The Mechanical Drawing of Cycloids, The Geometric Chuck

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Abstract

This paper discusses cycloids and their construction using the 19th century mechanical drawing instrument known as the Geometric Chuck. The first part of the paper is a brief history and description of the Geometric Chuck. The last part of the paper is devoted to a discussion the definition of cycloids and examples showing the results that various settings of the Geometric Chuck have on the cycloid patterns produced. This paper is an attempt, in part, to respond to the comment in the Savory book "As this book does not aim at giving a scientific account of the principles on which it works. It might be an exceedingly interesting subject for the scientific person, ….the scientific knowledge required to understand a three-part chuck would be so great that I doubt if there is the person existing who could describe the course of a line that would be produced…."[6]

1. History

1.1 Definition. "The Geometric Chuck is an arrangement of mechanism for producing two or more circular movements in parallel planes. The combination of these movements with different velocity ratios and different radii results in the formation of a great variety of highly interesting curves and geometric figures"[1]. The Geometric Chuck is just one of many devices that were designed to be used as accessories to the Rose Engine Lathe. This lathe, similar to the modern lathe in name only, was designed to produce ornately decorated items. The craft of producing such items on the Rose Engine Lathe is known as Ornamental Turning. A foot treadle was used to power the lathe but some larger workshops used steam power.

There are two main differences between the Rose Engine Lathe and the modern lathe. First, the rate of rotation of the Rose Engine Lathe was very slow. With a Geometric Chuck attached the rotation would not be much more than 100 revolutions of the drive spindle per minute. This slow rate of rotation was required by the fact that the mass of the Geometric Chuck is not centered on the drive chuck and a fast rotation would destroy the out of balance Geometric Chuck. Second, in the modern lathe the work piece is turned and the tool is held relatively stationary in a tool rest and applied to the spinning material. The work piece in the Rose Engine Lathe may or may not be rotating when the tool is applied and often the tool is mounted in a device that moves the tool in a prescribed manner to produce a design. These devices are driven by another set of belts and pulleys off of the foot treadle.

1.2 Development. The Geometric Chuck, as it is known today, is the result of the designs of a number of inventors. They are generally known by the inventors' names, Plant's Chuck (Fig. 1), Ibbetson's Geometric Chuck, and Holtzapffel Geometric Chuck (Fig. 3,4). All of these devices are very similar having various improvements added to make the setup and execution of patterns easier and more consistent. According to Northcott, "Ibbetson himself says he derived the idea and the name from the geometric pen (Fig. 2) of Suardi, who published an account of it in 1752[1]. The *Manuel du tourneu* shows a device, "La Machine Epicycloie", that has many of the basic parts of the Geometric Chuck[2].

The Holtzapffel Chuck is arguably the most advanced in features and flexibility. There are a number of problems in maintaining consistent starting and stopping locations when scaling patterns and changing gear ratios that the Holtzapffel Chuck solved by various adjustment devices that were incorporated into the design.

Ibbetson attempted to keep the details of his Geometric Chuck a secret, believing the use of it for the engraving of spirals on bank notes, to prevent forgery, of utmost importance[1][3][4]. The Geometric Chuck was and still is used in this application, but Ibbetson's attempt at secrecy was a failure.

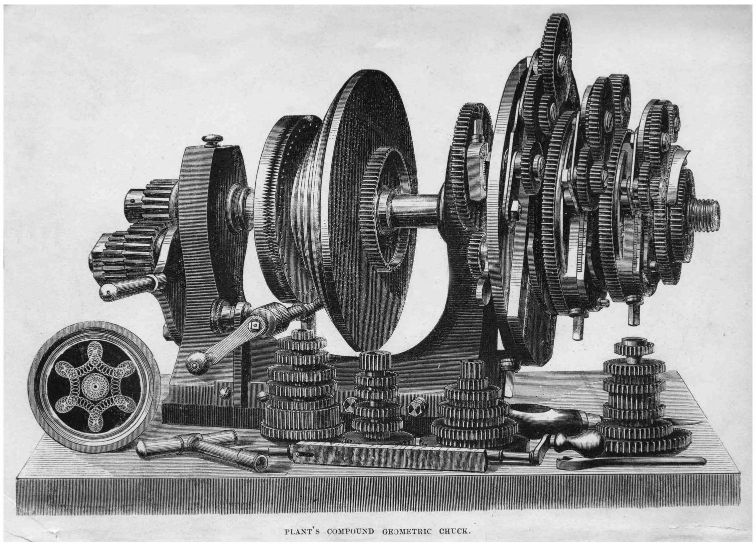

Figure 1 Plant's Compound Geometric Chuck[6]

