RHYTHMOS: An Interactive System for Exploring Rhythm from the Mathematical and Musical Points of View

Jakob Teitelbaum and Godfried Toussaint*
School of Computer Science
Center for Interdisciplinary Research in Music Media and Technology
McGill University
Montréal, Québec, Canada
E-mail: godfried@cs.mcgill.ca

Abstract

This paper introduces RHYTHMOS: an interactive software system designed as a tool-kit for the visualization, exploration, understanding, analysis, praxis, and composition of musical notated (symbolic) rhythms. As such it provides user-friendly bridges between art (music composition), performance (praxis), mathematics (cyclic polygons and the distance geometry of point sets on a circle), and science (the psychology of music perception). A description is provided of the system's capability and interactive graphical user interface. Applications to teaching, learning, and practicing rhythms are discussed. Examples are given of the kinds of research that RHYTHMOS facilitates. These include the testing of rhythmic features for the classification, clustering, and phylogenetic analyses of families of rhythms.

1. Introduction

RHYTHMOS is an interactive software system designed as a tool-kit for the visualization, exploration, understanding, analysis, praxis, and composition of musical notated (symbolic) rhythm. It has a graphical user interface (GUI) that allows the user to enter, view and analyze rhythms in a wide variety of geometric representations [12]. Its sound system allows the user to hear the rhythms at different tempos and volume. More than one rhythm may be heard simultaneously, each with a different timbre. One key innovative feature of the system is the ability to view both the rhythm represented as a cyclic necklace on the circle of time, and the full-intervallic content (histogram) of all the inter-onset durations [15]. This allows the user to dynamically control the shape of the histogram and to listen to the resulting rhythms corresponding to the shapes displayed. Another key innovation is the ability to display a rhythm in vertical *braid* notation so that the user can play along following the left-hand and right-hand beats as they occur on the screen [10]. The system contains a library of rhythms, as well as a list of different rhythmic distance (similarity) measures [14] that may be computed to obtain a distance matrix for any family of rhythms selected from its library, thus facilitating the use of phylogenetic analysis programs such as *SplitsTree* [2], [3], [6], [13].

2. The Main Features of RHYTHMOS

2.1. Graphical User Interface. RHYTHMOS is a multi-window system. In the main window, pictured in Figure 1, the user selects one or more rhythms to be explored, and then clicks on the desired view or type of analysis. This, in turn, is displayed in a new window, which may be moved, resized or minimized.

