*** were used with an 11½" ***edge***. Once each ***edge*** had a ***vertex*** positioned near its mid-point a tetrahedron was assembled**.** The ***half-edges*** were inserted into the holes of the mid-edge ***vertices*** **Figure 2b.** An octahedron is formed with these 12 ***edges*** **Figure 2c.**



**(a)** **(b)** **(c)**

**Figure 2**: vertex at mid-edge (**a),** ***vertex*,** full-***edge*** & half-***edges*** **(b)**, tetrahedron & octahedron **(c)**

**3.3. Mid-Face Tetrahedron.** The next model drew attention to the ***mid-face*** of the tetrahedron. This model consisted of 6 11¼" ***edges,*** 12 6½" ***edges***, 6 3¾" ***edges*** and 8 ***degree*** 6 ***vertices***. Four of the 8 ***vertices*** were combined with the 6 shorter ***edges*** to form a tetrahedron as seen in the center of **Figure 3a**. These ***edges*** will use every other hole in the ***degree*** 6 ***vertices***. Each of the 12 mid-length ***edges*** will have one of their ends inserted into the open holes in the 4 ***degree*** 6 ***vertices*** of the recently assembled smaller tetrahedron. These ***edges*** will be collected into groups of three and then inserted into the remaining ***degree*** 6 ***vertices*.** Finally, the long ***edges*** were inserted into the open holes of the 4 ***degree*** 6 vertices to form an outside tetrahedron **Figure 3a*.***

**3.4. Mid-Cell Tetrahedron.** The next model drew the viewer's attention to the mid-cell of the tetrahedron. This model had 6 9 ¼" ***edges***, 4 6" ***edges***, 5 ***degree*** 4 ***vertices***. The 6 longer ***edges*** and 4 ***degree*** 4 ***vertices*** were assembled into a tetrahedron **Figure 3b**. The remaining 4 shorter ***edges*** would have one end inserted into each of the 4 ***vertices*** of this tetrahedron. The 4 free ends of these shorter ***edges*** were inserted into the remaining ***degree*** 4 ***vertex.***

**3.5. Dual Tetrahedron.** For this model 6 11" ***edges*** and 24 5½" ***edges*** were cut with 8 ***degree*** 3 ***vertices*** and 6 ***degree*** 8 ***vertices***. This assembly is very similar to the Mid-Edge tetrahedron assembly previously.

On 6 of the longer ***edges*** place a ***degree*** 8 ***vertex*** near the mid-edge **Figure 3c**. These 6 ***edges*** were assembled into a tetrahedron with 6 ***degree*** 3 ***vertices***. With 12 of the shorter ***edges*** were inserted into the ***degree*** 8 ***vertices*** as in ***Figure 2c***. The remaining 12 shorter ***edges*** will be inserted into the 4 ***degree*** 3 ***vertices*** in groups of three. Their free ends were inserted into the ***degree*** 8 ***vertices*** to form a second tetrahedron that was the same size as the initial tetrahedron for this model**.**

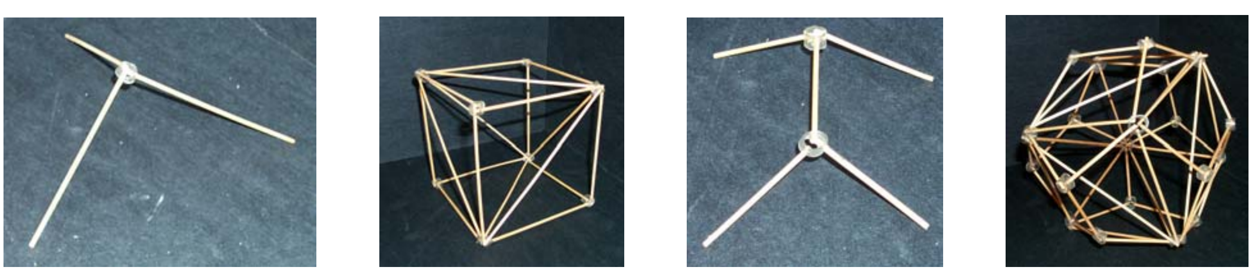
**(a)** **(b)** **(c)**

**Figure 3**: mid-points of faces **(a),** mid-point of fcell **(b),** stella octangula **(c)**

When a single tetrahedron is joined together with a second tetrahedron that is identical in size in the manner described above a stella octangula is formed. This model is also the stellation of a regular octahedron. A stellation is formed when non-adjacent faces are extended so that they intersect each other and the edges resulting form star polygons as in Coxeter [4] Section 6.2 Page 96.

