Mathematics and the Architecture: The Problem and the Theory in Pre-Modern Cultures

Zafer Sagdic
Architecture Department
Yildiz Technical University
80630 Besiktas
Istanbul, Turkiye
zafersagdic@hotmail.com

Abstract:

There is always a mystery on pre-modern architecture practice on the relation between dimensions and ratios. The reasons of using certain proportions used on the design of religious buildings/ spaces are the result of the application of numerical symbolism and Pythagorean triangle. Thus, the paper will be focused on the unity of theory in pre-modern architecture practice via giving some special examples of pre-modern architecture through the human history, such as Antique Egyptian and Antique Greek temples, Roman churches, Gothic cathedrals, and so on.

1.The Problem and The Theory

In modern architectural practice, it is known that, the formation of spaces are the result of diverse needs and requirements, such as; being suitable for certain activities, offering enough space to accommodate needed number of users, giving answer to technical regulations and parallel to reality resisting the influence of climate, earthquakes, storms, etc. Also, in modern thought, scientific thinkers interpreted the nature has an inherent order beyond that which man brings to his observations. In pre-modern cultures, theologians transformed nature to mean god, and through this they were able to give architecture a formalised higher purpose. This way of thinking allowed them to interpret nature through the use of power of algorithms- mathematics. The artist- form makers- usually found himself somewhere between the two positions, making the best use of each for the purposes determined by his culture. At certain times in history the artist relied heavily on religious scholasticism for interpretations of natural phenomena. It is seen on the pre-modern cultures that some certain numerical values are used to serve a religious purpose, especially on the design of religious buildings, such as mosques and churches. It should not be forgotten that the mathematics is a kind of language used through the mystery of existence. According to Galileo, "the big book of nature only can be read by the one who knows the language of it; the language of mathematics". If we are using mathematics to the formation of result of "real" needs and requirements, it should be zoomed on the nucleus of designing purpose of the religious buildings in the pre-modern world. Parts of the main religious building may be grouped to symbolise religious beliefs, and some symbolic numbers have been utilised for centuries in these spaces.

"The form of a building is its internal physical structure, as described under some appropriate conceptualisation". This definition is in the spirit of the general usage of the term aesthetics. According to Clive Bell (1914), all of the relations and combinations of line, spaces and even the colours are build up the "significant form". So, what architects did in pre-modern space formulation, is that, they always use the organisations and completion of their formal experiences are mixing with the mathematical and geometrical rules of architecture, the rules such as proportion, balance, line recession and so on. Architects are the coordinators and organisers of these experiences and finally the ones who definite form in a

building. According to Arpat [1], certainly after so many years of research and hundreds of analytical calculations performed on plans of religious architecture, an answer to this enigma had to start to take shape: “Logical Thinking”. It is a possible source and driving force, powerful enough to generate a secret worldwide influence in many fields of human activities, including architecture, regardless of differences between cultures and religions, over several centuries. In 1958, Monroe Beardley’s “Aesthetics” named text suggests that, “the form of an aesthetical object is the total web of relations among its parts”. Numerical symbolism, which is the nucleus of pre-modern aesthetic, beauty understanding and architectural practice, is nearly as old as history itself. And definitely, it existed nearly as far back as Babylonian and Antique Egyptian designs. The rules, that nature has in, were symbols of “beauty”. If we get a view of historical development of human being, we can understand that architects designing artificial-environment and living-spaces by using these rules; from the Antique Egyptian, Antique Greek periods, till medieval architecture and Renaissance and maybe till today.

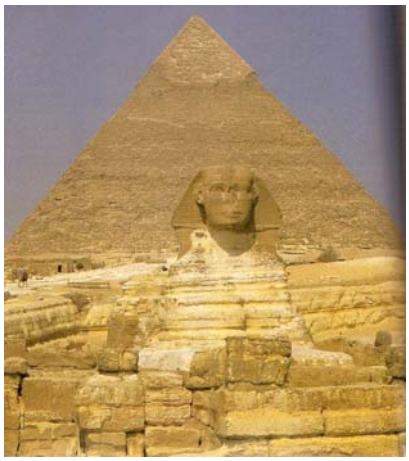
Here, the definition of design should be given. Design is the process of generating form fort he purpose of enriching human existence. In modern life, while design process designers are facing with some needs and requirements as the basic point of their designs, which is an effective rationale way. Obviously, pre-modern design creating based on geometric models of natural phenomena form designates inanimate physical entities; pattern will designate religion.

