New ways in symmetry

María Francisca Blanco Martín[1] y **Elena Elvira Nieto**[2]
[1] Universidad de Valladolid, ESPAÑA
[2] Universidad del Nordeste, ARGENTINA
Colaborators: Raúl Capretini, Pablo Estévez
E-mail: fblanco@maf.uva.es, elenanieto14@yahoo.com

Abstract

This proposal presents the continuation of the task assumed some years ago by this interdisciplinary research team about the relations between Mathematics and Design.

The basic objectives in this proposal are:

1. To research about the syntactic, generative and methodological possibilities of mathematical models and fundamentally, geometric structures, as a base for the morphologycal definition of the objects, in their widest significance.
2. To study the transference of these knowledges to the educational level, through the implementation of learning situations that imply not only to offer the model, but also the ways of manipulation, extracting from it all its compositive possibilities. The idea is to establish a work methodology that can be applied to different situations, moving the students to be involved in each possible stage of the search.
3. To develop a systemic approach that allows the use of different informatical programs to promote creative development of students in the teaching- learning tasks.

Introduction

In this paper we present a synthesis of the work carried out on the possibilities of basic figures partitions arisen from their own geometric structure, using as a "design tool" the symmetry in connection to other such as, *tesselations*, *crystallographic groups*, and *regular divisions of the plane*, in order to develop different periodic mosaics.

About the teaching, we adopt a constructivist methodology: to present a design problem (containing something of game, something that implies creativity) that the student should solve making use of mathematical models in a quasi intuitive way, to explain rigorously the scientific justification, in each case.

As a conclusion, we believe it is possible to demonstrate the validity of this educational work methodology, in which the student is the main actor and can be motivated to inquire into the ways that geometry and symmetry offer, penetrating in the dialectical game of its rules, discovering relationships and generating forms in order to incentivate and potenciate the creative act.

1. Development

This work was previously started in the universities involved by putting into practice some guidelines related to the implementation of a body of learning situations which allow the student to develop from a mathematic model, different morphologic interpretations tending towards a deeper conceptualization of the design problems, both for the Architecture and Graphic Design courses of studies.

Our purpose is to complete this proposal in order to enable its realization, always under assessment, in the different courses involved. This implies the systematization of the integrated theoretical contents for the courses of Mathematics and Morphology of the Design which are ordered according to the increasing complexity of the theoretical and practical developments.

Since the current computer programmes allow different degrees of formalization in the graphic systems, it is our purpose to make the best of the possibilities offered, including the animation as the incorporation of movement, used from the perspective of the didactic strategies as well as the expression of the generation of the bi and tri dimensional form.

The first part of this work consisted in relating the morphologic operations of subtraction and transposition in the generation of the emerging forms to the geometric transformations corresponding to a given figure, mathematically formalizing such operations.

Analysing the possibilities of the square, as one of the regular polygons that tiling the plane, we chose it as the initial figure and, from its axis of symmetry, we performed the various partitions generated by the two axis of reflection. (**[3])**

2. Partitions of the square

The set of movements of the plane that leave the square fixed, its group of symmetry, is a dihedral group of order 4, D_4 which is a Leonardo's group or group of rosettes:

$$D_4=\{ s_1, s_2, s_3, s_2, g_{O,\pi/2}, g_{O,\pi}, g_{O,3\pi/2}, g_{O,2\pi} \}$$

In which s_1, s_2, s_3, s_4, are the reflections about the lines M_1, M_2, M_3, y M_4, y $g_{O,\pi/2}$, $g_{O,\pi}$, $g_{O,3\pi/2}$, the rotations of centre in the centre of the square and angles $\pi/2$, π, $3\pi/2$ y 2π.. A set of generators of this group is formed by $\{s_3, s_2\}$.

From the axis of symmetry of the square M_1, M_2, M_3 and M_4 we proved that it has only four different partitions, those generated by M_1 and M_2 (I); M_1y M_3 (II); M_4 y M_2 (III) y M_1 y M_4 (IV). (Fig. 1)

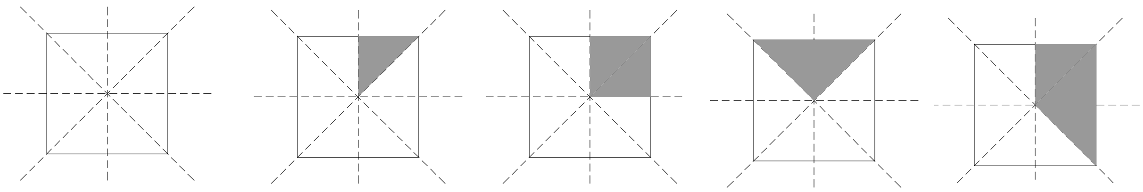


Figure 1

The ordered application of the morphological operations of subtraction and transposition to each of this partitions results in emerging figures that have the same area as the square but a different perimeter.