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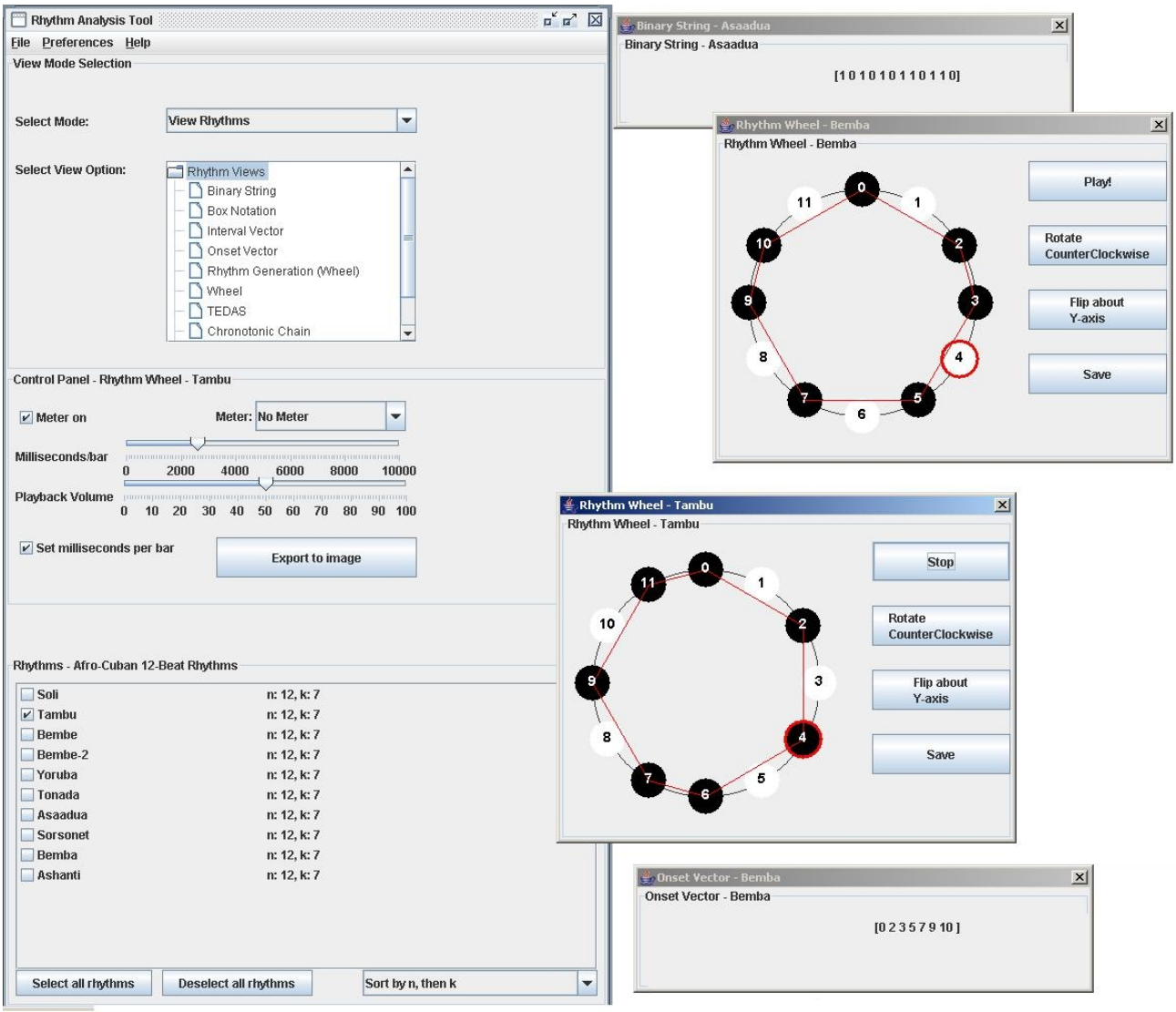

Figure 1: The main window (left) shows view options, a control panel, and a family of rhythms. The control panel has slide bars for tempo and volume adjustment. A variety of underlying meters can be turned on that accompany the rhythm with a different timbre. A window may also be exported as a figure. The four windows on the right show various rhythms represented from top to bottom, respectively, as a binary string, as polygons on circles of time, and as an onset-time-coordinate vector. The highlighted circle on pulse number four of the wheels indicates the pulse being played. Thus the user sees which pulse is being played at the same time it is heard. The position of the windows may be rearranged on the fly as desired.

