

# On Mathematics, Music and Autism

Ioan James  
Mathematics Institute  
University of Oxford  
United Kingdom  
E-mail: imj@maths.ox.ac.uk

## Abstract

A discussion of research into the psychology of mathematicians, especially in relation to autism, and possible links with the psychology of musicians.

Every science has its own culture and that of mathematics is quite distinctive. As Henri Poincaré said ‘mathematics is the activity in which the human mind seems to take least from the outside world, in which it acts or seems to act only of itself and on itself, so that in studying the procedure of geometric thought we may hope to reach what is most essential in the mind of man.’ When Bertrand Russell asserted that ‘mathematics rightly viewed possesses not only truth but supreme beauty — a beauty cold and austere, like that of good sculpture ... supremely pure, and capable of a stern perfection such as only the greatest art can’ he was expressing an extreme view which applies, if at all, only to certain kinds of mathematics. Although Russell was writing a long time ago and expressing his personal feeling for the discipline there are modern mathematicians who would agree with him that mathematics has a special quality, different from that of any other kind of science. In fact many do not regard it as a science at all, rather as one of the arts, and research in the discipline as art for art’s sake. For the pure mathematician, the interest seems to lie in the mathematics itself, rather than its application to the real world. Poincaré wrote about ‘the feeling of mathematical beauty, of the harmony of numbers and forms, of geometric elegance. This is a true aesthetic feeling which all real mathematicians know.’ The contemporary French mathematician Alain Connes explained that ‘exploring the geography of mathematics, little by little the mathematician perceives the contours and structure of an incredibly rich world. Gradually he develops a sensitivity to the notion of simplicity that opens up access to new, wholly unsuspected regions of the mathematical landscape.’

Psychologists, especially cognitive psychologists, have long been interested in mathematicians, and mathematicians have long been interested in cognitive psychology. The Leipzig neurologist Paul Möbius (grandson of the mathematician, August Ferdinand Möbius), an enthusiast for the pseudoscience of phrenology, wrote about mathematicians in his book *Die Anlage zur Mathematik*. in the late nineteenth century. Just a century ago, Henri Poincaré gave a famous lecture on the ‘Psychology of Mathematical Invention’ at a conference of psychologists in Paris, while about the same time Felix Klein ran a seminar on the subject at Göttingen. Hadamard’s well-known monograph *The Psychology of Invention in the Mathematical Field* [1] refers extensively to Poincaré’s lecture during which reported extensively on his personal experiences. Poincaré explained that he relied greatly on his unconscious mind. Hadamard’s investigations confirmed that many creative mathematicians also rely on intuition although perhaps not to the same extent. Cognitive scientists tell us that most thought is unconscious. They distinguish between verbal thinkers and visual thinkers. My impression is that many mathematicians think in pictures, for example geometers and mathematical physicists, but many others are more verbal thinkers, for example mathematical logicians, but no research has been done on this, as far as I know.

There has been some research into the incidence of psychological disorders among mathematicians. The conclusion seems to be that the incidence of mental illness is much the same as in the general population. Depression is quite common. However, whereas in literature, especially poetry, the incidence of manic-depression is remarkably high, this is not so in mathematics. The same is true of other forms of mental illness. There has also been some research into the incidence of myopia, and one study concludes that this is exceptionally high among mathematicians. Research is being done to try and discover whether there really is a mathematical gene, as has sometimes been suggested, or more plausibly that genetic factors may be involved in the development of a module in the brain which is associated with mathematical activity.

However, whereas the incidence of mental illness is not especially high among mathematicians the situation is quite different in the case of disorders on the autistic spectrum, especially the mild form of autism known as Asperger's syndrome. Hans Asperger was a Viennese paediatrician who, in 1944, wrote about the disorder to which his name has been attached. The symptoms of Asperger's are generally grouped under six headings, namely impairments of social interaction, all-absorbing narrow interests, repetitive routines, speech and language peculiarities, problems of non-verbal communication, and possibly motor clumsiness. Under each heading there are a bewildering variety of ways in which the disorder can manifest itself; no individual will exhibit more than some of them, although there should be at least one under most of these broad headings. The disorder usually shows itself in early childhood, and is present throughout life. It is estimated that it affects about 1 in 200 of the general population, males much more than females.

