Steve Reich's *Clapping Music* and the *Yoruba* Bell Timeline

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Abstract

Steve Reich's *Clapping Music* consists of a rhythmic pattern played by two performers each clapping the rhythm with their hands. One performer repeats the pattern unchangingly throughout the piece, while the other shifts the pattern by one unit of time after a certain fixed number of repetitions. This shifting continues until the the performers are once again playing in unison, which signals the end of the piece. Two intriguing questions in the past have been: how did Steve Reich select his pattern in the first place, and what kinds of explanations can be given for its success in what it does. Here we compare the *Clapping Music* rhythmic pattern to an almost identical *Yoruba* bell timeline of West Africa, which strongly influenced Reich. Reich added only one note to the *Yoruba* pattern. The two patterns are compared using two mathematical measures as a function of time as the piece is performed. One measure is a dissimilarity measure between the two patterns as they are being played, and the other is a measure of syncopation computed on both patterns, also as they are played. The analysis reveals that the pattern selected by Reich has greater rhythmic changes and a larger variety of changes as the piece progresses. Furthermore, a phylogenetic graph computed with the dissimilarity matrix yields additional insights into the salience of the pattern selected by Reich.

1 Introduction

The history of music is often the history of humanity's reactions to it. A good example of this may be observed in Minimalism. Since the Second World War, mainstream classical music has been dominated by composers such as Boulez, Berio, Cage, Ligeti, and Stockhausen, among others. These composers represent postwar Modernism, either through postserialism, Boulez being its most prominent figure, or through indeterminacy, where Cage is its most notable figure. Although the term Minimalism was originally used for the visual arts, it was later applied to a style of music characterized by an intentionally simplified rhythmic, melodic and harmonic vocabulary (see [19]). Its main representatives are LaMonte Young, Philip Glass, Terry Riley and Steve Reich. Their music and ideas become the major reaction to the Modernism epitomized by the aforementioned composers. Indeed, whereas Modernism is decisively atonal, Minimalism is clearly modal or tonal; whereas Modernism is aperiodic and fragmented, Minimalism is characterized by great rhythmic regularity; and whereas Modernism is structurally and texturally complex, Minimalism is simply transparent.

Minimalism has different materializations depending upon the particular composer, but minimalist works share concern for non-functional tonality and reiteration of musical phrases, often small motifs or cells, which evolve gradually. For example, while Young uses sustained drones for long periods of time, Glass chooses recurrent chord arpeggios, and Riley and Reich incorporate repeated melodies and quick pulsating harmonies. No less significant is the fact that minimalist music possesses almost

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none of the main features of Western music (at least since the time of the Romantic period) that is, harmonic movement, key modulation, thematic development, complex textures, or musical forms with well-designed structures. On the contrary, this music deliberately skirts around any sense or awareness of climax or development, and seems to ignore the dialectic of tension and release, at least as it is usually posited in the classical music tradition.

It is probably Reich, the minimalist composer who most unhesitatingly repudiates the Western classical tradition. Reich objects to both European serialism and American indeterminacy because in these traditions the processes by which the music is constructed cannot be heard and discerned clearly by the listener. This rejection, formulated not only by Reich but by other minimalist composers as well, may be the reason why minimalist music has been so incomprehensibly ignored by many critics and scholars. In his essay *Music as a Gradual Process*, included in [20], Reich states his principles as follows: "I am interested in perceptible processes. I want to be able to hear the process happening throughout the sounding music." For such processes to be accessible to the listener, they must flow in an extremely gradual manner. The process itself must be related to the idea of shifting phases. First, a melody is played by two or more players, and after a while one of them gradually shifts phase. At the beginning of the phasing a kind or rippling broken chord is produced; later, as the process moves forward, the second melody is at a distance of an eighth note, and a new interlocking melody arises. The process continues until the two melodies are in phase again.

These ideas are fulfilled in many of Reich's works composed between 1965 and 1973. This experimentation starts with *It's Gonna Rain* and *Come out* (both composed in 1966), where he uses phasing on tape music; it continues with *Piano Phase* (1967), and *Violin Phase* (1967), where he experiments within an instrumental context (no electrical devices); and finally, Reich reaches the highest development in *Drumming* (1970-71), *Clapping Music* (1972), and *Music for Mallet Instruments, Voices and Organ* (1973), where he incorporates gradual changes of timbre and rhythmic augmentation, among other musical resources. By the end of 1972, he abandoned the gradual phase shifting processes, because "it was time for something new" [20].