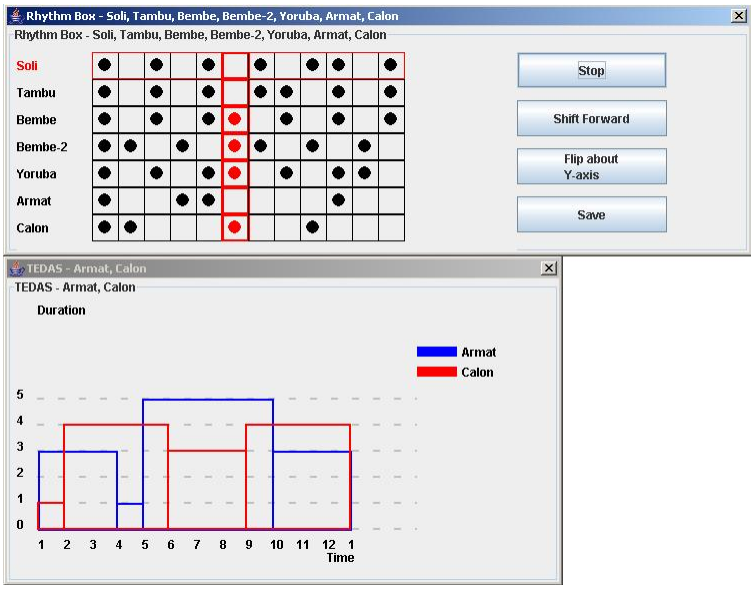

Figure 2: A screen shot of the box and TEDAS representations of some rhythms. The highlighted sixth column in the upper figure indicates that the sixth pulse is being "played" in all the rhythms simultaneously.

There are three basic modes to the program: Input Rhythm, View Rhythms, and Analyse Rhythms. The mode is chosen via the main window, which displays all relevant choices for that mode.

The Input mode allows the user to create rhythms manually, using any of six visual representations. Alternatively, the user may load a predefined library of rhythms from a file, or use existing rhythms to generate new ones, either manually or algorithmically, in the View mode.

Figure 2 is a screen shot illustrating two such visual representations of rhythms: the box notation and the TEDAS representation. The box notation is commonly used by ethnomusicologists. It was popularized in the West by Philip Harland at the University of California in Los Angeles in 1962, and is also known as TUBS (Time Unit Box System). However, it has been used in Korea for hundreds of years [7]. The TEDAS representation was proposed in 1987 by Kjell Gustafson, at the Phonetics Laboratory of the University of Oxford with the aim of visualization [4], [5]. Each temporal element (interval) is a box and both the x and y axes represent the time-durations of the interval.

In the View mode, the user may view one or more rhythms, and a variety of their properties, in any of ten displays. Via these displays, the user may also modify rhythms, generate new rhythms, and listen to rhythms. As mentioned, each display exists in its own window; these windows are dynamically linked. An example of one of the main innovative features of RHYTHMOS is the computation of the inter-onset interval histogram of a rhythm. For example, if the user modifies a rhythm called *Sorsonet*, then the *Sorsonet* histogram will instantaneously be recalculated with the altered rhythm. See the illustration in Figure 3.

The Analysis mode supports a variety of analyses, divided into Rhythm Properties and Distance Measures (dissimilarity measures). These properties may be used to compare rhythms in order to better understand them. Such comparisons help the user to remember rhythms and their inter-relationships, thus serving as a valuable pedagogical tool. RHYTHMOS has the capability of computing a wide variety of standard distances such as the Hamming distance between the rhythm's binary sequences, or the *chronotonic* distance discussed in [14], as well as several new distance measures, such as the *directed swap-distance* analysed in [1]. The chronotonic distance between two rhythms is the total area of the region contained in between their TEDAS functions of time.