The Greek Philosopher, Pythagoras (560-480 BC) who travelled to Babylonian and Antique Egypt acquired some secrets of numerical symbolism and mathematical based design formulation in these countries and founded a religious philosophy. The numerical relations of the movements of heavenly bodies and of the relations inherent in the 3-4-5 triangle. Thus, the Pythagorean theory was discovered afterwards. Pythagoras also found that there are two other important way of creating numerical symbolism in pre-modern cultures: first is a relationship between sound and number, and the second is gematria. Gematria is a mystical art that has played a key role in the history of architecture, says Arpat [1]. Gematria involves assigning numerical values to the letters of the alphabet, therefore numbers are deriving from words, names and passages of main script. It is used in all three major religions to calculate the corresponding symbolic numbers of the holiest name, especially the name of God.

The pre-modernite accepted and developed rather than studied and restored the heritage of the past, unlike today. Even they stuck with using of the same religious based designs through the universal language mathematics century by century, they used new geometrical patterns and new architectural orders on their creations. The creative experience of a work of art depends not only the natural sensitivity and the visual training of the spectator, but also on his cultural equipment. This cultural perfection shows how the artist-form maker- can be used his geometry knowledge and/or gematria knowledge on his art pieces. In this point of view, patterns, iconography, hieroglyph, miniatures, making pictures of especially religious scenes in art and etc. concern themselves with the subject matter or meaning of works of art, as opposed to their form. Thus, all of these branches of art are methods of interpretation which arise from synthesis rather than analysis in modern cultures.

2.Antique Egyptian Architecture

Complex religion of Egyptians was the reason of making huge temples and monumental sized tombs dedicated to after-life mentality and half animal gods. They gave importance to buildings symbolised after-life. So that, there are two different important tomb types in Egypt: **1.**monumental sized tombs named **Pyramids** of Pharaohs, **2.mastabas** for rich and important people, monks and merchants.



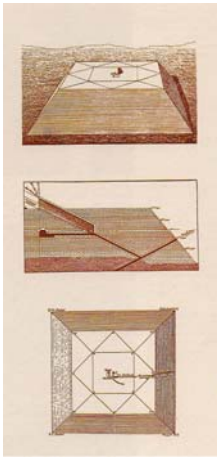
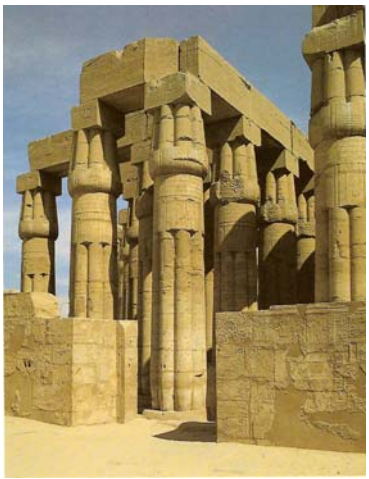


Figure 1: *Pyramids of Giza and Sphinx, Egypt.* **Figure 2.**: *Details of Egyptian mastabas*

In Egyptian culture nearly every image is symbolic, but that did not change the truth of mathematical and geometrical rules applied on designs came aside. It is known that Egyptians used very complex mathematical and geometrical knowledge on their royal tombs (Pyramids), which were temples for the name of worship of the deceased pharaoh. There are many labyrinths, mathematically well organised on 3rd dimension, inside pyramids and also the location of each pyramid is very specifically chosen according to religion based geometrical rules. The second important building type in Egypt is the temple. Each of Egyptian temples planned according to symbolical zoning: praying zone for ordinary people, praying zone for young monks, and zone of Pharaoh. All of Egyptian temples also designed according to symbolically based geometrical rules. The Khonsu Temple of Karnak (1198 BC.) and the Amman Temple, which is quite similar to Khonsu Temple, but six times in size, were stood inside a large walled enclosure which had to be also containing service buildings and sacred lake. Both have the similar mathematical rules used in site plans with a large door between two tall pylons, an open court and a colonnaded room, or hyposytle hall. Maybe the Egyptian style was seen too specific and too conservative for the needs of the other civilisations such as more poorer ones like Minoans and Mycennaeans in the Aegean sea till Antique Greek style appeared, no huge and some meanings, menthalitical and religious reasons loaded buildings made.