This paper is concerned with a mathematical comparative analysis of *Clapping Music* and the bell rhythmic timelines of West African *Yoruba* music. This relation is not as distant as it might seem at first glance. During the summer of 1970 Reich traveled to Ghana where he studied African drumming. He learned *Gahu*, *Agdabza* and other musical styles, which influenced his music (later he also studied Gamelan music). Such an influence can be perceived in works such as *Drumming* and *Clapping Music*, where the phasing is discrete rather than continuous, as in his previous works. More specifically, we study *Clapping Music* with regards to syncopation, inasmuch as it forms an essential part of that piece.

2 *Clapping Music*

Clapping Music is a phase piece for two performers clapping the same pattern throughout the duration of the piece. The phasing is discrete, with one performer advancing an eighth note after several repetitions of the pattern, while the other imperturbably remains playing the pattern without shifting. See Figure 1 for further details. In the following, variations produced by shifting are numbered in ascending order as $\{V_0, V_1, \cdots, V_{11}, V_{12}\}$, where $V_0 = V_{12}$ indicates that the two performers play the pattern in unison.

This piece, in spite of its apparent simplicity, does not lack musical interest. First of all, *Clapping Music* constitutes a synthesis and a refinement of Reich's ideas by means of a piece with few but well combined elements. Secondly, *Clapping Music* enjoys a profound metrical ambiguity (something common to Reich's pieces) as well as a great deal of interlocking rhythmic patterns. The analysis

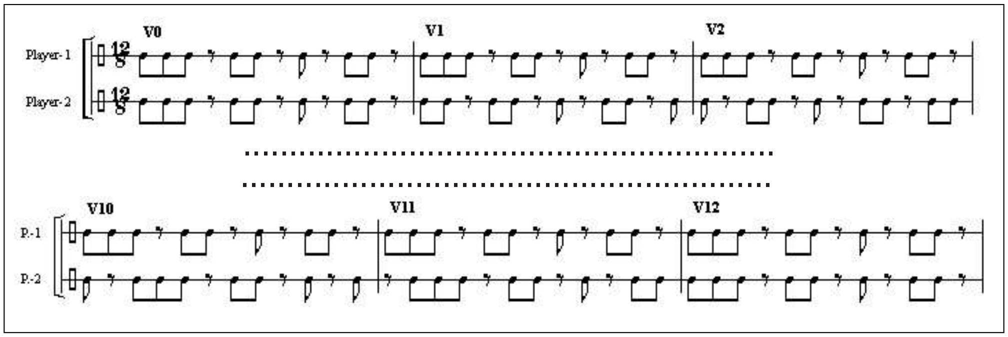

Figure 1: The first and last few bars of *Clapping Music*.

of those interlocking rhythmic patterns and its distribution along the piece is quite elucidating on its musical structure, as we will see later. Paradoxically enough, in spite of this rich structure, *Clapping Music* has received little *musical* analysis, in sharp contrast to the much more attention received from a *mathematical* point of view [8, 4, 9, 21].

3 Measuring Features of *Clapping Music*

When one listens to *Clapping Music*, a question that arises naturally is how Reich came to select that particular pattern. As the pattern shifts, a series of interlocking rhythms emerge, creating great variety. Furthermore, there is a sense of balance in the whole piece, between the resulting variations, as they create and release rhythmic tension. Once the pattern is defined however, the rulebook does not allow us to change it. Therefore, the pattern must be carefully chosen to begin with.

Phylogenetic graphs have been already used to analyze musical rhythms. In [21], a phylogenetic analysis of binary *claves* from Brazil, Cuba and some parts of Africa was carried out. *Claves* are rhythmic patterns repeated throughout a piece whose main functions include rhythmic stabilization as well as the organization of phrasing [16]. Subsequently such an analysis was extended to ternary claves taken from the African tradition, and to the hand-clapping metric rhythms of Flamenco music ([22] and [5], respectively). In all cases, worthwhile conclusions were drawn from the phylogenetic analyses. In this paper we use phylogenetic graphs to both, analyze the structure of *Clapping Music* itself, and to compare it to the *Yoruba* rhythmic timeline.

