Photography and the Understanding of Mathematics

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Abstract

This paper considers ways in which photographs help our understanding and teaching of mathematics. Some historical landmarks are considered from Muybridge's galloping horses to mathematics trails snapped with mobile phones. The possibilities have always been limited by the available technology and have been shaped by changing attitudes to mathematics teaching. It is argued that in mathematics teaching, photographs are not just for illustration. They provoke discussion, pose problems and provide data. We can measure them and model them with graphs. The approach adopted for developing the *Problem Pictures* calendars and CD-ROMs is described together with some of the ways these resources are used.

1. A Little History

From the earliest days of photography, the medium has had mathematical possibilities. A simple frontal photograph of a building could be measured to extract angles and ratios with an accuracy that had not been possible when working from a drawing or painting. The first photographers may not have recognized the potential but it has always been there. [1]

In 1878, Eadweard Muybridge took a famous series of photographs of a trotting horse. Its owner, Leland Stanford, had proposed, that during a horse's running stride, there is a moment of suspension where no hooves are touching the ground, and Muybridge's pictures proved that he was right. Today Muybridge's work is seen as a landmark in motion photography, and by biologists, as pioneering work in understanding animal movement. To ask 'In what order does an animal place its feet on the ground?' is also a mathematical question, demanding analysis and modeling of a complex situation.

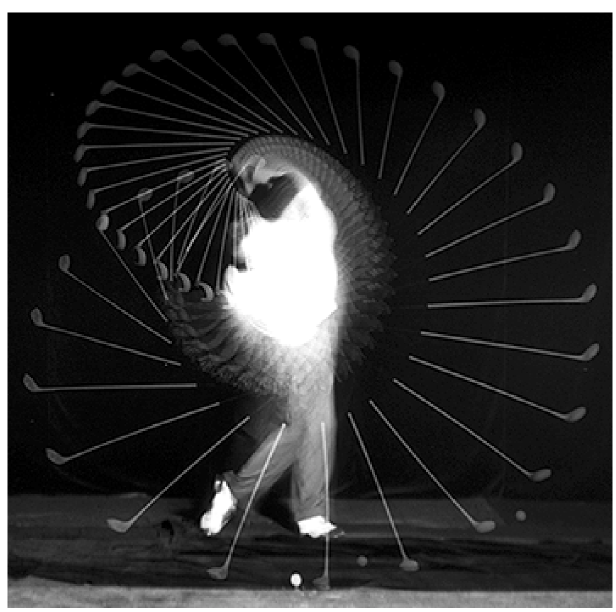


Figure 1: *Harold Edgerton's 'Multiple-flash photographs of the golfer Dennis Shute', about 1935.*

In the 1930s, Harold Edgerton's experiments with multiple-flash photography created stunning images that seem to pose their own mathematical questions. Edgerton's picture of the golfer Dennis Shute was taken at 100 flashes per second. How fast was the club moving when it struck the ball? How fast does the ball move? How long does the golf shot last? How close is the motion to a circle?

It has taken a long time for photographs to begin appearing regularly in mathematics textbooks and journals. One imagines that it was easy for an author to convince a publisher of the value of photographs in geography textbooks but the case for mathematics was harder to make. A small number of photographs were used to good effect in *Mathematics: A Human Endeavor* published in 1970 [2]. In 1980, the classic Dutch publication *Schaduw en Diepte (Shadow and Depth)* [3] is an early example of teaching material using photographs to pose mathematical questions. The book explores ideas about light, space and perspective.

In the UK an important landmark year was 1983 when Ray Hemmings and Dick Tahta took over the editorship of the journal Mathematics Teaching [4]. Their impact has been considerable and introduced many changes that have lasted to this day. They chose to fill their journal with photographs. Interspersed among the articles are images of coal hole covers, scaffolding, balustrades and more, mostly in grainy black and white. There is a double page spread of car wheels contributed by Marion Walter. Very few of the images had captions, and those that did are very minimal. The photographs were not accompanied by explanations, questions or activities. These were left for the reader to invent. Nothing was written to justify the pictures as being in any way mathematical but there was no doubt that they were.

