Seville's *Real Alcázar:* Are All 17 Planar Crystallographic Groups Represented Here?

B. Lynn Bodner
Mathematics Department
Cedar Avenue
Monmouth University
West Long Branch, New Jersey, 07764, USA
E-mail: bodner@monmouth.edu

Abstract

Contemporary with the Alhambra, the *Real Alcázar* of Seville, Spain was rebuilt in 1364 as a palace for Dom Pedro, Christian king of Castile (1334 - 1369) in the *Mudejar* style. (Muslims who chose to live under Christian rule were known as *mudéjares*). Although there have been alterations and additions over the centuries, this remarkably well-preserved palace was originally built by a Christian ruler in the Islamic style of Andalusia and retains its Islamic character, containing some of the most beautiful examples of *Mudejar alicatado* (Spanish, for cut tiles, derived from the Arab verb *qata'a*, "to cut") from this time period. Since all 17 planar crystallographic groups are now believed to be represented in the tilings of the Alhambra, one wonders if the same may be said of the ornament found in the *Alcázar*. This paper will briefly discuss the history of the *Alcázar*, illustrate and classify some of the planar designs as to the isometries they permit and then attempt to answer the salient question broached in the title of this paper.

The *Real Alcázar*

As early as 712, palaces were erected by the invading Islamic armies near the shores of Seville's *Rio Quadalquivir* and at the current site of the *Real Alcázar* (Spanish for "royal palace or fortress" from the Arabic word, *al-qasr*). 'Abd al-Raman III, ruler of the Córdoban caliphate, was also known to have ordered the building of a fortified palace here in 913. Although few remains of these Moorish structures exist, portions of the high encircling walls, built in the 11th century, may still be seen today (see Figure 0. below).

Figure 0. *View of the Lion's Gate in the wall surrounding the Real Alcázar*

On November 2, 1248, after a siege that lasted 16 months, Seville was reconquered by the Christians under King Ferdinand III of Castile and León. However, for several hundred years after the fall of the caliphate, Seville remained under the influence of Islamic artistic models, due in part to the continuing existence of the Nasrid dynasty in Granada, the last sultanate on the Iberian Peninsula. During the second reign of Muhammad V (1362 – 1391), considered the Golden age of Nasrid rule, many buildings of the Alhambra were erected and lavishly decorated with geometric mosaic tiles, vegetal patterns, inscriptions and stucco, giving that palace complex its present appearance. This Nasrid style continued to evolve in the *Mudéjar* style after the Reconquest. (Muslims who chose to live under Christian political authority were known as *mudéjares*, from the Arabic word, *mudajjan*, meaning "settled" or "tamed" [1]. The *mudéjares* "continued to wear distinctive garb, adhere to a religiously prescribed diet, follow social mores different from those of the Christians, among whom they lived, speak privately in Arabic, and heed their own religion." [2])

Dom Pedro, the Catholic king of Castile (1334 – 1369) valued the Moorish heritage, and so, in 1364, when he made significant additions to and remodeled the *Alcázar* as his palace, he used the best available craftsmen from Seville, Toledo, and Granada. Muhammad V, said to be a friend of King Pedro, sent Islamic artisans and materials for the construction and decoration of the *Alcázar*. Hence, the *Real Alcázar* is contemporary with and evokes the Alhambra. In 1492, when the Nasrids were driven out of Granada, it was the *mudéjares* tilers of Seville who made the repairs to the damaged dados of the Alhambra and who were also called upon to produce the *alicatado* for their own *Alcázar*. (*Alicatado* is Spanish for cut tiles, derived from the Arab verb *qata'a*, "to cut").

The *Alcázar* has been used as a residence by the Spanish royal family ever since 1248 and as such, is the oldest royal residence in Europe still in use. It is a remarkably well – preserved palace, having undergone many alterations and additions over the centuries, including renovations in the 15th century by Ferdinand and Isabella. The *Real Alcázar* also retains its Islamic character, containing some of the most beautiful examples of *Mudejar alicatado* from this time period.

Classifying Two-Dimensional Patterns

To analyze and classify patterns in the plane, we assume that the motif repeats infinitely in two directions. Using group theory, we find that there are only 17 possible categories, based on the isometries (distance-preserving transformations) permitted by the pattern. The isometries include translations, rotations, reflections across a vertical, horizontal or diagonal axis, and glide reflections, which involve translation and reflection. The classification notation used here was devised by Carl Hermann and Charles-Victor Mauguin, where each category begins with either a ***p*** for a primitive cell, or a ***c*** for a face-centered cell. The ***p*** or ***c*** is then followed by a digit ***n***, which indicates the highest order of rotational symmetry the pattern exhibits. For example, patterns with no rotational symmetry have ***n = 1***; patterns with ***n = 2*** have 2-fold rotational symmetry; and so on, for ***n = 3, 4,*** or ***6***. The third and fourth symbols indicate the existence of mirror reflections, glide reflections, or no reflections, and use the symbols ***m, g*** or ***1,*** respectively.