The key mystery in *Clapping Music* is how Reich was able to find a pattern that would work so well within the constraints of process music. We believe that his inspiration for the pattern came from his study of African drumming. In particular, we note an extraordinary resemblance between the pattern of *Clapping Music* and a clave bell pattern used by the *Yoruba* people of West Africa: only one additional seemingly inconsequential note has been added by Reich! The two patterns are shown in Figure 2.

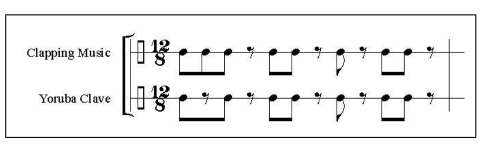

Figure 2: The pattern of *Clapping Music* and the *Yoruba* clave.

The *Yoruba* people live on the west coast of Africa, mainly in Nigeria, although they can be found also in the eastern Republic of Benin and Togo. Because most of the slaves were taken from West

Africa, a diaspora took place and the descendants of the *Yoruba* people can also be found in Brazil, Cuba, The Caribbean, the United States and the United Kingdom. They are one of the largest cultural groups in Africa, and musically speaking are of great relevance. *Yoruba* music has exerted much influence on the music of the surrounding countries.

The clave considered here is widely employed as a timeline in the sacred music among the *Yoruba* people [18]. Bettermann [3] calls this rhythm the *Omele*. It is also found in Cuba, where it is used in several styles [17] like the *Columbia*.

It might be argued that there could likewise exist other clave patterns very close to that of *Clapping Music*. This is not the case, as we will see in the next section. Once we formally introduce the measure of distance between a pair of rhythms, we will verify that the *Yoruba* clave is the rhythm closest to the *Clapping Music* pattern among a great number of African ternary claves. These reasons constitute our primary motivation for comparing these two patterns. In fact, an intriguing natural question is whether the *Yoruba* clave itself would work just as well as the pattern employed in *Clapping Music*.

The musical effectiveness of *Clapping Music* is partly due to the way in which syncopation is dealt with. The problem of defining a mathematical measure of syncopation has not been addressed until recently. In [7], Gómez et al. reviewed several measures of syncopation, and proposed a new measure, the so-called *weighted note-to-beat distance* measure (*WNBD* measure from here on). That measure will be used here to analyze the syncopation of *Clapping Music* and the *Yoruba* clave pattern.

4 Phylogenetic Analysis

Phylogenetic graphs were originally used in Biology to determine the proximity and evolution of species. Biologists measure the degree of proximity between two species by comparing their genes. In our context, rhythmic patterns take the place of genes, and as a consequence, we define a new measure of proximity (similarity) between rhythmic patterns (measures used in Biology are not appropriate in the context of Music). The question of how to define similarity measures for rhythms has already been considered [21, 22, 24, 5]. Among the many existing similarity measures (Euclidean interval vector distance, interval-difference vector distance, swap distance, etc.), the most satisfactory one in these studies has been the directed-swap distance, first introduced in [5]. For further information on measures of rhythmic similarity the reader is referred to [24].

The directed-swap distance is a generalization of the swap distance to handle comparison of rhythms that do not have the same number of onsets. Let P and Q be two rhythms such that P has more onsets than Q. Positions of the rhythm that contain an onset will be referred to as occupied positions. Thus, the directed-swap distance is the minimum number of swaps required to convert P to Q according to the following constraints: (1) Each onset in P must move to some occupied position of Q; (2) All occupied positions of Q must receive at least one onset from P; (3) No onset may travel across the boundary between the first and the last position in the rhythm.

For example, the directed-swap distance between Player-1 and Player-2 in variation V_1 is 4, since we have to perform 4 swaps in Player-1 at positions $3, 6, 8$ and 11 to convert Player-1 to Player-2; see Figure 1. In our case, since all rhythms have the same number of onsets, computing the directed-swap distance is easy because it reduces to the computation of a sum of a linear number of terms [5, 24].

As mentioned before, one of the reasons for comparing the pattern of *Clapping Music* to the *Yoruba* clave is that this clave is the rhythm closest to it. The distance between them was measured with the directed-swap distance, and the set of claves used in the comparison was taken mainly from well-established African musical traditions. To obtain precise details about these claves consult [22] and the references therein.