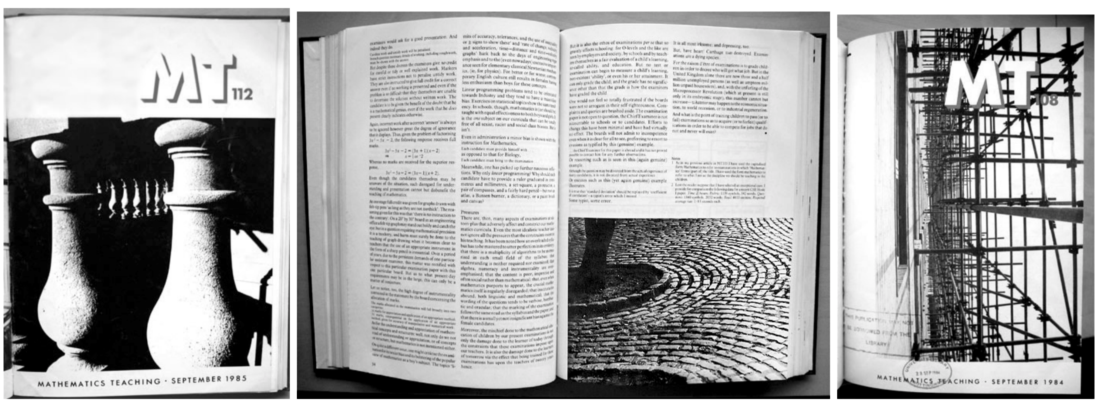


Figure 2: *Use of photographs in the UK journal Mathematics Teaching between 1983 and 1985.*

This generous use of photographs became possible because of changes in printing costs. In the 1970s a separate printing plate had to be made for every photograph at considerable expense. By the 1990s, with the arrival of computer filmsetting, it made almost no difference whether a page had one photograph or a hundred photographs. When Laurinda and Tony Brown took over the editorship of Mathematics Teaching in 1987 they enthusiastically continued the imaginative use of photographs including some playful juxtaposition of text and image. Photographs started to appear more often in other mathematical journals and books. Also in 1987 the Mathematical Intelligencer's Mathematical Tourist column began publishing photographs of mathematical significance from around the world. In the same year the NCTM published 'Geometry in Our World,' a collection of colour slides as a classroom resource. In 1988, *Geometric Patterns from Roman Mosaics* was the first of a series of photo books by Robert Field from Tarquin Publications.

In more recent years, a different type of photograph has become more common in several of the mathematics teaching journals. These could be described as 'role model' photographs. They show teachers and students working at mathematical activities in a wide variety of ways. Often the pictures show activities away from the classroom or using practical apparatus. Editors seem to use these photographs to convey several types of message, such as:

- There are many ways of learning mathematics.
- Mathematics can be fun.

- Mathematics can be engrossing.
- You can do your own mathematical investigations.

Some editors in the late 1980s were cautious about using pictures of this kind because they had received frequent criticisms of the kind ‘Why are there more boys than girls in this picture?’ or ‘Why are there more white faces than black faces in this picture?’ Fortunately this has not seriously inhibited the use of role model photographs, which are now an established way of communicating about ways of learning mathematics.

A recent development in print publications is the Mathematical Lens column in NCTM's Mathematics Teacher. This is edited by Ron Lancaster from Ontario, an enthusiastic user of photographs for mathematics teaching. Each article typically consists of a single photograph which is used to pose numerous questions. Some questions are quite simple and others require considerable work, sometimes using resources like interactive geometry software. The questions are followed by answers and a commentary.

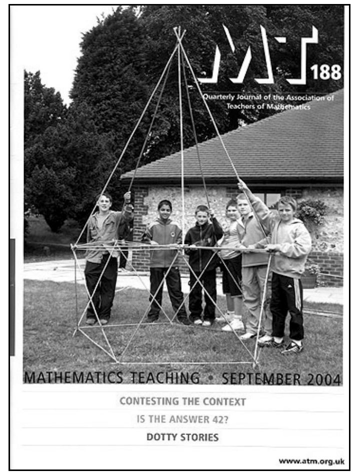


Figure 3: *A ‘role model’ photograph from cover of Mathematics Teaching, 2004.*

About 1990 the ‘digital multimedia’ revolution made it much easier to work with photographs, sound, and video on a computer. A range of technological advances opened up new possibilities including the first affordable digital cameras and the arrival of CD-ROM and other low-cost, high capacity storage media. The world wide web followed soon after. The revolution continues to this day embracing new technologies along the way, such as mobile phones that take photographs.