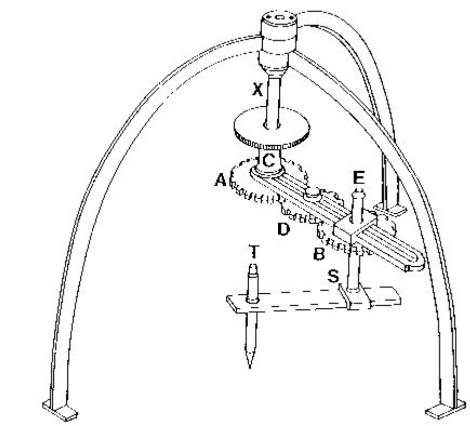

Figure 2 Suardi's Geometric Pen[11]

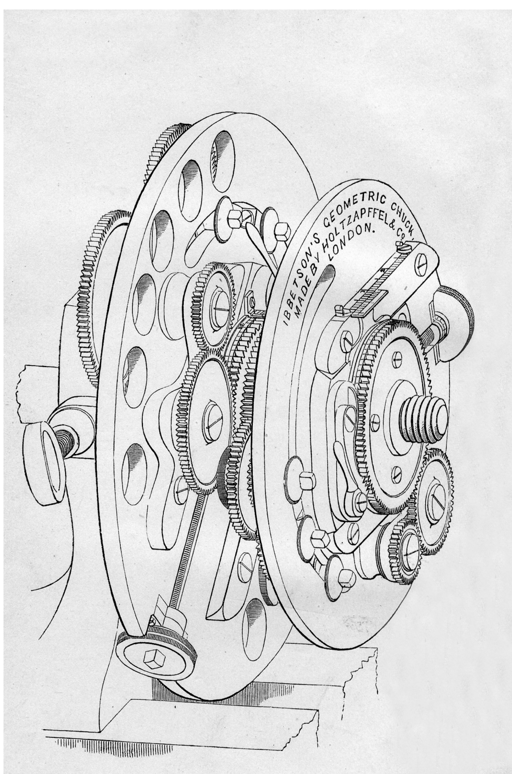

Figure 3 Holtzapffel Geometric Chuck 2 stages[5]

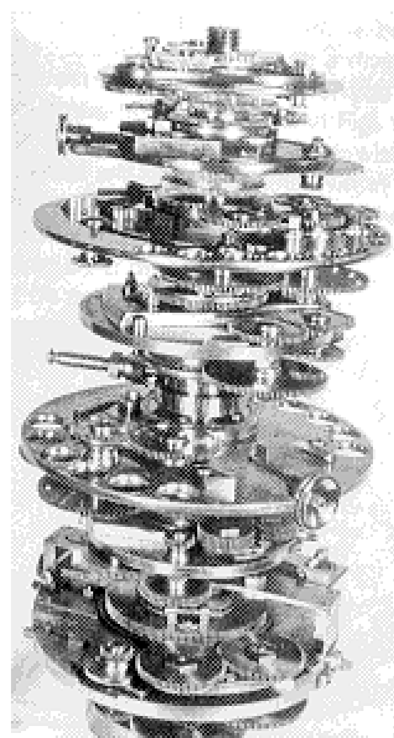

Figure 4 Holtzapffel Geometric Chuck 7 stages[12]

The number of Geometric Chucks produced was low and it was quite expensive due to the precision and handwork required to build such a complex mechanism. Its popularity was also limited because of the complexity of the device and amount of time and experimentation that was required to produce pleasing patterns. Thomas Bazley in *Index to the Geometric Chuck* states, "The impression of this work consists of 150 copies only, probably about equivalent to the number of persons who take an interest in this peculiar branch of amateur mechanism"[5]. This book contains detailed directions on the workings and settings of the chuck including 3500 patterns and the parameters used to produce them. The book *Geometric Turning* by Savory is another text that contains many examples and instructions[6]. This book is less scientific and more artistic in its approach than the Bazley. The author states, "If rules are meant to make a subject difficult and to frighten, they are very successful rules….." This comment is in response to the 39 rules by Mr. Perigal in the Northcott book[1]. Mr. Perigal makes these rules even more difficult to understand by using terminology that was unfamiliar to the average person and by the absence of any drawings, figures, or examples.