In 1944, E. Muller found patterns at the Alhambra which represented 11 of the 17 planar crystallographic groups. In a 1986 paper, B. Grunbaum and Z. Grunbaum of the University of Washington (in the USA) & G. C. Shepard of the University of East Anglia (in the UK) reported to have found 13 of the patterns there. The four groups they did not find were *p*2, *pg*, *pgg* and *p*3*m*1. Subsequently, R. Perez-Gomez of the University of Granada & J. Montesinos of the University of Madrid have reported to have found the missing ones. Hence, one cannot help but wonder if all 17 of the planar crystallographic groups may also be found at the *Real Alcázar*. This paper will attempt to answer that question by first illustrating some of the recognizably Islamic planar patterns, and then classifying them as to the symmetry elements they permit. Note that colorings and interlacings may not be considered.

The first pattern (if color is not considered) may be classified as ***p*111** (or ***p*1** for short) to indicate that the pattern may be contained within a primitive cell, and there are no rotations, mirrors or glide reflections present. Notice the trapezoidal polygonal tiles at the top of the tiling, which seem to "point" to the left. This *alicatado* (see Figure 1. below) may be found as part of a dado (a mosaic found on the lower portion of a wall) in a hallway leading to the *Patio de las Doncellas*.

Figure 1. *A* ***p*1** *pattern*

Figure 2. *A* ***pm*** *pattern*

The second pattern (if color and interlacing are not considered) may be classified as ***p*1*m*1** (or ***pm*** for short) to indicate that the pattern may be contained within a primitive cell, there are mirror reflections but no rotations, nor glide reflections off the mirror reflection axis. This *alicatado* may be found as part of a dado in the *Patio de las Doncellas* (see Figure 2. above).

The third pattern (see Figure 3. below), a floor tiling showing a zigzag motif, may be classified as ***p*1*g*1** (or ***pg*** for short) to indicate that the pattern may be contained within a primitive cell, there are no rotations nor mirror reflections, but there are glide reflections.

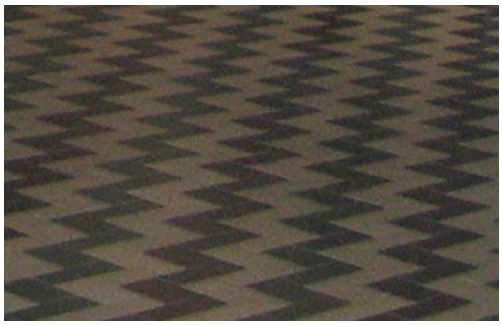

Figure 3. *A* ***pg*** *pattern*

Figure 4. *A* ***cm*** *pattern*

The fourth pattern (see Figure 4. above), part of an archway in the *Patio de las Doncellas* (and if idealized), may be classified as ***c*1*m*1** (or ***cm*** for short) to indicate that the pattern may be contained within a centered cell, there are no rotations, but there are mirror reflections and a glide reflection off the mirror reflection axis.

The fifth pattern (if one discounts the heraldic shields within the octagons) may be classified as ***p*211** (or ***p*2** for short) to indicate that the pattern may be contained within a primitive cell, there is a two-fold rotation but there are no mirror reflections nor glide reflections. Note that the interlacing (see Figure 5. on the next page) precludes the mirror reflections.

Figure 5. *A* ***p2*** *pattern*

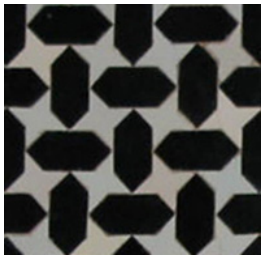

Figure 6. *A* ***pmm*** *pattern*

The sixth pattern (see Figure 6. above), part of a dado, may be classified as ***p2mm*** (or ***pmm*** for short) to indicate that the pattern may be contained within a primitive cell, there is a two-fold rotation and mirror reflections, with all the rotation centers falling on reflection mirrors.

The seventh pattern (if one disregards the coloring of the pattern) may be classified as ***p2gg*** (or ***pgg*** for short) to indicate that the pattern may be contained within a primitive cell, there is a two-fold rotation and a glide reflection but no mirror reflections. This (see Figure 7. below) is part of a dado.

Figure 7. *A* ***pgg*** *pattern*

Figure 8. *A* ***cmm*** *pattern*

The eighth pattern, part of a dado (see Figure 8. above), may be classified as ***c2mm*** (or ***cmm*** for short) to indicate that the pattern may be contained within a centered cell, there is a two-fold rotation and mirror reflection axes both perpendicular and parallel to the main axis, with not all rotation centers on the mirrors.

The ninth pattern, part of a dado, may be classified as ***p311*** (or ***p3*** for short) to indicate that the pattern may be contained within a primitive cell, there is a three-fold rotation and no mirror reflection nor glide reflections. Note that the motif (see Figure 9. below) consists of one white and one adjacent black shape.