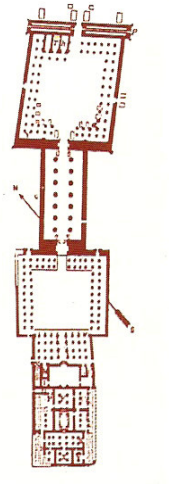


Figure 3-4: View from and plan of *Luksor Temple, Egypt.*

3.Antique Greek Architecture

The Ancient Greeks were fascinated by the concept of form and in their consideration of the forms of things they drew a fundamental distinction between chaos and cosmos. In their architectural products, the attempt at discovering a cosmology and disciplines in compositions is seen. All these definitions about Antique-Greek architecture insist on influential tradition of formalist criticism by the creation and usage of round column, graceful symmetry, repetition of rhythms, well-used proportions, harmonies and etc. It is not possible to add a further comment about "roundness" for the technical reason that round is a predicate symbol, not the identifier of an object in the universe of discourse.

This can be remedied by treating "round columns" as the value of the function. While the symmetry was used in Antique-Greek buildings such as temples, rationality was used for impressing evaluating functions. In other words, the symmetry was impressed by the gracefully of roundness; so what they had "the graceful symmetry".

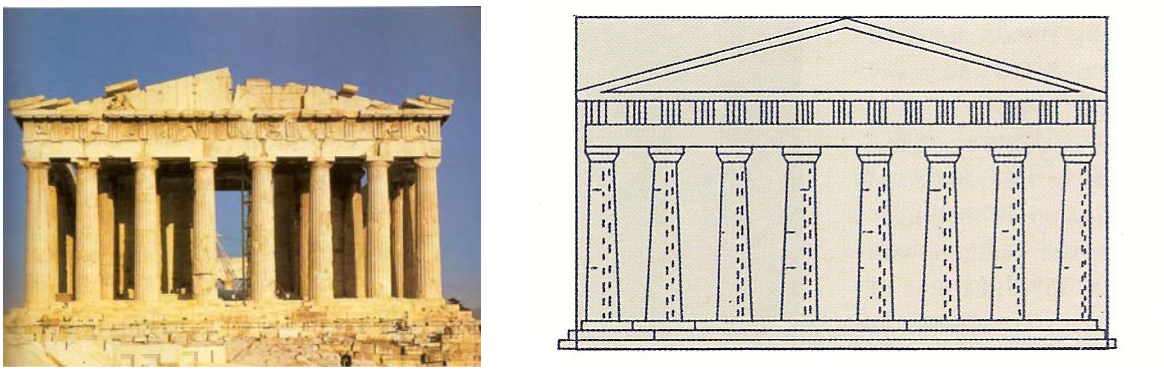


Figure 5-6: *View of façade of Parthenon Temple, Greece.*

By the repetitions of the columns, they also caught the rhythm; and by the dimensional compositions they caught the ratios, well-used proportions and finally harmony.

4.Roman Architecture

Roman Empire brought a new administration-system to Mediterranean world, its own city state democracy. Within this democracy, they built lots of monumental sized buildings, especially basilicas for law, amphitheatres, Roman baths and so on. These huge and monumental seem buildings were built by Emperors for inheriting the democratic obligation to maintain the buildings, their own popularity and Empire's power. Perhaps the most famous building of them, is the Coliseum in the capital, Rome; began in AD. 70 in the Emperor Vespesian period. This monumental sized structure has 50.000 spectator places for the brutal entertainment of the amphitheatre. Doric, Ionic, Corinthian and Composite columns orders are set in an ascending sequence. Although this arrangement was not universal, but adoption of this vertical sequence of the orders is seen in Coliseum.