2. Description of the Geometric Chuck

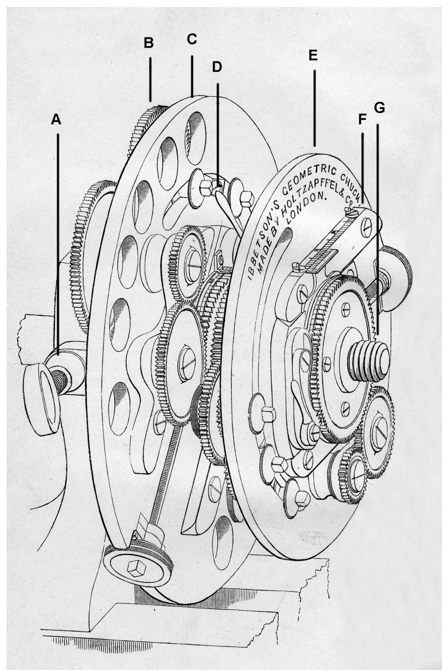

Figure 5 [5]

The figure above is a drawing of a two part Geometric Chuck. It consists of the following primary parts. A. Attachment to the head of the lathe. B. Train of wheels on arbors. C. Foundation plate of first stage. D. Spring detent. E. Foundation plate of second stage. F. Eccentric slide. G. Front terminal wheel and mandrel of stage two.

A brief description of the purpose of these parts follows:

A. The foundation plate of the first stage is screwed onto the mandrel of the lathe. The first motion wheel is clamped to the headstock of the lathe with the large thumbscrew. Without this attachment the whole mechanism would be free to rotate and no drive motion would be transmitted to the train of wheels.

B. The train of wheels consists of various sizes of wheels on arbors that are attached to the curved slot in the foundation plate. The slot allows space for the different sizes and numbers of wheels used in making the patterns. The motion of these wheels is transmitted through the foundation plate just below "D". The motion then is transmitted to the front terminal wheel by the two wheels on the front face of the foundation plate. They are mounted on a movable link whose purpose is to keep the wheels in contact with the front terminal wheel as it is moved on the eccentric slide (F, second stage). The train of wheels is what gives rotation to the first motion wheel of the next stage and determines the number, direction and spacing of cusps generated by this stage.

C. The foundation plate has a number of functions. The arbors of the wheels are attached to it, the eccentric slide is attached to the front face and the first motion and front terminal wheels are attached to it. The spring detent (D) has its base attached to the foundation plate.

D. The spring detent. There are two types of these. The one whose lever is just visible behind the foundation plate of the second stage is used to clamp the front terminal wheel of stage one to first motion wheel of stage two. By doing this the chain of wheels from stage one are connected to stage two. This detent also allows the train of wheels to be thrown "out of gear" which effectively stops the transmission of motion from the train of wheels from this stage on. The second type of detent is visible near the front terminal wheel of stage two. This detent allows the front terminal wheel to be locked so that it will not spin freely. This would be used when facing the surface of a plate for engraving.

E. The foundation plate of the second stage, similar in function to the foundation plate of the first stage.

F. Eccentric slide. There is an eccentric slide on each foundation plate. This slide is used to give the eccentric motion to each stage. It is adjusted by the large knob, shown at the back of stage two and at the bottom of stage one. The eccentric slide has a graduated index engraved on it, usually in 100ths of an inch. The position of the eccentric slide determines the size of the loops and cusps generated by stage.

G. Front terminal wheel and mandrel of stage two. This is where the next stage is attached or the piece of material to be engraved is mounted. Often a flat plate with paper is mounted to this and a pen or pencil held in the slide rest of the lathe is used to draw the pattern. This was done as a way to test the setup and function of the Geometric Chuck before the actual engraving was done.