These morphological operations, geometrically correspond to the application of a movement to the parts taken from the square in such a way that some vertex of the partition coincide with some vertex of the square, having a side in common.

3. Emerging Figures

Each of the partitions generates the following non- isomorphic figures:

a) The partition I generates 14 figures: In (3) we studied this partition generating 14 emerging figures that tile the plane (except for I.3) and classifying the corresponding crystallographic group.
b) The partition II generates only 3 non- isomorphic figures (Fig. 2)
c) The partition III generates only 2 new different figures (Fig.3)
d) The partition IV generates 24 new different figures, which study will be the object of a forthcoming paper.

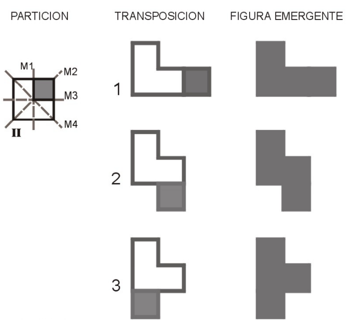


Figure 2

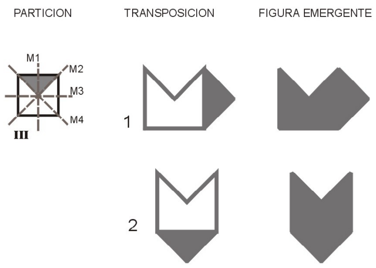
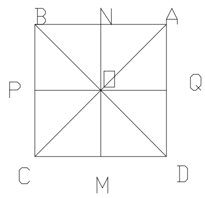


Figure 3

4. Geometric Generation of the Emerging Figures

We consider {O; **u**, **v**}, **u** = (1,0)= OQ y **v** = (0,1) = ON, as a system of reference, bound to the square.



The emerging figures corresponding to the partition II (Fig.2) are obtained from the following movements:

1: Rotation of centre Q and angle π
2: Translation of vector **-2v**
3: Rotation of centre P and angle $3\pi/2$

The emerging figures corresponding to the partition III (Fig.3) are obtained from the following transformation:

1: Rotation of centre A and angle $-\pi/2$
2: Translation of vector **-2v**

5. Regular division of the plane

5.1.–The following step of this work consisted in taking each of the figures that are called emerging figures and verifying what type of movements of the plane (rotation, translation, reflection or reflection with displacement) is required by each of them to cover the plane by repetition and which ones allow that possibility.

When studying the partition II, we found that the three generated figures have the capacity of tiling the plane by using operations of symmetry

II.1 The tiling of II.1 is a crystallographic group type p1, the tile has no symmetry. The group is generated by the translations of the vectors **a** = **u** + **v** (diagonal of the square) and **b** = 4**u**. (Fig. 6).

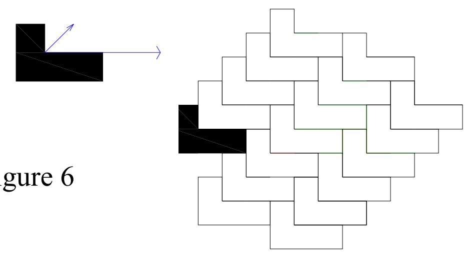


Figure 6

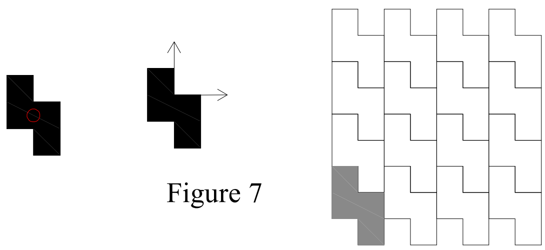


Figure 7

II.2. The tile corresponding to this partition is invariable by a rotation of angle 180º and centre of the midpoint of OM.
This partition gives rise to different tilings:

The tilings of figure 7 is a crystallographic group type p2, generated by the rotation previously mentioned and the translations of the vectors: **a** = 2**u** y **b** = 2**v.**

The tiling of the figure 8 is a group of type p2: generated by the rotation of angle 180º and centre of the midpoint OQ and the translations of the vectors: **a** = 2**u** y **b** = 2**v.**