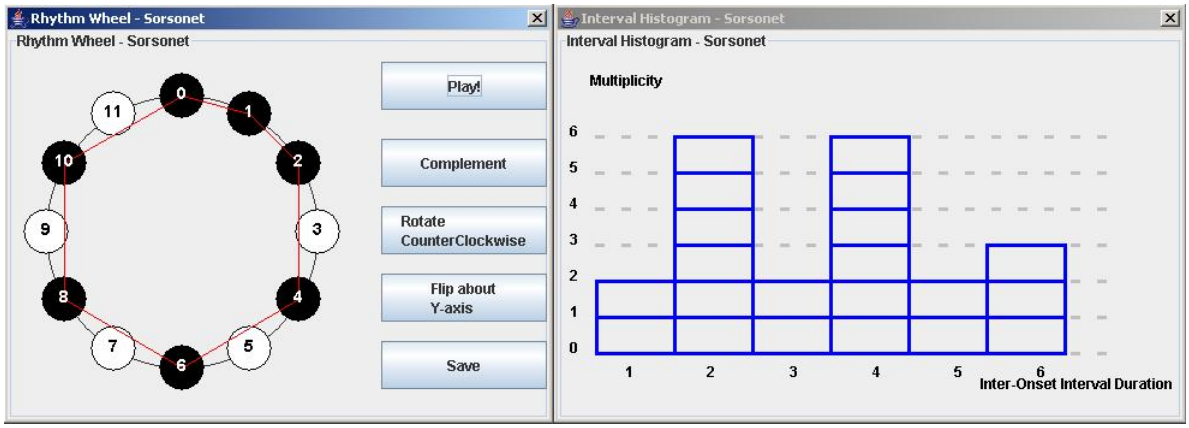

Figure 3: The wheel showing a *Sorsonet* bell-pattern, and its inter-onset-interval histogram. This histogram shows, for each inter-onset interval duration value, the number of times (multiplicity) it occurs in the rhythm.

While some view-specific buttons, such as 'Flip about Y-axis', are contained in the display window, most actions are performed via the main window. Similarly, although the main window shows some of the settings of the display windows (such as tempo), virtually all the information is contained in the display window itself. Thus, the interface is neatly divided into input and output: the user enters information in one window, and receives information from the others.

In addition to the interactive features described in the preceding, RHYTHMOS also helps the user by means of a comprehensive popup system: if the user, while modifying a rhythm, happens to turn it into another rhythm that is already in the library, a popup window will let the user know of the overlap and provide the name of the rhythm in question.

2.2. Audio Playback with Visual Update. RHYTHMOS also supports realtime visual feedback for audio playback of up to eight rhythms, each with an individual timbre and volume, as illustrated in Figure 2. Thus, as a displayed rhythm is played, a highlight visually traces the playback in the display window. Also supported is the simultaneous playback of multiple rhythms, even those having different values of n (number of pulses in the cycle) - this is handled by specifying the time duration of one cycle, regardless of the value of n. In addition to these parameters, the user may also control the tempo, and may choose a background meter.

2.3. System Features. Space limitations allow us to only briefly touch upon three features: Cross-Platform Conformance, Extensibility, and Error Handling.

2.3.1. Cross-Platform Conformance. RHYTHMOS is written entirely in Java. Windows, UNIX, Macintosh, and other computers need only install Java 1.5.

2.3.2. Extensibility. The system's modular, object-oriented design reflects an emphasis on extensibility: new analyses and views may be easily added without disrupting other components. Furthermore extra features, such as the exportation to image and audio files, were implemented with little reprogramming needed for the existing version.

2.3.3. Error Handling. RHYTHMOS contains a robust error handling system, designed to take care of invalid user-input without crashing or damaging existing data. When an error occurs, an informative message appears notifying the user, and describing the problem.

3. The Value of RHYTHMOS to the User

The above features illustrate only some of the capabilities of RHYTHMOS which underpin the true benefit that a wide variety of users will derive from the system. RHYTHMOS introduces new users to an unfamiliar field of study in a visual, practical, gentle, intuitive, exploratory way. In addition, scientists and mathematicians will appreciate its time-saving analytical tools, while users interested in understanding and learning musical rhythm, without learning music theory, will enjoy the multiple visual and aural representations of rhythm that RHYTHMOS offers, as well as its possibilities for playing along with such feedback.

3.1. Wide Accessibility. Much of the value of RHYTHMOS lies in the fact that it is so easy to use; anyone with the most rudimentary of computer skills may, within minutes, create, view, modify, listen to, and play along with rhythms. Furthermore, the user is also exposed to a diverse array of rhythms: on the RHYTHMOS website, rhythmos.cs.mcgill.ca, libraries of rhythms from around the world will be made freely available. Finally, anyone interested in learning about analytical tools, such as rhythmic properties or rhythm similarity measures, may directly apply these tools to rhythms of their choice. This learning-by-doing method provides an engaging and instructive complement to the existing literature in the field.