The figures show the Geometric Chuck mounted horizontally, as it would be in a lathe. Compound Geometric Chucks such as the one shown in Fig 4 are often used in a vertical position. The reason for this is that the weight of the numerous stages becomes more than the head of the lathe can safely support. Another reason for this using this orientation is the fact that when the numerous stages are set at various eccentricities, the whole mechanism is thrown out of balance and rotation in the horizontal is unstable. Rotation of the Geometric Chuck in the vertical orientation is accomplished by attaching the first stage to a base, which usually contains some type of hand wheel mechanism. Since the Geometric Chuck is always driven at low revolutions per minute this is quite adequate.

3. Types of Cycloids

The common definitions of the epicycloid and hypocycloid are:
Epicycloid: The path traced out by a point *P* on the edge of a circle of radius *b* rolling on the outside of a circle of radius *a* [7]. (Fig. 6)
Hypocycloid: The curve produced by fixed point *P* on the circumference of a small circle of radius *b* rolling around the inside of a large circle of radius *a* [8]. (Fig 7)

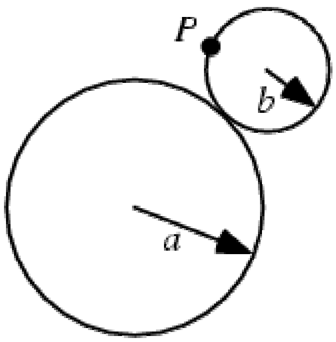

Figure 6 Epicycloid

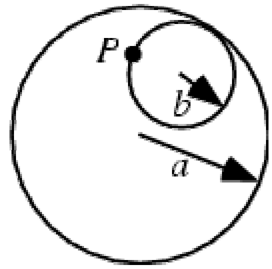

Figure 7 Hypocycloid

Two important points should be noted here.

First, *circumference* of the circle with radius *b* rolls without slipping inside or outside the *circumference* of circle of radius *a*. Thus, the rotational velocity of the circle with radius *b* is fixed by the relationship of radius *b* to radius *a*.

The specific relationship of rotational velocity to radius, while possible, is seldom maintained when creating patterns with the Geometric Chuck. The variety of patterns is limited if this relationship must be maintained. The actual physical size of the patterns seldom exceeded 4-5 inches and the smallest patterns were determined by the minimum line thickness a tool could inscribe or draw and still maintain clarity.

Second, for an epicycloid, the direction of rotation of the point *P* on the circumference of the circle of radius *b* is the same as the direction of rotation of the *center* of that circle.

Figure 8 shows the relationship of the stages of the Geometric Chuck as it is most often used.

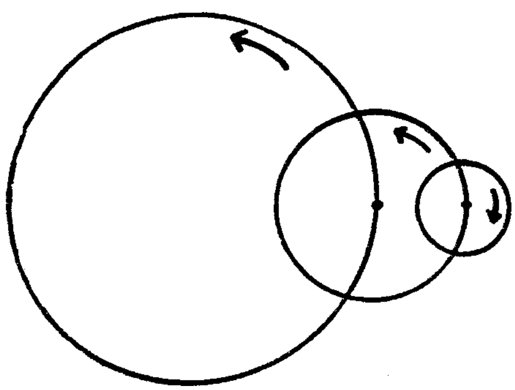

Figure 8 Wheels on Wheels[9]

Note that the *centers* of the wheels follow the *circumference* of the wheel to which they are attached. The direction of rotation of the wheel is *independent* of the direction that the center of that wheel is moving. Thus, the direction of rotation direction of a point on the circumference of a wheel may or may not be the same direction as the rotation of the center of that wheel.

These last two points are often confused at worst or unclearly stated at best. The cycloids generated the Geometric Chuck or other wheels on wheels devices are not governed by the same rules as those generated by the popular drawing toy the Spirograph® Hasbro Inc. [9][10]

The definition of the epicycloid as it relates to wheels on wheels and most settings of the Geometric Chuck is as follows:

Epicycloid: The path traced out by a point *P* on the circumference of a circle of radius *b* whose center is following the circumference of a circle of radius *a*.