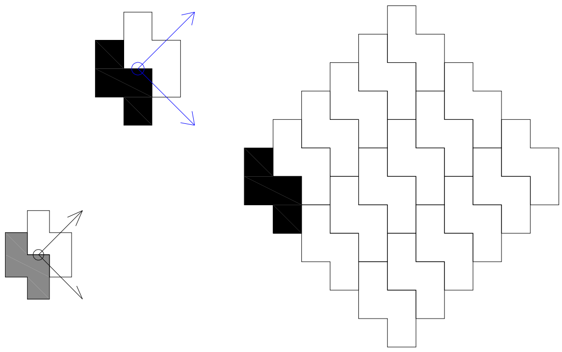


Figure 8

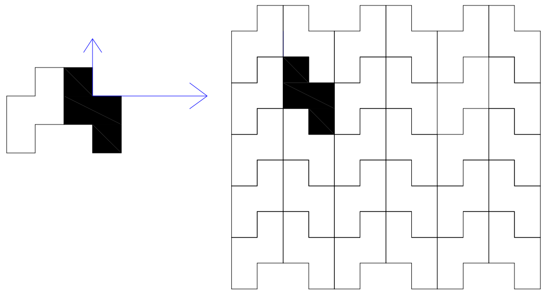


Figure 9

In the tiling of the figure 9, there is a symmetry with respect to the axis BC, (which) it is a group of type pmg: generated by the rotation of 180º and centre of the midpoint of OQ, the symmetry of the axis BC and the translations of vectors : **a** = 2**v** and **b** = 4**u.**

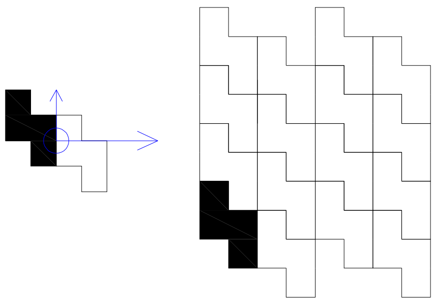


Figure 10

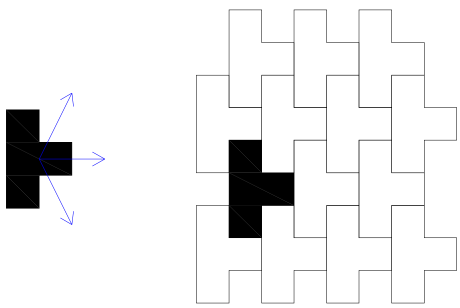


Figure 11

The tiling of the figure 10 is a group p2, generated by the rotation of centre D and angle π and the translations of vectors: **a** = 2**v** y **b** = 4**u.**

II.3 The tile corresponding to this partition is symmetric respecting to the straight line *r* that joins the midpoints of CP y DQ. Two different tiling with this tile are shown as follows.

The tiling of figure 11 is a crystallographic group of type cm, generated by the symmetry previously indicated and the translations of vectors: **a** = 2**u** y **b** = **u** + 2**v.**

The tiling of figure 12 is a group of type p4: generated by the rotation of the centre Q and order 4 and the translations of the vectors: **a** = 4**u** and **b** = 4**v.**

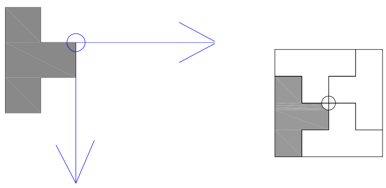
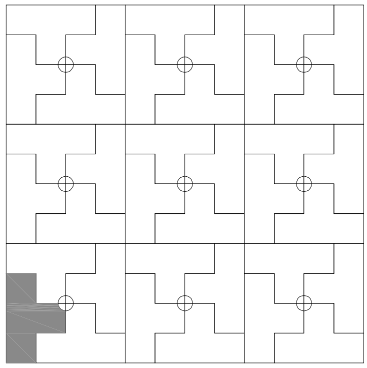


Figure 12



Partition III

The partition III gives rise to two emerging figures, the second one with an axis of symmetry.

III.1 The partition III.1 tiles the plane, figure 13, forming a crystallographic group of type p4, generated by a rotation of order 4 and centre shown in the figure and translations of vectors: **a** = 4**u** and **b** = 4**v.**

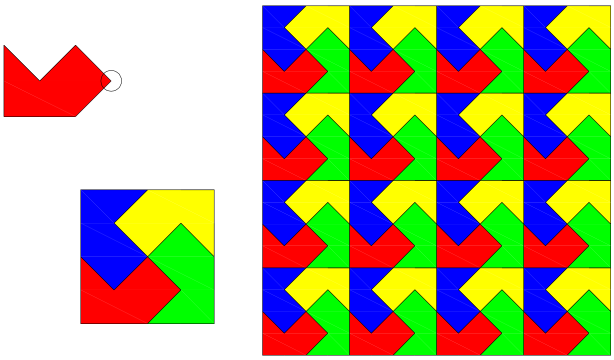


Figure 13

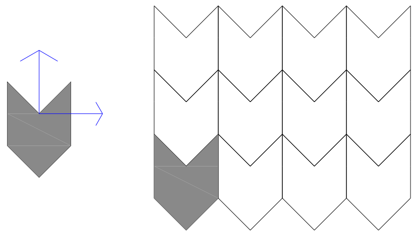


Figure 14

III.2 The emerging figure of this partition has a reflection of axis, the straight line MN.