**3.6. A Tetrahedron Inside a Cube and that Cube Inside a Dodecahedron.** For this model there were 6 6 7/8" ***edges*** with 4 ***degree*** 9 ***vertices*** to build an initial tetrahedron. Use every third hole in these 4 ***vertices.*** Subsequently, 12 5 1/8" ***edges*** were cut with 4 ***degree*** 6 ***vertices.*** Three of the shorter ***edges*** were inserted into each of the ***degree*** 3 ***vertices*** forming a corner **Figure 4a**. Each of the three free ends of this corner were inserted into every third open holes of the 4 ***degree*** 9 ***vertices*** on one face of the tetrahedron **Figure 4b.** When this model is completed there is a cube outside of a tetrahedron.

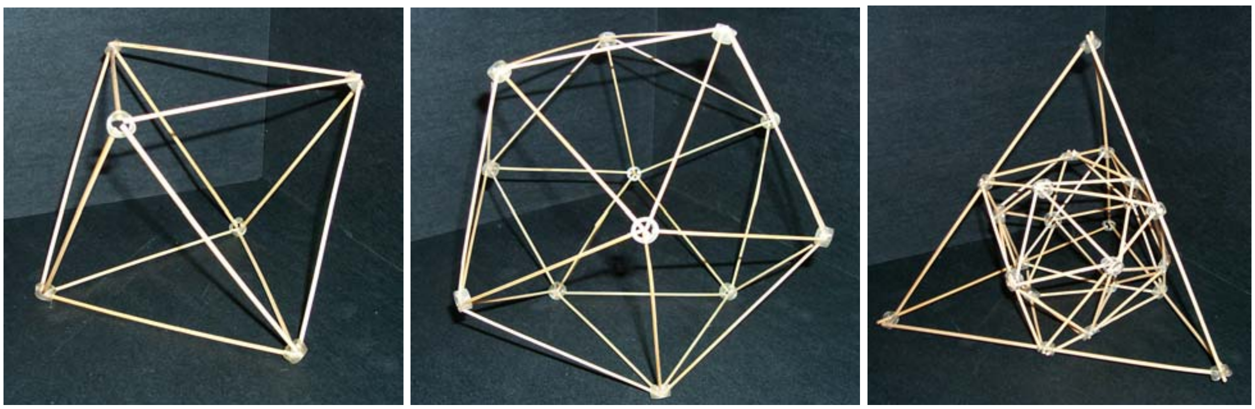
To form a dodecahedron outside this cube 30 2 7/8" ***edges*** were cut along with 12 ***degree*** 3 ***vertices***. Five of these shorter ***edges*** and two ***degree*** 3 ***vertices*** formed a ***winged edge*** assembly **Figure 4c**. The four free ends of this ***winged edge*** assembly were inserted into open holes on the four ***vertices*** on one ***face*** of the cube. The central ***edge*** of the ***winged edge*** assembly needed to alternate its direction so that there are five of these shorter ***edges*** surrounding a single ***edge*** of the cube **Figure 4d.** These five shorter ***edges*** combine to form a pentagon and the twelve ***edges*** of the cube are inside twelve pentagons that form a regular dodecahedron outside the cube **Figure 4d.**