As Asperger himself observed: 'To our own amazement, we have seen that autistic individuals, as long as they are intellectually intact, can almost always achieve professional success, usually in highly specialized academic professions, often in very high positions, with a preference for abstract content. We found a large number of people whose mathematical ability determines their professions.' It is well-established that people with Asperger's syndrome are drawn to mathematics and similar subjects. Mystified by the social world they take refuge in the certainties of mathematics. They tend to enter professions such as computer science, also certain types of engineering. Among creative mathematicians of the past, it is thought that Isaac Newton, Norbert Wiener, Alan Turing, Ronald Fisher, Kurt Gödel and Paul Erdős, amongst others, exhibited Asperger traits, perhaps also Sophie Germain and Emmy Noether. But there is quite a difference between saying that someone exhibited such traits and saying that they had Asperger syndrome, as I shall now explain.

In psychiatry the assessment procedure for a patient with personality problems is fairly standard. Some disorders have obvious physical or behavioural signs but for autism there is no single sign which would uniquely secure the diagnosis. The whole history of the patient has to be considered from birth, the nature of the impairments, their severity and their change over time. Then the person would be observed directly, usually by more than one professional, and would be given a range of standard tests. To facilitate the work a standardised interview procedure is used, so that leading questions are avoided and to ensure that alternative diagnoses are excluded. There are handbooks which contain the currently agreed diagnostic criteria in the form of lists. They are updated from time to time in the light of increasing knowledge. This underlines the fact that a complete set of scientifically objective criteria for the diagnosis of mental disorders has not yet been established. Although a good diagnostician relies as much on experience and intuition as on textbook knowledge nevertheless clinicians need objective procedures or else their conclusions may fail to convince.

Obviously it is impossible to carry out these procedures fully in the case of someone no longer alive. The best that can be done is to search, in the biographical literature, for evidence which is relevant to the standard tests, and then make a judgment based on experience and intuition. Most of the authors of books on autism identify cases of Asperger syndrome, often from far back in history. Uta Frith gives some

examples in her book *Autism: Explaining the Enigma*, such as one of the original followers of St Francis of Assisi is an example. Michael Fitzgerald gives some others in his books *Autism and Creativity* and *The Genesis of Artistic Creativity* [2]. Temple Grandin, who has personal experience of the disorder, has also identified people she believes had Asperger syndrome in her *Thinking in Pictures*. Amateurs have also entered the field, for example Norm Ledgin has suggested some Asperger possibles in his *Asperger's and Self-Esteem*, while I have combed the literature for others in my recent book, *Asperger's Syndrome and High Achievement* [3]. I have given Erik Satie and Bela Bartok as examples of composers with the syndrome, Jonathan Swift and Patricia Highsmith as examples of writers, and Vincent van Gogh as an example among painters — there are plenty of others.

What light does this throw on the nature of mathematical creativity, and of creativity generally? Asperger wrote: 'It seems that for success in science or art a dash of autism is essential. For success the necessary ingredient may be an ability to turn away from the everyday world, from the simple practical, an ability to rethink a subject with originality so as to create in new untrodden ways, with all abilities canalised into the one speciality.' Those who have the syndrome live very much in their intellects and certain forms of creativity benefit greatly from this. Certain aspects of the syndrome e.g. workaholism and an extraordinary capacity for persistence can accompany many forms of creativity. When Isaac Newton was asked how he conceived the theory of gravitation he replied that, 'It was through concentration and sheer dedication. I keep the subject constantly before me, till the first dawning opens slowly, little by little and little into the full and clear light.' The Asperger ability to focus narrowly on a topic and resist distraction is particularly important in mathematics. Mathematicians tend to have strong and narrow interests, and a high degree of focus. An enormous capacity for curiosity and a compulsion to understand are evident in those who have the syndrome, as is a tendency to reject received wisdom and the opinions of experts.

The reader may well wish to be presented with qualitative evidence of the link between science and autism. Fortunately this has recently become available. Simon Baron-Cohen [4] has devised a self-administered questionnaire for measuring the degree to which an adult with normal intelligence has the traits associated with the autistic spectrum. From the answers to the questions a number is obtained, which he calls the autistic-spectrum quotient, and this gives an estimate of where a given individual is situated on the continuum from normality to autism. When the questionnaire was given to 4,175 students at Cambridge University it was completed and returned by over one fifth, with no significant difference in the return rate between disciplines. Natural scientists (including engineers and mathematicians) scored significantly higher than both humanities and social sciences students, confirming the general belief that autistic traits are often associated with scientific skills. Briefly, scientists scored higher than non-scientists; and within the sciences, mathematicians, physical scientists, computer scientists and engineers scored higher than the more human or life-centred sciences of medicine and biology. Further investigation has revealed that a disproportionate number of mathematics students have received a diagnosis of autism, and that autistic individuals tend to have an unusually high proportion of engineers in their families.. Details will be found in a forthcoming article by Baron-Cohen et al [5].