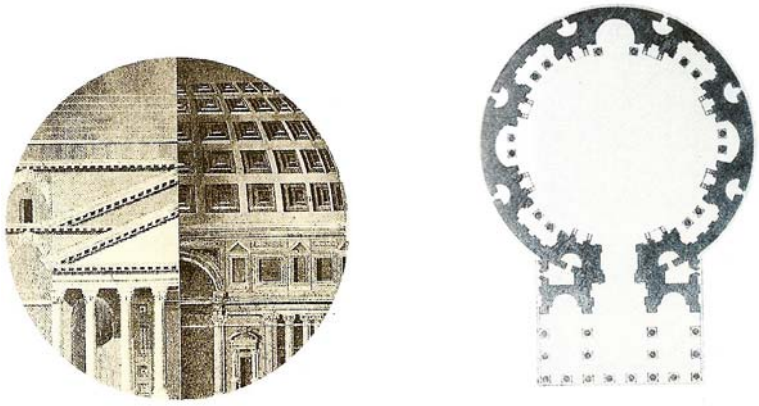
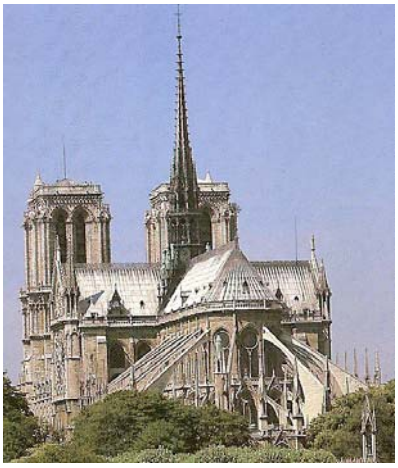


Figure 7-8: *Façade-section and plan of Panthenon Temple, Rome.*

With the systematic adaptation of columns orders, high capacity carried structure and monumental sized façades, Coliseum is still a memorial of mathematical perfection. Pantheon was one of the biggest and mathematically well-proportioned temple of Roman architecture, was built in AD.120 in Emperor Hadrian's period. This temple gives us the most complete impression of great buildings of Emperors. The huge brick clay concrete drum covered with 40 meters wide dome creating a dramatic top-lit space decorated with rows of coloured marble columns and arches.
So, it can be seen that, lots of high capacity carried, monumental sized façades having and mathematically well-proportioned buildings in the Roman architecture.

5.Gothic Architecture

Maybe products of Gothic were more menthality, more objective than all the other architectural styles' products; designs of Gothic Cathedrals were based on mathematical rules and complex than this, geometrical rules. Gothic architecture was based on logical solutions: using of one unique element, such as using of the rose window at the entrance façade or the symbol exists at the first stone step of the cathedral. This formation has a logical systematic, which is not only on the structure of the cathedral, but also on the layout and elevations with even the smallest details of ornaments. While designing a Gothic Cathedral, to make a good design, there was not any certain formula. However, there were some certain ways within classical architecture of organising the layout and detailing cathedrals. The plan is usually symmetrical on either sides or axis, starting from the main entrance and through the cathedral. It was believed that the symmetry reflected the balance of nature and the human form. The axis is more than the geometric structure of the plan and described the way, that, someone moves through, views and understands the cathedral. It can be seen in Greek temples focused on the image of god with a single axis, while Gothic cathedrals have a series of secondary axes. If Roman type of buildings are checked, it is seen that a rigid axial layout could be more informal; have individual groups of spaces at various angles by a series of secondary axes (=AD120). The design of an enclosed area of Gothic cathedrals, can make the space comprehensible by defining its geometry. Space elements; walls, floors and ceilings can be decorated to emphasise the geometry of the space. The construction of the cathedral introduces the geometric framework and all interior volumes of the cathedral are the parts of external design. The modification of detailed façade gives emphasis to any part of cathedral, not only for indicating its functional importance but also for improving the overall composition.



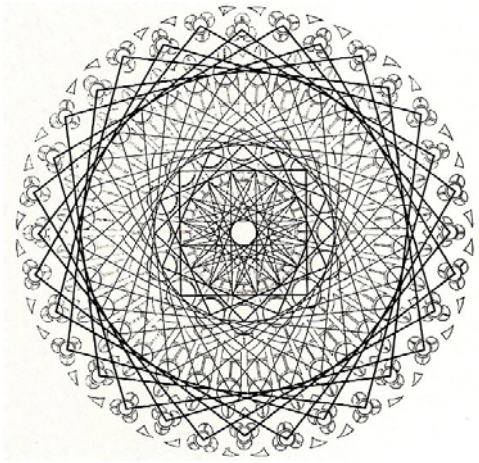
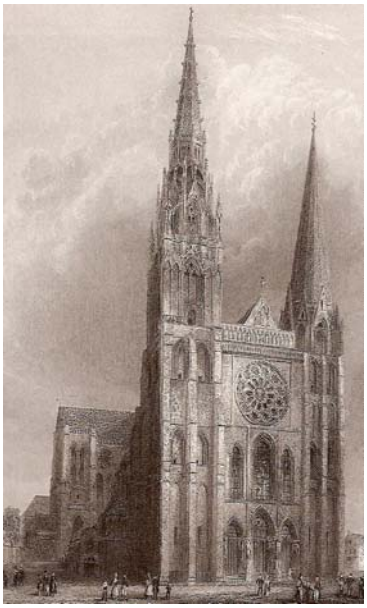