"World of Number” was an early UK multimedia project using interactive video discs under computer control to deliver a variety of classroom resources. In theory it was possible to put 45,000 photographs on one side of a laser vision disc. The project’s Picture Gallery offered a large collection of photographs, mostly taken frontal parallel to make it easy to measure them with the toolbox software provided. [5]

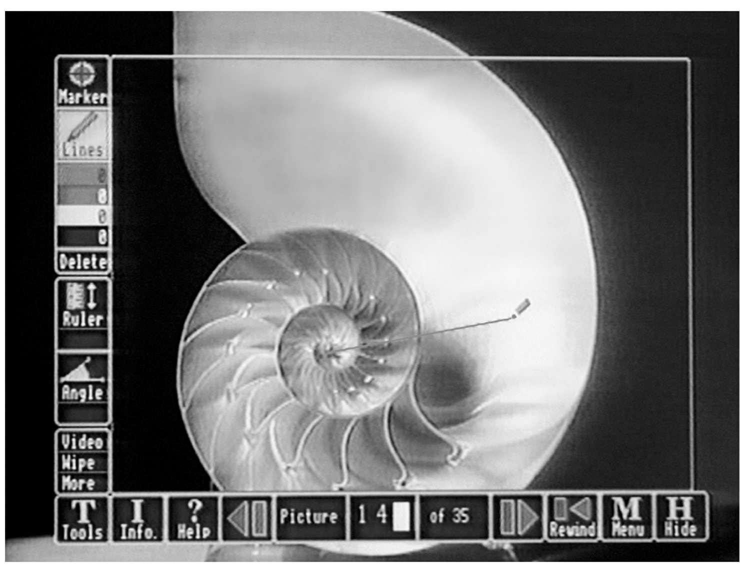


Figure 4: *Angle measurements being made on a photograph using the ‘World of Number’ software toolbox.*

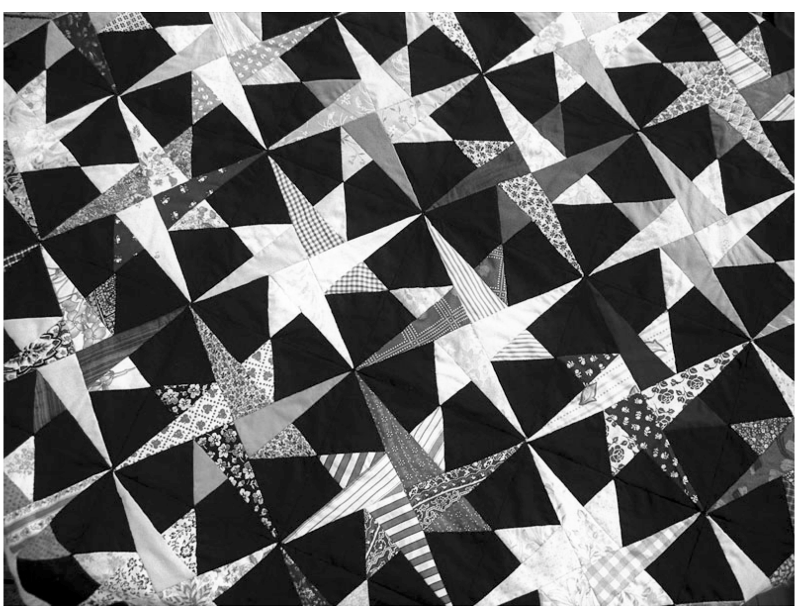
As well as multimedia publications, the revolution opened the way for students and teachers to take their own photographs. Primary age students at Roch School in Wales have developed a photographic mathematics trail around their village and published it on the internet [6]. A teacher at another primary school arranged her class into a 'living bar chart'. Only by taking a photograph could she show the class the pattern that they had made.

Figure 5: *Students make a human barchart to show their favourite fruit during a Maths Day at Mosley Primary School, Staffordshire.*

2. Problem Pictures

Over the last 25 years photography has grown significantly more important in understanding mathematics. There have been many different approaches to using the medium and its impact has touched teachers and students at every level. Building on this tradition I have developed an approach to publishing photographic resources for mathematics which I call *Problem Pictures.*

The photograph of the patchwork quilt and the accompanying text is an example.



This beautiful patchwork quilt has a complex design.
Try to see the basic units from which it is constructed.
Can you see any right angles here?
If the whole quilt has 100 black quadrilaterals, how many triangles does it have?

Figure 6: *Example of Problem Pictures photograph and text.*

The choice of the text that accompanies the photograph is of some importance. Because it is quite short (just four sentences) it is likely to be read. The language is chosen to be readable and informal. It suggests questions and activities, but the reader is quite at liberty to do something different with the photograph.

The instruction ‘Try to see the basic units’ is ambiguous. The basic units could be the pieces of cloth from which the quilt is constructed, or the names for their geometric shapes. It could also mean the ‘repeat’ unit within the pattern. Such ambiguity would be unacceptable in a task that was designed for assessing students. But the game is not to measure students but to engage their interest. By being ambiguous the phrase works for a very wide range of ages and abilities. ‘Try to see the basic units’ could be tackled by a seven year old, but could also be quite challenging to readers of this paper.