There is far more to be said about this kind of thing than can possibly be included in a short lecture; so may I now refer members of the audience to the forthcoming book *The Mind of the Mathematician* by Michael Fitzgerald and myself [6]. In this there is a section about mathematicians and musicians which I thought might be especially relevant to this conference, and so I will now say something about this. I will also refer to Fitzgerald's *The Genesis of Artistic Creativity* [2] which deals more specifically with possible links between autism and musical talent. Hadamard emphasizes that there are strong resemblances between the psychology of invention in different fields of the arts and sciences and specifically quotes a letter of Mozart's on musical invention.

There seems to be such a contrast between the science of mathematics and the art of music, yet there is a long-standing belief that they are related in some way. As Leibniz wrote to his friend Goldbach, music is a hidden exercise in arithmetic, of a mind unconscious of dealing with numbers. In the words of von Helmholtz ‘mathematics and music, the most sharply contrasted fields of intellectual activity which can be found, and yet related, supporting each other, as if to show forth the secret connection which ties together all the activities of our mind.’ Some composers seem to have been interested in numerology, if that counts as mathematics. Mozart was one of these and we know that his interests extended into elementary number theory. Stravinsky remarked that [musical form] is at any rate far closer to mathematics than to literature — certainly to something like mathematical thinking and to mathematical relationships. However it is not easy to think of many musicians, whether composers or performers, who were particularly interested in mathematics.

On the other hand, it is easy enough to name people of mathematical ability who were also musical. For example, there was Georg Cantor, who might have followed in the footsteps of his mother’s family of musicians and sometimes regretted that he had not. There was James Joseph Sylvester, who believed he had a fine voice and took singing lessons from Gounod. He is quoted as saying ‘may not music be described as the mathematics of sense, mathematics as the music of reason?’ The many-sided Olinde Rodrigues had some talent as a composer. Richard Dedekind was an accomplished pianist and cellist who composed a chamber opera. Janos Bolyai and A.C. Aitken were exceptionally-fine violinists. Further back we can cite the example of the little-known Hamburg mathematician, Johann Georg Busch, at whose home some of the compositions of Carl Philipp Emmanuel Bach were performed for the first time. Leopold Kronecker was an accomplished pianist and vocalist. Hermann Grassmann was a pianist and composer, some of his arrangements of Pomeranian folk-songs were published. He was also a good singer and conducted a male voice choir for many years. Felix Hausdorff was an excellent pianist and occasionally composed songs; he aspired to be a composer rather than a mathematician. There seems to have been a tendency for mathematicians to marry into musical families, for example Richard Courant and Jacques Hadamard did so, and when they entertained at home there was always music. The famous concerts of chamber music held at the home of the amateur mathematician and musician Emile Lemoine exerted a great influence on the musical life of Paris in the latter part of the nineteenth century.

Albert Einstein had a passion for music, as a way of experiencing and expressing emotion that is impersonal. He was an enthusiastic (but not very good) violinist; Mozart, Bach and Schubert were his favourite composers. Photographs of him playing the violin show a different Einstein from the more familiar images. When he was world-famous as a physicist he is reported to have said that music was as important to him as physics: ‘it is a way for me to be independent of people’; on another occasion he described it as the most important thing in his life.

Just after the Second World War the psychologist Geza Revesz [6] conducted a survey of 180 mathematicians, 220 physicists, 206 doctors, and 136 writers, who were asked to complete a short questionnaire. This revealed that 24% of the mathematicians were completely unmusical, as compared with 16% of the physicists, 19% of the doctors, 13% of the writers; 44% of the mathematicians were unmusical, as compared with 33% of the physicists, 41% of the doctors and 29% of the writers; 56% of the mathematicians were musical, as compared with 67% of the physicists, 59% of the doctors, and 71% of the writers; while 9% of the mathematicians were very musical, 9% of the physicists, 6% of the doctors and 11% of the writers. These results do not support the widespread belief that mathematicians generally are particularly musical.

If I had more time I would have liked to describe what has been written about the psychology of musicians, especially the significance of autistic traits in relation to musical talent, but there seems to have been relatively little research in this area. I have already mentioned some well-known composers

who are believed to have suffered from Asperger syndrome. Other musicians who may have had the syndrome include the pianist Glenn Gould and the conductor Carlos Schreiber.

Finally a few remarks about savants, which appear from time to time in certain fields; for example there are the lightning calculators. Savant skills seem to appear quite suddenly — they do not need to be consciously learned — and may vanish again equally suddenly. They are most striking when they occur in individuals with otherwise poor intelligence. Savants of various kinds have been much studied, and one conclusion of the research seems to be that they usually, but not always, are in some degree autistic. For further information see Beata Hermelin's book, *Bright Splinters of the Mind* [7].

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