# Interdisciplinary Bridges: A Novel Approach for Teaching Mathematics

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## Abstract

This paper describes examples of interdisciplinary exploration opportunities that encourage students to use critical and creative thinking skills as they gain understanding and ownership of mathematical ideas. Students enjoy a stimulating journey on a road of discovery.

"All genuine learning comes about through experience. . . Only by taking a hand in the making of knowledge, . . does one ever get a knowledge of the method of knowing."
John Dewey

## 1. Philosophy of Discovery Based Interdisciplinary Learning

The primary focus of my professional life is the development and dissemination of dynamic approaches to learning mathematics. This focus is built on the belief that the ideal course of study combines the inseparable partners of thought and content, and an essential part of education is to merge the teaching of both. Students need to learn how to think. The thought process must blend analysis with creativity. The goal of both secondary and university classrooms must be to create energetic, engaged learners. To achieve this goal, the ideal faculty member must utilize teaching strategies which necessitate student involvement in the process.

Interdisciplinary discovery projects involve students in a dynamic learning experience. These projects are group oriented; the focus of each project is on exploration, guided by leading questions which encourage mathematical discourse amongst the students. Students much more readily grasp theory that they have "discovered" independently. In the words of one of my students, "You really understand the concept well by the time you finish. In order to finish, . . you need to come up with the concept yourself and this makes it easier for you to use it in the future . . .. The element of discovery involved gives you self confidence . . . " Students taught in this manner experience a unique and extraordinary mathematical journey of learning and become more mathematically powerful.

This paper focuses on two projects. The first is based on *Flatland*, a mathematical novel written by Edwin A. Abbott, and enables students to see connections between mathematics, philosophy, and sociology. The second project is a portfolio assignment providing opportunities for creative writing.

## 2. Mathematics and Literature Project

**2.1. Description of the Project.**  Consider for a moment a geometry unit including philosophy, sociology, art, theology, and advanced mathematics.  In a freshman level high school geometry class, my students read *Flatland*, a mathematical novel written in the 1800s.  It is the story of a two dimensional world where woman are straight lines, and men are polygons with various numbers of sides depending on their social status.  The initial paragraphs of the novel immediately challenge our perception of reality.

> I CALL our world Flatland, not because we call it so, but to make its nature clearer to you, my happy readers, who are privileged to live in Space.
>
> Imagine a vast sheet of paper on which straight Lines, Triangles, Squares, Pentagons, Hexagons, and other figures, instead of remaining fixed in their places, move freely about, on or in the surface, but without the power of rising above or sinking below it, very much like shadows - only hard and with luminous edges - and you will then have a pretty correct notion of my country and countrymen. Alas, a few years ago, I should have said "my universe": but now my mind has been opened to higher views of things. . . .
>
> Place a penny on the middle of one of your tables in Space; and leaning over it, look down upon it. It will appear a circle.
>
> But now, drawing back to the edge of the table, gradually lower your eye (thus bringing yourself more and more into the condition of the inhabitants of Flatland), and you will find the penny becoming more and more oval to your view; and at last when you have placed your eye exactly on the edge of the table (so that you are, as it were, actually a Flatlander) the penny will then have ceased to appear oval at all, and will have become, so far as you can see, a straight line.

A sphere comes to visit the main character of *Flatland*; "A Square" is his name.  Conversations ensue as the Sphere tries to convince Square that there is a world beyond two space.

After the students have read the novel, a colleague from the humanities department leads the class in a discussion of various aspects of the novel, suggesting four categories of inquiry: philosophical, social, mathematical, and cross cultural.  The class is divided into groups; each group prepares a presentation explaining the novel in terms of their assigned category.  The oral presentation must include specific quotes supporting their ideas as well as a visual aid.  The open-ended assignment yields astonishing results.  Let us examine the ideas that the students develop.