On the Geometric Chuck point *P* is physically the center of the front terminal wheel and mandrel of a stage. The radius of the circle, which point *P* is on, is the relationship of the center of the stages first motion wheel and the center of the front terminal wheel and mandrel. The point *P* of the final stage in a compound Geometric Chuck is additionally determined by the relationship of the center of the front terminal wheel and mandrel of the last stage and the point of the cutting or drawing tool.

For the remainder of this paper the generation of cycloids using the common definitions located at the top of section 3 and with the restrictions that they imply will be referred to as "Type A" and cycloids generated by the definition above will be referred to as "Type B".

4. Effects of Changing Wheel Sizes

This section will show the effects of changing wheel sizes. For Type A cycloids both the number and position of cusps are changed. (Fig. 9) For Type B cycloids only the relationship of the cusps to one another is changed. (Fig. 10) The patterns in figures 9 and 10 were generated using a two-stage system for the sake of clarity. A multistage (compound) system would have the same characteristics. Part of the "art" of creating pleasing patterns using the Geometric Chuck is the selection of wheel sizes. The smaller the wheel size of a stage, relative to the other stages, the less impact on the pattern that stage will have. For complicated patterns using several stages the pattern can become very cluttered if too many stages have similar wheel sizes.

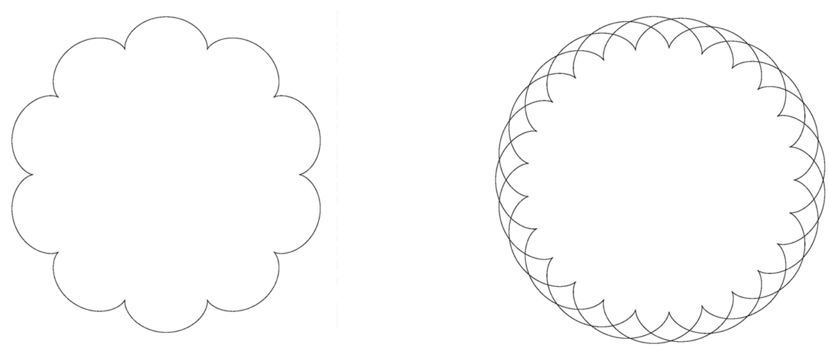

Figure 9 Type A changing second stage wheel from 10 to 12

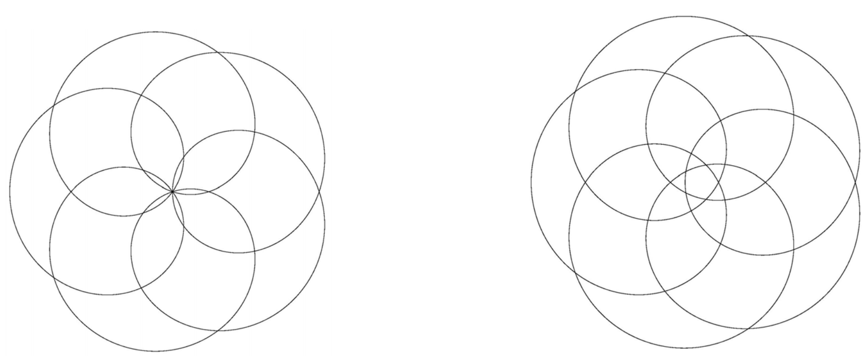

Figure 10 Type B changing second stage wheel from 10 to 12

5. Effects of Changing Wheel Rotation Direction

The changing of the direction of rotation of a stage causes inward-facing loops to face outward and vice versa. Figure 11 is an example of a Type B cycloid. The patterns in figure 11 were generated using a three-stage system, in this case not only did the orientation of the cusps change but also the symmetry of the loops is modified by the direction change. When using the Geometric Chuck, changing the wheel rotation direction is accomplished by adding an idler wheel in the chain of wheels of the stage that needs to be reversed. The idler wheel, being of the same size as the wheel that precedes or follows it in the chain, has no effect on the rotation rate of the stage. This aspect of the setup and usage of the Geometric Chuck often has a profound impact on the visual appeal of the patterns produced as seen in figure 11.