‘Can you see any right angles here?’ is a more closed question. It is also a kind of hint, helping the observer to structure their viewing of the pattern.

The final question about the number of triangles is more difficult. It is reasonable that the questions asked should get progressively more difficult. But this question can also be approached in different ways and is therefore open to students with different degrees of knowledge. Primary age students should be able to estimate an approximate answer.

This pyramid is built entirely from pieces of Turkish delight. The base of the pyramid is made from a square 16 by 16, using 256 pieces of Turkish delight. On top of this is a second square 15 by 15, a third square 14 by 14, and so on, up to the single piece of Turkish delight at the top.
How many pieces are there in the top three layers?
How many pieces were used altogether?

Figure 7: *A second Problem Pictures photograph.*

A second example shows a pyramid built from pieces of Turkish delight. The questions are about number patterns. Although no algebra is needed, it could be used. Students who know the formula for summing squares or the formula for the volume of a pyramid might find these useful. It is the kind of problem where you bring to it whatever knowledge and skills you have.

Most Problem Pictures photographs come with about four sentences of text. These offer questions and activities that work at a wide range of ability levels. The materials are designed to be as open and inclusive as possible.

It is useful to contrast this with the approach taken by Ray Hemmings and Dick Tahta in the 1980s with Mathematics Teaching. Their photographs generally came without captions or questions. But their intention was much more than to decorate the empty spaces in their journal. We know from other aspects of their work that they were committed to a very open approach to education where teachers ask very few questions and, as much as possible, the responsibility for learning is taken by the students. A photograph without a caption gives the learner the widest scope to pose their own questions. But it also greatly increases the chances that the photograph will simply be ignored.

Different again, Ron Lancaster in his Mathematical Lens column, accompanies one photograph with a whole article. With so many words, it is possible to explore in much more depth. There is considerable ingenuity in devising a wide range of interesting questions that emerge from one picture. The drawback here is that the reader is less likely to pose their own question. With so many words and so many possibilities, the ownership of the problems belongs more to the author than the reader.

Problem Pictures limits the text to about four sentences. This formula also suits the media in which the photographs are published – as CD-ROMs [7] and as calendars and posters from AAMT [8]. The open character of the material means that the photographs have been used by teachers in several different ways. One possibility is to lead a whole class discussion with photographs displayed on an electronic whiteboard, usually to begin a lesson. Some teachers use the photographs to prepare their own worksheets. Most work is with students aged between 9 and 16 years, but there are examples of use with older and younger students too.

Peter Finch is a teacher from South Australia. He assigned his year 8 students "research tasks" as project work. They had to choose one of seven photographs on Problem Pictures CD-ROM and answer the questions in the form of an A4 poster that would be understandable by someone who knows nothing about the task (see Figure 8).

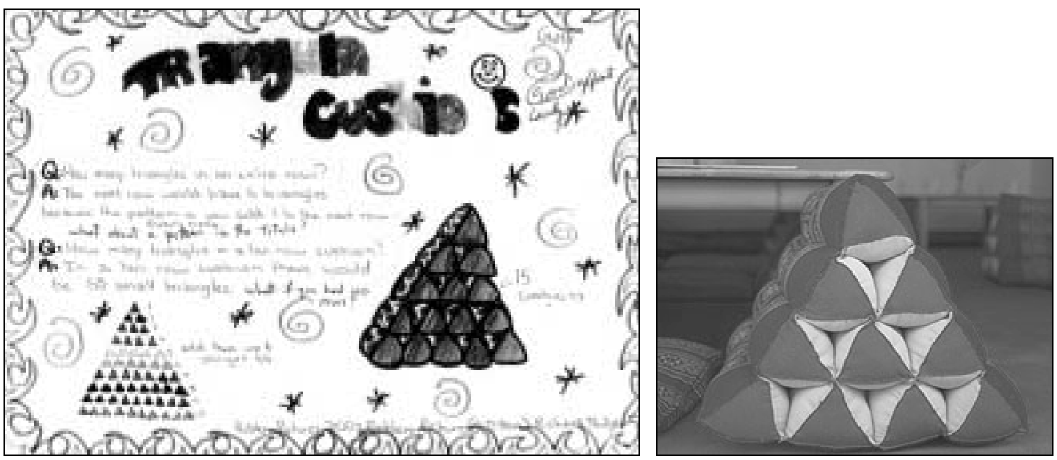


Figure 8: *One of Peter Finch's students produced this poster about triangular numbers.*

Adrian Oldknow [9] has developed some activities that involve modelling photographs with graphs. A photograph can be copied from a webpage and pasted into software that allows a graph to be superimposed. Recent versions of The Geometer's Sketchpad and Cabri Geometry are suitable for this. Three sections of the Problem Pictures Themes CD-ROM have photographs specially taken to be used in this way. This modelling activity offers a learning environment that encourages experimentation.