The distance matrix corresponding to the directed-swap distance is shown in Figure 3. Box notation is used for the variations of *Clapping Music*. For each rhythm the bottom of each column indicates the sum of the swap distances to all other rhythms. Surprisingly, the sums take on only two values, 48 and 74, where V_0, V_3, V_6 and V_9 are the variations that obtain the highest score. Also noteworthy is the diagonal below the zeros in Figure 3, that is, $\{4, 4, 4, 8, 4, 4, 8, 4, 8, 4, 4\}$. This diagonal gives the directed-swap distances between consecutive variations. It takes on only two values, 4 and 8, but these values differ considerably, and change 6 times over a total of 12 variations.

Variations	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}
V_0=xxx.xx.x.xx.	0											
V_1=xx.xx.x.xx.x	4	0										
V_2=x.xx.x.xx.xx	8	4	0									
V_3=.xx.x.xx.xxx	12	8	4	0								
V_4=xx.x.xx.xxx.	4	2	4	8	0							
V_5=x.x.xx.xxx.x	8	4	2	4	4	0						
V_6=.x.xx.xxx.xx	12	8	4	2	8	4	0					
V_7=x.xx.xxx.xx.	4	4	4	8	2	4	8	0				
V_8=.xx.xxx.xx.x	8	4	4	4	4	2	4	4	0			
V_9=xx.xxx.xx.x.	2	4	8	12	4	8	12	4	8	0		
V_{10}=x.xxx.xx.x.x	4	2	4	8	4	4	8	2	4	4	0	
V_{11}=.xxx.xx.x.xx	8	4	2	4	4	4	4	4	2	8	4	0
$\sum$	74	48	48	74	48	48	74	48	48	74	48	48

Figure 3: The directed-swap distance matrix of the *Clapping Music* pattern.

In Figure 4 the phylogenetic graph associated with the above matrix is depicted. The graph returns the degree of fit with the data, listed as a percentage. If the percentage reaches 100%, then the minimum distances between nodes in the graph correspond exactly to the entries in the matrix. The algorithm initially tries to impose a tree structure on the distance matrix. If this is not possible, it introduces extra nodes in order to maintain a high fit value. See [11] for further details on the construction and properties of these graphs. Black dots correspond to the variations, while the rest of the nodes are without dots; the central node is labeled as A.

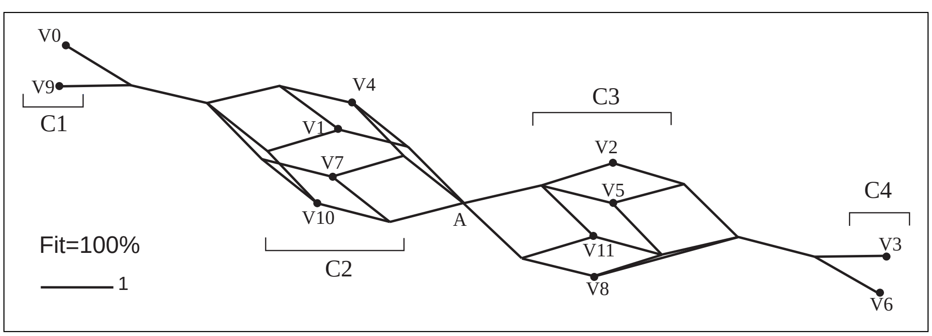

Figure 4: The phylogenetic graph of the *Clapping Music* pattern.

This phylogenetic graph provides valuable information about the structure of *Clapping Music*. For the moment, we consider the central node A. Such a node corresponds to an "ancestral" rhythm, and is also the center of the graph (i.e., it is the vertex that minimizes the maximum distance to any other vertex in the graph). Therefore, it seems that this central node plays a key role in the piece. There is as yet no known algorithm to compute the "ancestral" nodes for rhythms in phylogenetic graphs constructed with our distance measure. However, in this case, given the small number of rather short rhythms involved, the "ancestral" rhythm can be reconstructed by hand without much difficulty. It turns out to be the rhythm in Figure 5:

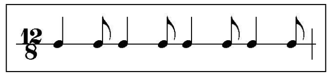

Figure 5: The "ancestral" rhythm of *Clapping Music*.