3.2. Education. RHYTHMOS may also function as a valuable educational tool. Many budding musicians have trouble grasping the abstract nature of musical notation; using the visual displays and audio playback, the rhythms become tangible, helping students learn to visualize and understand rhythms, and link them to Western music notation.

At present there is great interest in teaching mathematics in elementary schools using movement and music, particularly in the Montessori and Waldorf education systems [8]. RHYTHMOS is an ideal tool for accomplishing these educational goals. For example, since polygons represented on the circle of time are in fact rhythms, children may be taught the different types of polygons by listening to how they sound. Although the present version of RHYTHMOS is not directed mainly at young children, by simplifying the user interface to solely emphasize the creation, display, and playback of rhythms, and by disabling the analyses, the system provides an ideal environment for children to explore rhythm on their own. Indeed, a children's version called RHYTHMOS-Junior is also being launched.

3.3. Play-Along Practice. RHYTHMOS offers a variety of possibilities for the user to play along with the audio-visual output, using fingers or hands on a table or other instrument. One important skill for students to learn, is to play a different regular rhythm with each hand, such as 3 beats against 2, 4 beats against 3, etc. For example, in the case of 4 against 3, the left hand could play the 3 = [x . . . x . . . x . . .], and the right hand could play the 4 = [x . . x . . x . . x . .]. Displaying and listening to both rhythms simultaneously on the wheel helps to learn this task more quickly.

Another important rhythm-learning technique is to play *all* the pulses, with both hands alternating, so as to make the rhythm *emerge* from the 'background' of pulses by accentuating the correct onsets. For example, the rhythm [x . x x . x . x x . x .] could be played by starting the pulses with the right hand so that the right hand plays all the odd pulses and the left hand all the even pulses. Then the right hand would accentuate the onsets in [x . x . . . . . x . x .] whereas the left hand would accentuate the onsets in [. . . x . x . x . . . .]. RHYTHMOS can decompose any rhythm into this left-hand-right-hand *braid* notation and display it in a vertical side-by-side fashion so that the user can easily follow along and acquire these skills more easily.

3.4. Mathematical Analysis. RHYTHMOS provides an efficient timesaving tool for intensive mathematical exploration and analysis of rhythms; with a few clicks of the mouse, hours of tedious calculations of

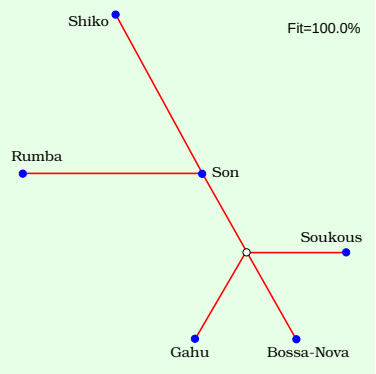

Figure 4: The *SplitsTree* of the six fundamental clave rhythms using the Hamming distance.

a variety of mathematical properies of rhythms, distance measures between rhythms, and histograms of inter-onset durations, are avoided. The dynamic linkage of display windows also encourages exploratory analysis: as a rhythm changes, its displayed properties reflect those changes in real-time. These features allow scientists, for the first time, to thoroughly examine an entire family of rhythms, using multiple tools, in minutes rather than days or weeks.