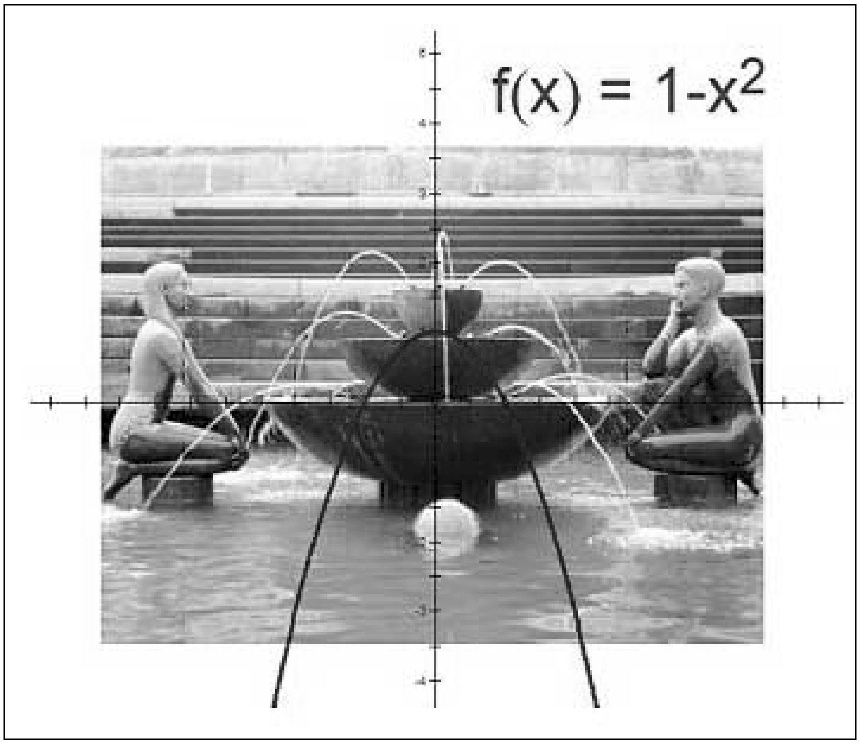


Figure 9: *Modelling the trajectory of water from a fountain.*

A workman has just finished painting the word STOP on the road. Four stencils were used to paint the letters.

Imagine the stencils are turned over. The letters T and O would still look like a T and an O. But what would happen to the other letters?

Suppose you had stencils for all 26 capital letters. How many of these could be turned over without changing the shape of the letter?

Figure 10: *A visualization task.*

3. Conclusions

Photographs are an excellent way of building bridges between mathematics and the real world. They bring relevance and interest to a subject that is often see as abstract and remote. There are many ways of using photographs in teaching mathematics, with exciting possibilities for students of all ages.

In the past, the cost of the technology has often limited the educational possibilities of photography. Today, this obstacle has been removed. There has never been a better time to use photography to improve the understanding of mathematics than now.

Acknowledgements

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[2] H. A. Jacobs, *Mathematics: A Human Endeavor*, Freeman, San Francisco, 1970.

[3] A. Goddijn, *Schaduw en Diepte (Shadow and Depth)* Rijksuniversiteit Utrecht, the Netherlands, 1980.

[4] The journal *Mathematics Teaching* is published by the Association of Teaching of Mathematics, Derby, UK. See also: www.atm.org.uk

[5] *World of Number*, interactive video discs, Shell Centre & New Media, UK, 1992.

[6] See www.ngfl-cymru.org.uk/vtc/ngfl/eng/ks2/mathematics/number/maths_trail_roch

[7] *Problem Pictures* and *Problem Pictures Themes*, collections of colour photographs with activities for mathematics teaching on CD-ROM, Badsey Publication, Badsey, UK, 2001, 2005. An American edition titled *Mathematical World* is published by Key Curriumum Press. See www.problempictures.co.uk

[8] *Problem Pictures* calendars have been published annually since 2003. There is also a set of four posters. Australian Association of Mathematics Teachers, Adelaide. See also www.problempictures.co.uk

[9] A. Oldknow, ICT - bringing maths to life and vice versa, *Micromath* 21(2), 2005. See also www.adrianoldknow.org.uk