This fundamental rhythmic pattern is none other than a group of trochees. A trochee is a rhythmic grouping consisting of a long note followed by a short note. This "ancestral" rhythm has a strong metric time-keeping character. The trochee, expressed in box notation as [x . x], is a common Afro-Cuban drum pattern, also found in disparate areas of the globe. For example, it is the conga rhythm of the (6/8)-time *Swing Tumbao* [14]. It is common in Latin American music, as for example in the Chilean *Cueca* [15], and the Cuban *coros de clave* [6]. It is common in Arab music, as for example in the *Al Táer* rhythm of Nubia [10]. It is also a rhythmic pattern of the Drum Dance of the *Slavey* Indians of Northern Canada [2]. Furthermore, the entire pattern of 8 onsets in a time span of 12 pulses shown in Figure 5 is also the *Euclidean* rhythm $E(8, 12)$ which distributes the onsets as evenly as possible [25].

The phylogenetic graph has four distinguishable clusters $C1, C2, C3$ and $C4$, that can be easily seen in Figure 4. When *Clapping Music* is performed these clusters appear in the order given in the following table in Figure 6:

Clusters	C1	C2	C3	C4
	V_0	V_1	V_2	V_3
		V_4	V_5	V_6
		V_7	V_8	
	V_9	V_{10}	V_{11}	
	V_{12}			

Figure 6: Clustering in *Clapping Music*.

From this sequence of clusters we may observe the evolution of the variations through time. There is a first section formed by variations V_0 to V_3; in it, each variation moves further away from V_0. In a second section, which goes from V_4 to V_6, variations are still kept away from V_0. In the third section, variations V_7 and V_8 remain around the center of the graph, and represent a turning-point after which the subsequent variations move towards V_0. The variations in section four, consisting of V_9, V_{10} and V_{11}, tend towards V_0. Lastly, *Clapping Music* closes by coming back to the main pattern ($V_0 = V_{12}$) played in unison. This evolution may be detected, although less visually, on the diagonal below the zeros in the directed-swap distance.

Let us now compare the *Clapping Music* pattern to the *Yoruba* clave. To start with, we present the swap distance matrix for the *Yoruba* bell pattern in Figure 7, and its corresponding phylogenetic graph in Figure 8. By looking at the bottom row of the matrix we see that the sums of the distances take many different values. The most similar rhythm is V_5 and the most different one is V_4. That is not surprising: V_5 is the so-called *Bembé*, a very popular ternary rhythm. Toussaint [22] already proved that it is one of the most similar of an important family of ternary claves. Here we note that the *Bembé* is the most similar rhythm in its own wheel (the wheel of a rhythm consists of those rhythms obtained by its rotations that begin on an onset). The diagonal below the zeros is $\{5, 7, 5, 7, 5, 5, 7, 5, 7, 5, 5\}$. The difference between two consecutive variations is smaller than in the case of *Clapping Music*. Changes are more frequent in the case of the *Yoruba* clave.

Variations	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}
V_0=x.x.xx.x.xx.	0											
V_1=.x.xx.x.xx.x	5	0										
V_2=x.xx.x.xx.x.	2	7	0									
V_3=.xx.x.xx.x.x	3	2	5	0								
V_4=xx.x.xx.x.x.	4	9	2	7	0							
V_5=x.x.xx.x.x.x	1	4	3	2	5	0						
V_6=.x.xx.x.x.xx	6	1	8	3	10	5	0					
V_7=x.xx.x.x.xx.	1	6	1	4	3	2	7	0				
V_8=.xx.x.x.xx.x	4	1	6	1	8	3	2	5	0			
V_9 =xx.x.x.xx.x.	3	8	1	6	1	4	9	2	7	0		
V_{10}=x.x.x.xx.x.x	2	3	4	1	6	1	4	3	2	5	0	
V_{11}=.x.x.xx.x.xx	7	2	9	4	11	6	1	8	4	9	5	0
$\sum$	38	48	48	38	66	36	56	42	43	55	41	66

Figure 7: The directed-swap distance matrix of the *Yoruba* clave.

The graph is a chain with a rather disappointing fit of 89%, with no ancestral nodes. Take into account that if the fit is not 100%, reasoning on the graph does not accurately reflect reasoning on the distance matrix, and accordingly, neither on the rhythms. For example, on the graph the distance from V_0 to V_2 is 2.5, but in the matrix it is actually 2. The role of A could be played by variation V_5 in this phylogenetic graph. It is the center of the graph, and as before, variations alternately go from left to right and from right to left around V_5. Nevertheless, this does not seem to yield anything in particular about the structure of the *Yoruba* clave. Hence, there is no remarkable clustering analysis to be discussed. In addition, the graph does not exhibit special symmetries or regularities of musical significance either. In reality, when the same musical process as *Clapping Music* is carried out, a rather awkward result is obtained.