Figure 9-10: *Notre-Dame Cathedral and pattern of north rose window.*

The Renaissance designs establish a tradition of separating volumes in space organisation while Gothic cathedrals have much more logically organised volumes because of the logical systematic that was told at upwards. Superstitions and mystic forms of Christianity were reasons of cryptic world of symbols and myth having figures used in Gothic. The architects of Gothic used more complex systematic of dimensions and numbers of architectural features of ancient buildings.

The bishops of northern France asked to architects to open up the walls of their churches for large areas of stained glass to cast a mystic light on elegant internal rows of classical columns. To achieved these idea, architects reduced the weight of stone vaults by crossing arches diagonally between columns. These arches or ribs carried outwards to half arches and flying-buttresses leaning against the outside walls.



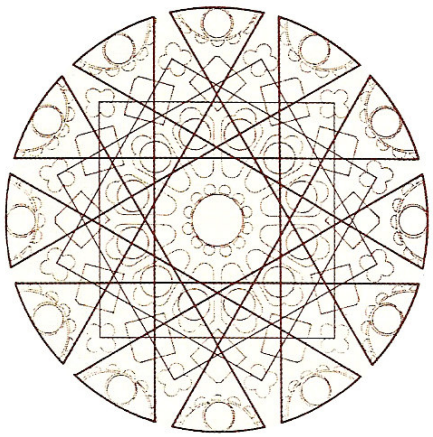


Figure 11-12: *Chartres Cathedral and pattern of North rose window.*

7.Renaissance Architecture

In the fourteenth century within the Italian city-life, an interest to the Rome led to the rescue of forgotten manuscripts from monastic libraries and a new philosophy, humanism, reconciling Christian and pagan thought.

Renaissance, the great artistic revolution blossomed from this fertile ground in the early fifteenth century. Architects studied the forgotten ruins of Rome to create buildings that served the needs of a society only gradually emerging from the feudalism of the Middle ages. The palazzo of Medici family, added classical order to the traditional scheme, in 1440. The whole composition is crowned by huge classical overhanging eaves and the cornice of traditional battlements. Alberti designed St. Andrea church in l470, he proposed on clear classical image to follow and drew his inspiration from large Roman vaulted interiors such as Constantine Basilica. He followed the established long church plan while changing side-aisles into chapels to support the heavy stone roof. And finally in the façades he used the design of a Roman triumphal arch, changing its role from the glorification of an empire to Christian God. St. Maria della Consolazione church, was designed by an architect Cola da Caprarola in 1508, has a series of circles and squares and one central domed central space plan. With this great artistic revolution, a fresh consciousness of history was infused with the vigour of originality to create a new classical architecture of great beauty. Proportion and symmetry are two basic concepts within the relationship of mathematics and design. Pascal, who is known as the genius of mathematicians, apparently thought of proportion and symmetry as "synonymous". The first written architectural source we have today, from Roman Empire period, is the "Ten Books of Architecture" which was written by Marcus Vitrivius Polyo. He wrote this book to take constructions activities under a specific control, in Julius Cesars' period. Vitrivius put 3 basic rules that could be maybe the theory of him. **1) Firmitas:** firmness of statics and construction, **2) Utilitas:** appropriateness to aims; functionality; suitablity, **3) Venustas:** aesthetical necessity and proportions. Also venustas insists some other sub-rules such as symmetry, decor, etc, in 1673, when Claude Perrault published his French translation of "The Ten Books of Architecture", he rendered "symmetria" as "proportion". But in his deep-notices, he commented that symmetry referenced to "the relationship which parts on the left side have with those on the right, those high up with those low down, those in back with those in front".