**2.2. Philosophical Aspects.**  The group focusing on philosophy explains that universal truths in two-space are quite different from universal truths in three-space.  In two-space, a sphere is a circle, or a point, or nothing at all.  Yet in three-space, a sphere is a ball with which to play.  To illustrate their point, the students refer to the following quote.  "Were a four-dimensional figure to appear to us, we would only be able to see three of his dimensions.  He would likely appear to us as some kind of three dimensional solid, such as a cube or sphere, but he would be able to change his size and form.  He would actually consist of an infinite number of cubes or spheres or whatever, just as the sphere consisted of an infinite number of circle."  The students link these ideas to the differing beliefs of human beings living in dissimilar cultures.

**2.3 Social Aspects.**  This group focuses on the hierarchy of society in *Flatland*.  Women are the lowliest of the low, straight lines.  Next are the isosceles triangles.  Then there are regular polygons, starting with equilateral triangles, then squares, then pentagons, etc.  The priests are circles; they are considered regular polygons with many, many sides.  "Circles are the formal names of figures with sides far too numerous to count.  This was done out of respect for the highest member of a society in which angularity was despised and smoothness respected."  Again, the students explore the notion of a limit.  As the number of sides of a regular polygon increases without bound, the object becomes a circle.

**2.4 Mathematical Aspects.** The group exploring mathematics describes the sphere's visit to two-dimensional space. Initially the sphere touches Flatland only in one point, but it becomes a circle as it moves through the land. The students provide visual tools of this visit for their classmates. A peep box with a slit on the box to represent Flatland and a sphere inside the box which moves up or down by well constructed strings allows the sphere to be seen as a circle until it disappears. See Figure 1. The students describe how the circle "becomes" a point. The students also slice Styrofoam balls to illustrate the idea in another fashion. See Figure 2. Students begin to gain an intuitive idea of a limit. Years later, when studying calculus, these students already possess a general notion of the meaning of limit.



**Figure 1:** *A student views a sphere in the same manner as an inhabitant of Flatland.*



**Figure 2:** *Students demonstrate that a sphere is made from infinitely many circles.*

The students explain that "in the two-dimensional world of Flatland the citizens and all inhabitants alike are part of a razor thin existence, less than that of a sheet of paper." They begin to understand that although we represent two dimensional space and think of it as a sheet of paper, this representation is flawed.

**2.5 Cross Cultural Aspects.** The group dealing with cross-cultural aspects discusses the almost insurmountable difficulty of inhabitants from one land communicating with those from another. In the novel there are four cultures, Pointland, Lineland, Flatland, and Spaceland. See Figure 3. The students describe these cultures and the challenges of communication between and among them. The inhabitants view what they can not understand as having supernatural powers. The square thought sphere was a god because he couldn't understand it. To support their ideas the students cite the following quote. "It seemed that this poor ignorant Monarch – as he called himself – was persuaded that the Straight Line which he called his Kingdom, and in which he passed his existence, constituted the whole world, and indeed the whole of space. Not being able either to move or to see, save in his Straight Line, he had no conception of anything out of it. . Outside his world, or line all was a blank to him; nay, not even a blank, for a blank implies space; say, rather, all was non-existent." When inhabitants travel into other worlds, they experience unexpected differences. "An unspeakable horror seized me. There was a darkness; then a dizzy sickening sensation of sight that was not like seeing; I saw a Line that was no Line; Space that was not Space: I was myself, and not myself. When I could find voice, I shrieked aloud in agony, 'either this is madness or it is Hell'." Again, the students discuss the challenges of exposure to cultures different from one's own.



**Figure 3:** *Students use each side of a box to represent the four cultures in Flatland.*

The class soon begins to recognize that the boundaries between the mathematical, philosophical, social and cross cultural categories are blurred. Perhaps, these students truly experience the bridges between mathematics and other disciplines!

**2.5 Sample Assessment.** Creating opportunities for students to explore the realm of philosophical ideas can happen with well structured assessment. Consider the following question and response from the unit test including *Flatland*.

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Examination Question:    Explain the meaning of this quote in terms of how it relates to *Flatland* and mathematics in general.  Include a description of how we can envision a land of four dimensions.  "I    am not a plane Figure, but a Solid.  You call me a Circle;  but in reality I am not a Circle, but an infinite number of Circles, of size varying from a Point to a Circle of thirteen inches in diameter, one placed on top of the other."