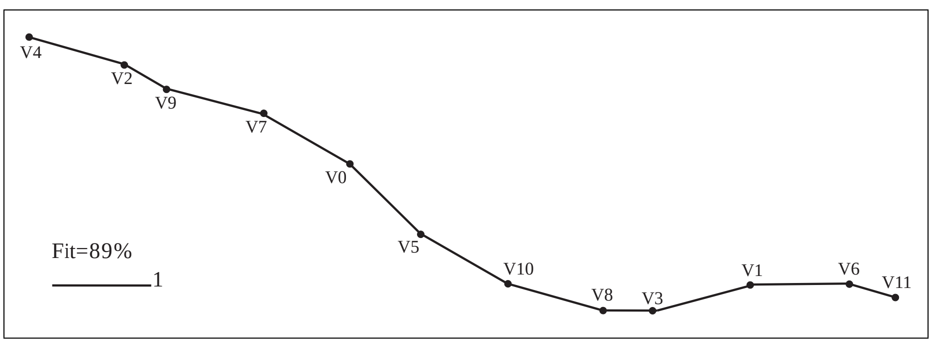

Figure 8: The phylogenetic graph of the *Yoruba clave.*

5 Syncopation Analysis of *Clapping Music*

The definition of syncopation includes a momentary contradiction of the prevailing meter [19]. According to this fact, the *WNBD* measure is based on the durations of notes and how they cross over the strong beats of the meter. This measure has a different approach than others, like Keith's measure [13], based on the structure of metrical levels, or Toussaint's off-beatness measure [23, 26], based on the underlying polyrhythmic structure of the meter. The *WNBD* measure has proven to be more flexible and precise than the others [7].

Now we introduce the *WNBD* measure, which will enable us to analyse the syncopation of *Clapping Music*. We assume that a note ends where the next note begins. Let e_i, e_{i+1} be two consecutive strong beats in the meter. Also, let x denote a note that starts after or on the strong beat e_i

but before the strong beat e_{i+1}. We define $T(x) = \min\{d(x, e_i), d(x, e_{i+1})\}$, where d denotes the distance between notes in terms of durations. Here the distance between two adjacent strong beats is taken as the unit, and therefore, the distance d is always a fraction (see Figure 9 (a)).

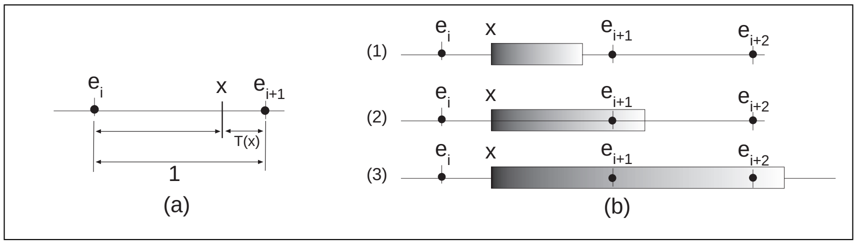

Figure 9: Definition of the *WNBD* measure.

The *WNBD* measure $D(x)$ of a note x is then defined according to the following cases: (1)$D(x) = 0$, if $x = e_i$; $D(x) = \frac{1}{T(x)}$, if note $x \neq e_i$ ends before or at e_{i+1}; (2) $D(x) = \frac{2}{T(x)}$, if note $x \neq e_i$ ends after e_{i+1} but before or at e_{i+2}; and (3) $D(x) = \frac{1}{T(x)}$, if note $x \neq e_i$ ends after e_{i+2}. See Figure 9 (b) for an illustration of this definition. Now, let n denote the number of notes of a rhythm. Then, the *WNBD* measure of a rhythm is the sum of all $D(x)$, for all notes x in the rhythm, divided by n.

Figure 10 plots the *WNBD* measure of the variations in *Clapping Music* with respect to a 12/8 meter. The measure produces only three different values, namely, $24/8, 21/8$ and $12/8$, but the graph is quite revealing about how syncopation works in *Clapping Music*. Variation V_0 by itself has a high value of syncopation. Two identical ascending-descending cycles, $V_1 - V_2 - V_3 - V_4$ and $V_4 - V_5 - V_6 - V_7$, follow after V_0. From V_7, we find a symmetric cycle with respect to the previous cycle, namely, $V_7 - V_8 - V_9 - V_{10}$. Finally, we discover an ascending path to V_{12} (which is a half of the previous cycle). If variation V_1 were moved after V_{12}, then the resulting graph would have a perfect symmetry about V_7. Therefore, a strong symmetry in musical form is evident in *Clapping Music*.