The tiling of the plane is a crystallographic group of type pm, generated by the symmetry with respect to the axis MN and the translations of the vectors: **a** = 2**u** and **b** = 2**v.** (Fig. 14).

6. Conclusion

As it was already mentioned, we are presenting another stage of an ongoing research work. Considering a probable extension of this work, we submit it for its validity as a didactic tool before finishing each stage. At this respect and taking into account the various formal possibilities that these geometrical structures generate, as well as the inclusion of the colour as a problem to be worked out, we are presenting only two of the results obtained by the students of Graphic Design, in the Universidad Nacional del Nordeste, Argentina.(Figure 15) In this case, the students worked with different regular figures, as the hexagon, tiling the plane and using different schemes of colour in the grid generated using the same methodology as for the square. As we already said, this is a proposal to invite you to follow this new way of the symmetry to enjoy the design.

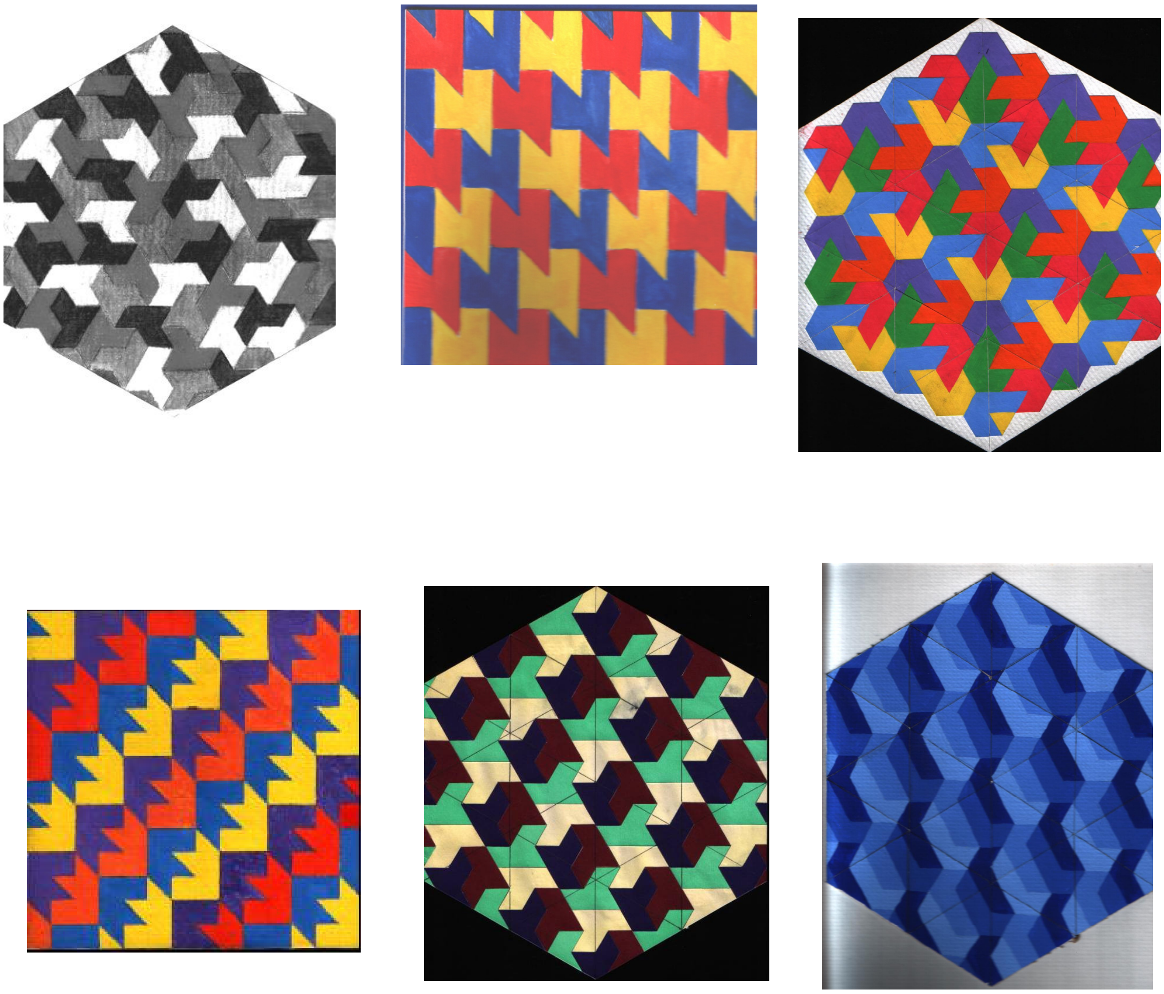


Figure 15

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