Student Response 1:  "This is an example of how the inhabitants of Flatland see things.  They are unable to imagine a solid figure of three dimensions, just as we are unable to imagine a figure of four dimensions.  The inhabitant view the world as a thin slice of the third dimension so perhaps we view the world as a thin slice of the forth dimension."  [Spelling and grammar as in original.]

Student Response 2:  "This quote is the sphere trying to explain his form to the square who is unable to see all three dimensions of the sphere.  Because the square can only see two dimensions, he sees the sphere to be a circle able to change his size.  This is one of the fundamental ideas in Flatland; how different worlds with different numbers of dimensions can interact and the confusion their interaction causes.  This quote also helps us understand how we would see a land of four dimensions.  Were a four-dimensional figure to appear to us, we would only be able to see three of his dimensions.  He would likely appear to us as some kind of three dimensional solid, such as a cube or sphere, but he would be able to change his size and form.  He would actually consist of an infinite number of cubes or spheres or whatever, just as the sphere consisted of an infinite number of circle."

## 3.  Mathematics and Creative Writing

**3.1   Introduction**.  Good teaching orchestrates the learner's interdisciplinary experience so that all aspects of brain operation are addressed, such as analytical thinking, imagination, and emotions.  A portfolio can be an excellent vehicle for connections, as well.  Consider the following writing assignments from a portfolio project given to a high school algebra class.

**3.2.  Example 1.**  The student creates a link between Shakespeare and mathematics.  The class had recently completed a unit on graphs and transformations of functions.

Assignment:  Explain how you used mathematics or mathematical thinking in another class.

Response:

> In English class we are reading Richard III and are split into groups to discuss the tragedy of Richard.  As we were discussing this, I drew a physical representation of Richard's demise.  We believed that Richard saw his destiny through his dreams.  He was afraid of this fate, which was ultimately his demise.   This fear drove him to try to control his fate.  What happened was that at the same exact time that his success was apparent as he was becoming a king, he was actually falling farther from being a good man.  All of this is represented by the line I graphed, an absolute value, $x = |y|$, that makes its "v" around the x-axis.  The positive ray shows his attempts to control his fate by reaching high goals.  The negative ray shows where his actions are actually taking him.

**3.3.  Example 2.**  One portion of the portfolio assignment is to provide an example of a problem that was totally misunderstood, and the process that eventually led to comprehension.  One student wrote,

When I first stared at this problem it looked like a menacing fire-breathing dragon who could never be conquered, because it looked like it was going to be hard to succeed. Immediately, I ran to Dr. Kaplan for help, hoping she had some kind of magic which would subdue the dragon. After talking with her, the dragon seemed friendly enough, but once out of range of her mathematical magic, the fires returned. Frustrated, I finished the problem to the best of my abilities, and handed it in. When it was given back to me, complete with many helpful comments, the magic was turned over to me and the dragon disappeared in a cloud of smoke.

What a fabulous way to connect mathematics with creative writing!

## 4. Summary

These interdisciplinary projects are unique learning tools for motivating and encouraging students in the energetic pursuit of mathematical knowledge. Instruction and student involvement are so firmly interwoven that they form inseparable partners in the learning process. Nontraditional interdisciplinary projects serve as genuine learning experiences, allowing students to actively pursue mathematical knowledge in a cooperative setting. Each student is engaged in the learning process; passivity vanishes. Mathematical discourse is an essential ingredient for success. Students explore ideas with one another. In the sharing process, each listens intently to the ideas of others and together they traverse various paths and detours until a successful route is found. The student not only learns the mathematics and its relationship to other disciplines, but also enjoys the satisfaction of playing an active role in an investigative learning process. The journey is challenging, filled with pitfalls of frustration and peaks of excitement until the final "Aha!" moment is achieved.

## Reference

Abbott, Edwin A. *Flatland A Romance of Many Dimentions*, Dover Publications, 1992, Unabridged republication of *Flatland*, first publication Seeley & Co., Ltd., London 1884