8.Conclusion

Actually, modern mathematics has further generalised and formalised the concept of geometric symmetry, grounding it upon the idea of a group of geometric transformations. It can be said that, an architectural composition is symmetrical to the extent, which has symmetry operations as isometric transformations such as translations, rotations, reflections and compositions. This symmetry is defined as a property of a set of transformations. It could be exemplified as a bilaterally symmetrical plan is transformed into itself by reflection across its axis or a pinwheel is transformed into itself by rotation and etc. Thus, not only symmetry, but also axes passing through a point, repeating linear patterns or two or more dimensional patterns could be given as examples. The rules that nature has in, were symbols of "Beauty". If we get a view of historical development of human being, we can understand that architects design artificial-environment and living-spaces by using these rules. From the ancient times, the "Beautiness" applied to compositions that have classical formal qualities of rhythm, proportion and symmetry; as Vitrivius mentioned in his book. He put "eurythmia" word for it which could be translated as "grace". He defined it as "...beauty and fitness in the adjustments of the members. This is found when the members of a work are of a height suited to their breadth, of a breadth suited to their length and in a word when they all correspond symmetrically". Such an idealist view can be also seen in St. Augustine's aesthetic theory; "...If I ask to a workman why, after constructing one arch, he builds another like it over against it, he will reply, I dare say that in a building like parts must correspond to like. If I go further and ask why he thinks so, he will say that it is fitting, or "Beautiful" or that it gives pleasure to those who behold it." Also in

Alberti's "Ten Books", it is seen that such a similar definition was used under drawing directly upon Aristotelian ideas: "... just as all the individual members harmonise in an animal organism, so all the separate parts of a building should harmonise...Each part of a building must correspond to all the others; so, as to contribute to the success and beauty of the whole. The building can not be beautiful in only one of its parts while the others neglected; all must harmonise in order to appear as a single, well-articulated body, not a jumble of unrelated fragments". In 1725, one of the British philosopher Francis Hutcheson wrote the "Beauty" to show how it depends on formal principles. In his doctrine, richly varied compositions are organised in accordance with some underlying unifying principle are "Beautiful".

In pre-modern thought, it is seen that artists- form makers- always used the organisations and completion of their formal experiences mixing w,th the mathematical and geometrical rules, the rules such as proportion, balance, line recession, symmetry, numerical symbolism, gematria and so on. Even sometimes the method has been changed on designs, never the menthalitic point of view on the theory was changed. Thus, it can be said that, there is a unity of design theory in pre-modern architecture practice.

An architectural design was based to visual symbolism on the multi-God focused religions in pagan period. Geometrical formulation were important in that point of view. However, numbers were not always coupled with geometry. Some cases they were applied to dimensions just by simple multiplication or divisions. Religious shaped numerical symbolism is shaped in centuries, where unity of God is believed. The planning of religious spaces in medieval era seems to have originated in different concepts, such as in the images corresponding in the human body proportions, in the musical harmony of cosmos, in regular polygons, in gematria, and etc.

So, all these and more are the proof of mathematical theorems, apply to extensive sets of apparently diverse figures or curves; the forms of plants and animal; that there is always a relationship, such a very precise relationship between "Mathematics and Design"; more than this, the relationship through "Mathematics, Nature and Design".

References

[1] A.Arpat, Secrets of Architecture, Evans Communications, Canada, 2004.
[2] C.Yildirim, Matematiksel Dusunme, Remzi Kitapevi, İstanbul, 1996.
[3] D.Wells, You are a Mathematician, Penguin Books, 1995.
[4] J. P. King, *Matematik Sanati*, tr. Nermin Arik, Istanbul, 1992.
[5] R.Adam, *Classical Architecture*, A Times Mirror Company, 1991.
[6] L.Mumford, *The City in History*, A Pelican Book, NewYork, 1979.
[7] M.Safdie, Form and Purpose, Houghton Mifflin Company, Boston, 1982.
[8] PG.Rowe,Design Thinking, The MIT Pres, Cambridge, 1987.
[9] T.Pappas, Yasayan Matematik, tr.Yildiz Silier, Sarmal Yay., İstanbul,1993.
[10] Vitrivius, Mimarlik Uzerine On kitap (The Ten Books of Architecture), İstanbul,1990.
[11] W.J.Mitchell, *The Logic of Architecture*, The MIT Press, Cambridge, 1994.
[12] Z.Sagdic, *Mathematics and Design Relationship in Architecture Within Chronological Period*, pp.383-390,Spain,1998.
[13] A.T.Mann, The Sacred Architecture, Element Pub., USA,1993.
[14] E.Panofsky, Meaning in the Visual Arts, Doubleday & Company Inc., Garden City, 1955.

*All of the images are from A.T.Mann, The Sacred Architecture, Element Pub., USA,1993.