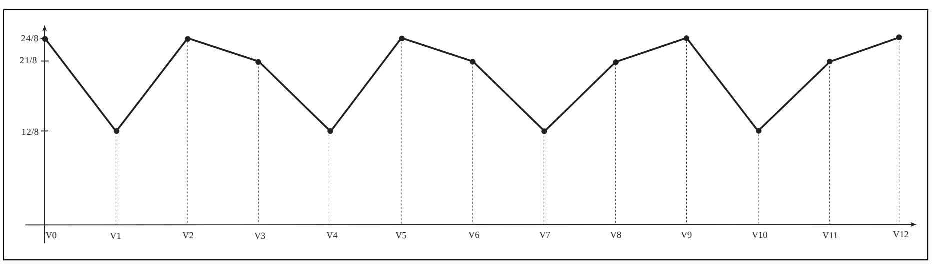

Figure 10: The graph of the syncopation measure of *Clapping Music*.

On the other hand, when looking at the graph of the *WNBD* measure for the *Yoruba* clave, depicted in Figure 11, several differences come out. As in *Clapping Music*, the measure only takes three values, namely, $21/7, 18/7$ and $15/7$. The range of the syncopation values is much smaller than in the case of *Clapping Music*, and consequently, so is its rhythmic variety. The smaller the range of the measure is, the less interesting the rhythms are from the syncopation standpoint. The graph of the *Yoruba* clave exhibits the same quasi-symmetry about V_7.

6 Concluding Remarks

Although phylogenetic graphs have been already used for analyzing families of rhythms, in this paper we use them for studying the musical structure of process music, in particular Reich's *Clapping Music*. The resulting graph allows us to explore a variety of musical properties of the piece, such

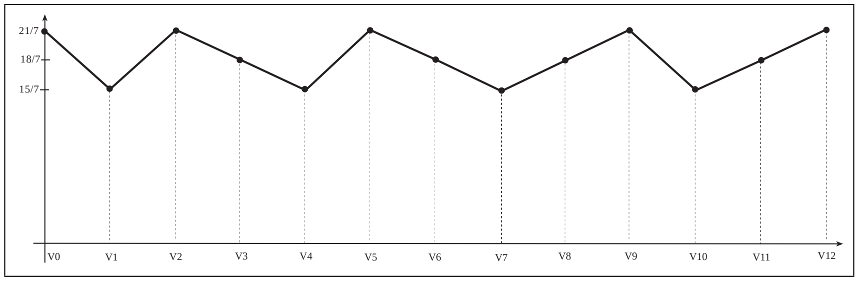

Figure 11: The graph of the syncopation measure of the *Yoruba* clave.

as the classification, evolution, and transformation of variations, or the structure of the musical form. We advocate the use of phylogenetic graphs as a useful tool for musical analysis in general, especially for musical styles as characteristic as Minimalism. Reich's other pieces in this style, such as *Music for Pieces of Wood*, may also be analyzed in this way.

We have compared *Clapping Music* to the *Yoruba* bell timeline since they are almost identical. One would expect that changing only one note could not make such a significant difference in musical terms. Quite to the contrary, as we have seen, the phylogenetic graph of *Clapping Music* has a richer structure than that of the *Yoruba* clave. This is a consequence of its inherent musical structure.

The *WNBD* measure produces interesting conclusions as well, mainly that the rhythmic variety, at least from a syncopation standpoint, can be measured in terms of the range and distribution of the syncopation values.

Previously, Haack [8] proved that *Clapping Music* is unique from a combinatorial point of view. In this paper we have added arguments of a geometrical nature to support this conclusion.

Finally, the properties exhibited by the phylogenetic graphs and the *WNBD* measure pose several open problems. For example, one may ask which rhythms yield nice graphs, that is, graphs with good properties (symmetry, clustering, 100% fit, etc.) The *WNBD* measure gives poor results when it is used to measure the interlocking melodies (for that we just use the *Clapping Music* pattern as the meter, and recompute the measure). Therefore, finding a function that measures the overall complexity of two rhythms is an interesting open problem. Also open is the question of which other rhythmic patterns would work just as well as the pattern used in *Clapping Music*.

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