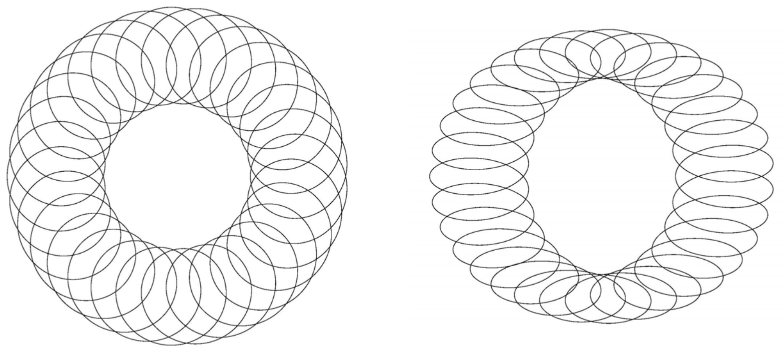

Figure 11 The rotation direction of stage three is reversed

6. Ratios, Cusps Loops and Symmetry

The subject of the relationships of rotational rates, wheel sizes and symmetry is well documented so it will not be covered in great detail here. References [1][5][9][10][11] all contain sections related to this. Rotation direction and the relationship of the wheel sizes of the stages determine Type A cycloid symmetry. The wheel size determines the rotation rate of the stage, which in turn controls the number of cusps or loops and how densely they are drawn. With Type B cycloids the number of cusps, loops and symmetry are determined by the relationship of the rates of rotation and direction of rotation of the different stages of the Geometric Chuck. Type B cycloid symmetry of compartments (the empty space between lines) can be controlled by the size of wheels that are chosen. As stated earlier if several stages have wheels of similar sizes the pattern often becomes very cluttered.

7. Asymmetrical Patterns

The majority of the patterns that were created in the 18th century were based on symmetric patterns. This was probably due to the tastes of the day. Considering the amount of time required to setup and generate patterns using the Geometric Chuck experimentation was not that easy to do. When more stages are added to the Geometric Chuck asymmetric patterns seem to be easier to find. Using computer software, experimentation and adding stages is quite simple. Most 18th century Geometric Chucks only had two or three stages.

Figures 12 and 13 are two patterns that are examples of asymmetry. Figure 12 is a Type A cycloid generated using the digits of the dates of Bridges 2006 for the wheel sizes, (84826, excluding the zeros). This is a five-stage pattern and all wheels are rotating in the same direction with the exception of the first one. Figure 13 is a five-stage Type B pattern. The wheel sizes are 100,11,35,48,70. The rotation rates are 22,66,42,22 and the fourth and fifth stages are rotating in the opposite direction of the others.

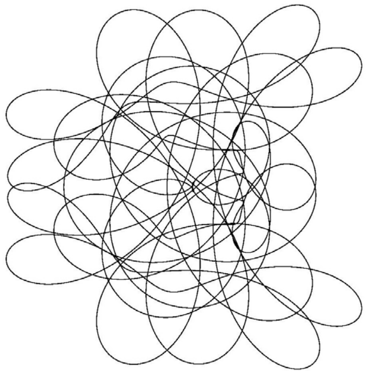

Figure 12

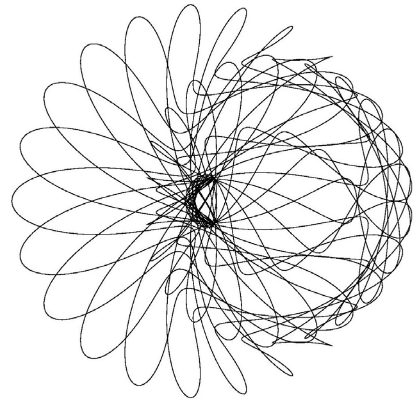

Figure 13

8. The Art of Using the Geometric Chuck

The art of the using Geometric Chuck has two components. The first is knowing how to set up the various variables of the device, wheel chains and wheel size (eccentric slides) to obtain the desired patterns. The second is the combining and scaling of patterns together on one engraving or drawing.

9. Conclusions

The Geometric Chuck and the cycloid patterns that it is capable of generating are an interesting combination of mechanics, mathematics and art. The relationship of direction of wheel rotation and rotational velocity of multistage Geometric Chucks, especially those of 4 or more stages is an area for more study. I would like to thank Craig Kaplan for math, programming and graphics advice and John Sharp for his assistance in the locating of reference materials.

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