Figure 9. *A* ***p3*** *pattern*

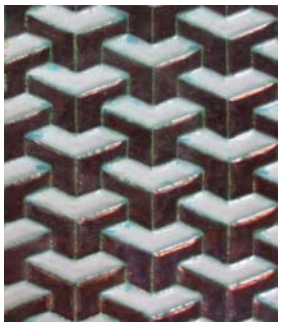

Figure 10. *A* ***p31m*** *pattern*

The tenth pattern (see Figure 10. on the preceding page), preserved in the *Alcázar*'s museum, may be classified as ***p31m*** if one disregards the following: the six-sided polygonal shapes have different colorings and the light colored polygons are slightly larger than the dark-colored polygons. If one considers an idealized tiling with all the polygons congruent and similarly colored, the motif for the tenth pattern may be contained within a primitive cell, with a three-fold rotation and a mirror reflection, not all rotation centers on the mirrors. Note that in the idealized pattern, there is a point about which one may rotate 120 degrees and this point is not on a mirror of reflection.

The eleventh pattern, part of a dado (see Figure 11. below), may be classified as ***p411*** (or ***p4*** for short) to indicate that the pattern may be contained within a primitive cell, there is a four-fold rotation but there are no mirror reflections nor glide reflections.

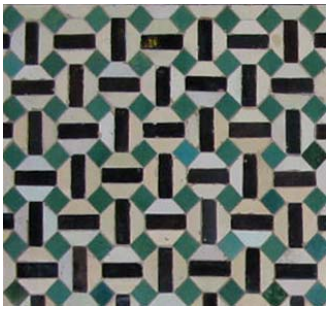

Figure 11. *A* ***p4*** *pattern*

Figure 12. *A* ***p4m*** *pattern*

The twelfth pattern (see Figure 12. above), part of a dado, (if interlacing is not considered) may be classified as ***p4mm*** (or ***p4m*** for short) to indicate that the pattern may be contained within a primitive cell, there is a four-fold rotation and mirror reflections 45 degrees to one another.

The thirteenth pattern (see Figure 13 below), preserved in the *Alcázar*'s museum, (if interlacing is not considered) may be classified as ***p4gm*** (or ***p4g*** for short) to indicate that the pattern may be contained within a primitive cell, there is a four-fold rotation, mirror reflections 45 degrees to one another, and a glide reflection perpendicular to the main mirror reflection axis.

Figure 13. *A* ***p4g*** *pattern*

Figure 14. *A* ***p6*** *pattern*

The fourteenth pattern, part of a dado (see Figure 14. above), may be classified as ***p611*** (or ***p6*** for short) to indicate that the pattern may be contained within a primitive cell, there is a six-fold rotation, but no mirror reflections nor glide reflections.

The fifteenth pattern (see Figure 15. below), part of a dado, (if interlacing is not considered) may be classified as ***p6mm*** (or ***p6m*** for short) to indicate that the pattern may be contained within a primitive cell, there is a six-fold rotation and mirror reflections.

Figure 15. *A* ***p6m*** *pattern*

Discussion

As mentioned earlier, Grunbaum, Grunbaum, and Shepard reported to have found patterns represented by 13 of 17 the planar crystallographic groups at the Alhambra. The four they didn't find were *p*2, *pg*, *pgg* and *p*3*m*1, which Perez-Gomez and Montesinos reportedly did find. This author was unable to locate examples of patterns representing two of the 17 groups: *p*2*mg* (or *pmg* for short) which displays 2-fold rotation, has reflections, but no perpendicular reflections, and *p*3*m*1 which displays 3-fold rotation, where all rotation centers are on mirrors. It is interesting to note that one of these, *p*3*m*1 was also missed by Grunbaum, Grunbaum, and Shepard.

In *Symmetries of Islamic Geometric Patterns* [3], Abas and Salman provide a chart showing the distribution of the various symmetry groups of the Islamic patterns they have compiled. The three rarest categories of patterns found in mosaic tilings in Andalusia and the *Maghreb* involve glide reflections, indicating that the least preferred tilings in Islamic cultures may be classified as *pg*, *p*1 and *pmg*. Thus, it is no surprise that the author was unable to find an example representing the *pmg* class at the *Real Alcázar*. This is consistent also with the findings of the author in a previous presentation [4], where the only unrepresented frieze pattern at *Seville's Real Alcazar* was *p*1*g*1, which also employs a glide reflection.

References

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[2] F. C. Lister and R. H. Lister. *Andalusian Ceramics in Spain and the New World.* The University of Arizona Press, 1987.
[3] S. J. Abas and A. S. Salman. *Symmetries of Islamic Geometrical Patterns*. World Scientific, 1998.
[4] B. L. Bodner. *Classifying the Frieze Patterns of Seville's Real Alcazar,* presented at the Joint Mathematics Meeting in January, 2006.