3.5. Phylogenetic Analysis. RHYTHMOS facilitates the application of techniques for generating phylogenetic trees from the distance matrices it calculates and outputs [6]. One of these methods, called *SplitsTree*, already used in several rhythmic studies, has the desirable property that it produces graphs that are not trees, when the underlying proximity structure is inherently not tree-like [3], [6]. Like the more traditional phylogenetic trees, *SplitsTree* computes a plane graph embedding with the property that the distance in the drawing between any two nodes reflects as closely as possible the true distance between the corresponding two rhythms in the distance matrix. However, if the tree structure does not match the data perfectly then new nodes in the graph may be introduced, as in Figure 4, with the goal of obtaining a better fit. Such nodes may suggest implied "ancestral" rhythms from which their "offspring" may be derived. In addition, edges may be split to form parallelograms, such as in Figure 5. The relative sizes of these parallelograms are proportional to an *isolation index* that indicates how significant the clustering relationships inherent in the distance matrix are. *SplitsTree* also computes the *splitability index*, a measure of the goodness-of-fit of the entire splits graph. This *fit* is obtained by dividing the sum of all the approximate distances in the splits graph by the sum of all the original distances in the distance matrix [3], [6]. The rhythms in Figures 4 and 5 comprise the six fundamental 4/4 time *clave* (and bell) timelines used in traditional African and Afro-Cuban music [12]. RHYTHMOS outputs a distance matrix in a format that may be directly plugged into the *SplitsTree* program.

3.6. Potential Future Applications. In addition to its main function, RHYTHMOS may also be employed in other fields: for instance, as a composition tool for generating new rhythms, or perhaps in cognitive science research on rhythm perception. The extensible design of RHYTHMOS makes any of these applications realistic. Adding a musical-notation function, optimizing the microsecond-accuracy of audio playback, or simplifying the user interface for young children, are all achievable customizations that require little alteration of the main system.

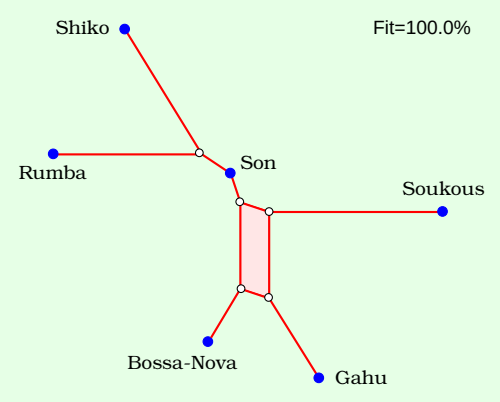

Figure 5: The *SplitsTree* of the six fundamental clave rhythms using the chronotonic distance.

3.7. Advantages over Standard Systems. In terms of visualization capabilities and mathematical analysis of rhythms, RHYTHMOS is the first of its kind: no other software system provides so many analytic tools. As a tool for simply creating rhythms and listening to them, however, RHYTHMOS does have its peers: Notepad [9] and Finale [11]. Notepad and Finale are much more extensive than RHYTHMOS in that they encompass pitch, staves, dynamics, and so on. However, this difference also accounts for the main advantage of RHYTHMOS : it is much less complicated than these systems. For instance, Finale 2004 has 245 different buttons and menu items on the main screen; RHYTHMOS has only 26.

The two other systems on the market that resemble RHYTHMOS (both use a wheel to represent rhythms) are the Flamenco Metronome Compás and the *O-generator*. The Flamenco Metronome Compás is severly limited by being applicable only to the 12-beat patterns of flamenco music, and functions only as a play-along system. The *O-Generator* allows the composition of music with a variety of instruments besides percussion. Unfortunately it is limited to 16-beat patterns, and is designed specifically for ages 10-16. RHYTHMOS on the other hand is useful to both a Ph.D., and an eight-year old child.

4. Conclusion

RHYTHMOS serves as a bridge between the art of music and the science of mathematics; it makes music theory more accessible to the student of music; it carries unacquainted users closer to the realms of structural and mathematical rhythmic analyses. It also serves as a bridge between theory and practice by providing pedagogical tools for playing along with audio-visual feedback. RHYTHMOS has a well-defined function: to aid scientists in the mathematical analysis of rhythms, and to aid all users in exploring and learning about rhythm. Describing *O-Generator*, David Mason, the Content Manager of London Grid for Learning, states “Most music software provides the sentence. *O-Generator* shows how the sentence is put together!” RHYTHMOS goes further by fleshing out the *meaning* of the sentence.

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