**(a)** **(b)** **(c)** **(d)**

**Figure 4**: cube corner **(a),** cube outside **(b)**, winged edged **(c),** dodecahedron outside **(d)**

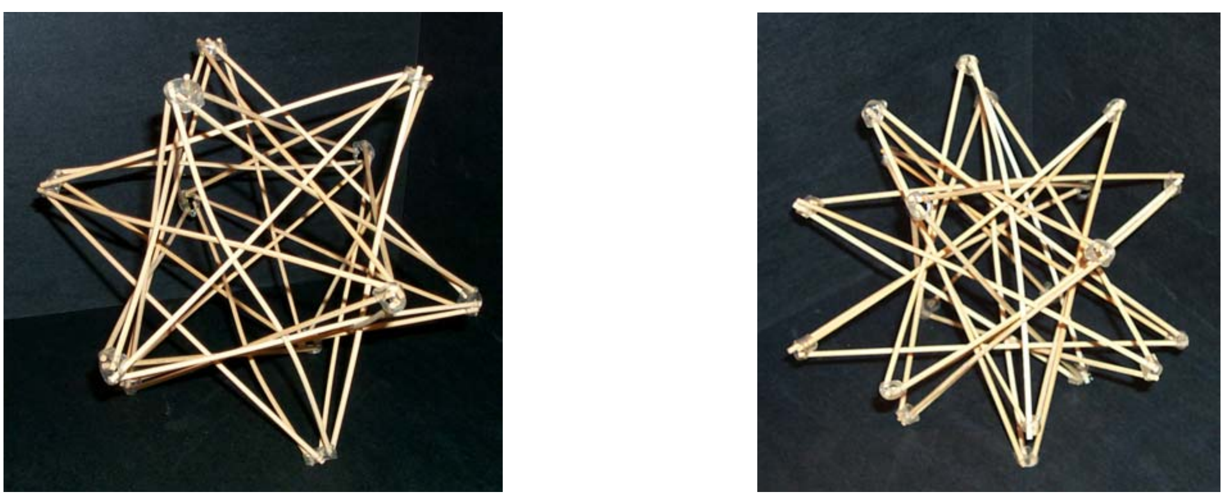
**3.7. Octahedron, Icosahedron, Tetrahedron with octahedron and icosahedron inside.** Building an octahedron with 12 ***edges*** and 6 ***drgree 4 vertices Figure*** **5a** and an icosahedron with 30 ***edges*** and 12 ***degree*** 5 ***vertices*** **Figure 5b** were very similar to building the initial tetrahedron. Taking the model from **Figure 2c** with the octahedron inside the tetrahedron an additional polyhedron was built inside this octahedron. For the icosahedron 12 ***degree*** 6 ***vertices*** and 30 3 5/16" ***edges*** cut. For this model the ***degree*** 6 ***vertices*** will be placed on an edge of the octahedron but not at its mid-edge. This ***vertex*** will be placed at a location that divides the ***edge*** into the golden ratio (approximately 3½" and 2¼" for the 5¾" ***edge***). Within each of the 8 ***faces*** of the octahedron a ***face*** of an icosahedron was formed. Near each of the 6 ***vertices*** of the octahedron 2 ***faces*** of the icosahedron were formed for a 20 face icosahedron **Figure 5c**.



**(a)** **(b)** **(c)**

**Figure 5**: octahedron **(a)**, icosahedron **(b),** tetrahedron , octahedron, icosahedron **(c)**

**3.8. Stellated Dodecahedron & Stellated Icosahedron.** Just for fun two stellations were built for a dodecahedron and an icosahedron **Figure 6**. These models were spatially and visually quite interesting as well as being less simple to build from shish kebob stick for ***edges*** and holes punched in vinyl tubing rings for ***vertices.*** Each has 30 ***edges,*** where one has 12 ***degree*** 5 ***vertices*** and the other has 20 ***degree*** 3 ***vertices.*** These models were assembled starting with a single ***vertex*** and observing star polygons.



**(a)** **(b)**

**Figure 6**: stellated dodecahedron **(a)**, stellated icosahedron **(b)**

## 4. Handling Stick Models

Each of the models in this paper has multiple axes of symmetry. The presence of so much symmetry leads the students into spending a considerable amount of time with the handling of these models. Using index fingers to help orient these models aligns the different axes of symmetry for vertices, edges and faces of each model to more clearly observe the symmetry. These models can have a vertex, an edge or a face set on a flat surface to study them individually. Some models can have a set of vertices set on a flat surface and studied from this orientation. All in all, a long time can be spent with this set of models observing them individually and using them as examples for other concepts to be studied by students at all levels of their education.

## 5. Conclusion

I have presented these simple stick models to students ranging from elementary school to graduate school for a period of more than twenty years. These presentations have always been well received by both the students and their teachers. The teachers have shown their appreciation by wanting to keep the models to hang in their classrooms and serve as constant examples for future study. Recently, I presented the models to eighth grade students at the McGillis School in Salt Lake City, Utah. This class has fifteen students interested in learning and enthusiastic about a presentation that includes models. They listened to my words about each model, but they were most interested in their opportunity to handle the models. I have been giving presentations to this group of students since they were in pre-school as our daughter is a member of this class.

Each group of students, depending on their individual maturity and their educational experience, takes away something different, even though the materials are exactly the same and the words used are very much the same. Emphasizing the tactile experience of handling each of the models provides the students with a recognizably different experience than merely viewing three-dimensional images of the regular solids on a piece of paper or on a computer screen

With the stick models, as opposed to the paper models [2], the students think of themselves as having x-ray vision, seeing from the near side to the far side of the models. Also, students can visually and physically align geometric relationships from both the near side of the model and the far side as they are study each of the models individually.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## References


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