Repeated Figures

Susan McBurney
211 Rugeley Road
Western Springs, IL 60558 USA
E-mail: smcburney@prodigy.net

Abstract

This paper illustrates the development of two types of design from the beginning concept through execution onto enhancement for final presentation. Emphasis is on a structured, modular process suitable for instruction in either an art or beginning programming curriculum.

1. Initial Concept

The power and flexibility of modern computers and software have opened new avenues of exploration for artists and mathematicians alike. Sometimes even a very simple concept can lead to unexpected yet stunning results. This paper will present two types of designs developed by the author from basic building blocks manipulated with simple algorithms. Additionally it will discuss the enhancement of one of these designs leading to a finished piece of artwork.

The overall concept can be denoted as "Repeated Figures" since all the examples are built from just one figure repeated and manipulated to yield a more complex design.

An example of repeated figures can be seen in Figure 1. These designs were presented and explained at the 2006 Joint Mathematics Meeting in San Antonio, Texas, USA [1].
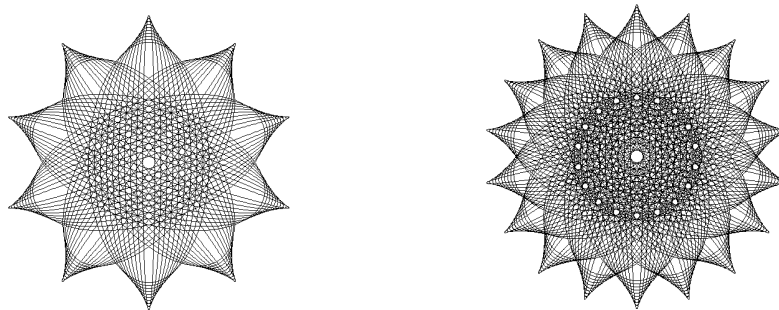

Figure 1

Each of these is made of "just ellipses", nothing else, grouped and rotated in a pattern. The surprise comes from the intricacy of the design that develops. No one could predict the rings of circles that appear in the finished design, yet all of the complexity derives from the interactions of very basic elements—ellipses.

2. The Sunflower Series

Now we will develop two additional types of figures step by step. First is the "sunflower" series. Once again the name comes after the development. That is, one doesn't start to draw a sunflower, but instead chooses a design element and then waits to see what happens. Only afterward does the finished product suggest a name.

In this case the underlying construct is a semi-circle and the language used to draw it is Logo, an easy-to-learn language freely available on the web. Logo has two basic semi-circle commands, arc and arc2, as illustrated below.
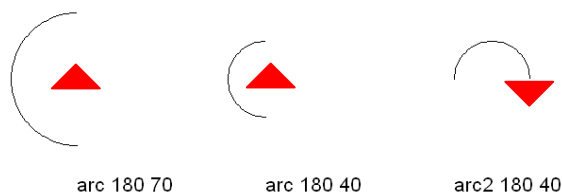

arc 180 70   arc 180 40   arc2 180 40

Figure 2: *Drawing semi-circles*

Drawing begins at the rear of the pointer. The first number specifies the number of degrees spanned by the arc and the second number determines the radius of the arc from the pointer. With the "arc2" command drawing begins at the base of the pointer which is then left at the termination of the arc.

Using the last command, arc2 180 40, consider what would happen if it were repeated five times, each time turning 108 degrees to the right before drawing the next arc. This is the result.
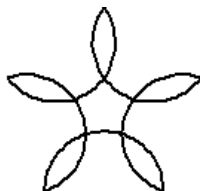

Figure 3: *repeat 5 [arc2   180   40   right   108]*

Some exploration and a little thought will allow students to discover a method to determine the amount of turn necessary at the end of each arc to complete a full figure after n turns. Thinking in terms of a regular polygon, draw one side, calculate the vertex angle of the polygon, and realize that you must turn 180° minus that number to continue. The vertex angle (VA) of any regular polygon is $180(n-2)/n$ so the turning angle (TA) = 180-VA degrees which reduces to $360/n$ degrees where n is the number of sides of the regular polygon. For the arc2 command, adjustments must be made to accommodate the position of the pointer at both the beginning and end of the arc. The desired TA is $180-360/n$ degrees, or 108° if n=5.

Now what would happen if this figure were repeatedly rotated about the beginning point two times, six, or twenty times?
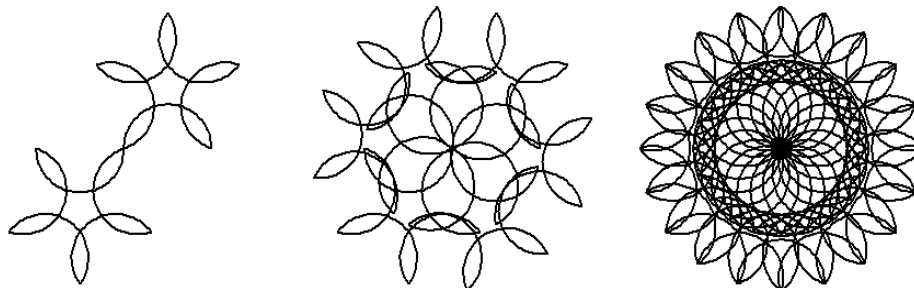

Figure 4:
repeat :n [repeat 5[arc2 180 40 right 108] right 360/:n]
n = 2, 6, 20

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In three steps we have gone from drawing a simple arc to creating a design of interest. In particular the last design has four distinct elements that can be emphasized in different ways. They are the outside rim, the interim band with holes, the inner overlapping arcs, and the small interior circle at the center of the design.

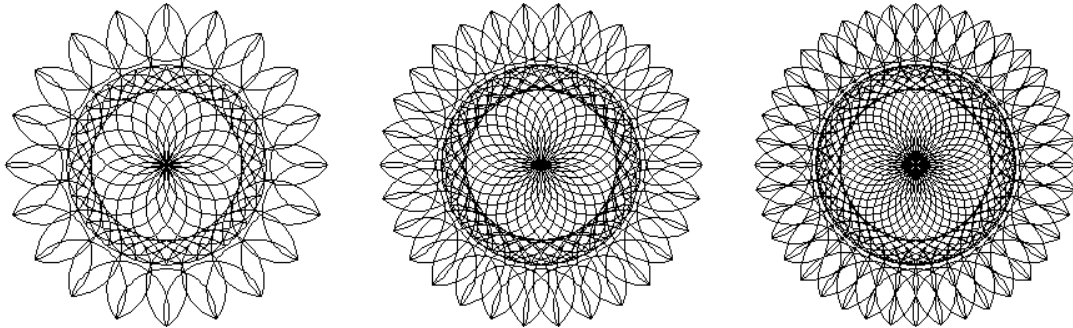Here are three "sunflower" designs of increasing complexity.

Figure 5: *Sunflowers, n=20, 30, 40.*

Continuing with this series, the design in Figure 6 is created by the command:
repeat :n [repeat 5[arc2 180 40 right 108] right 360/:n] with n = 36. Notice the change in shape of the outermost petals as well as the heavier interior band.

Once the design is obtained, it can be enhanced in other programs. This last design was imported into PhotoShop LE and manipulated in layers. The background is a solid dark green. On top of this there are two layers, each with the basic design transformed in some way. On the first layer the figure was resized, the background deleted, and the transparency set to 44%. The second layer contains a clipped, transparent copy of the figure moved toward the bottom right hand corner. Then a square was selected, filled with white, and the Texture, Grain filter applied. The final design is shown below in grayscale.
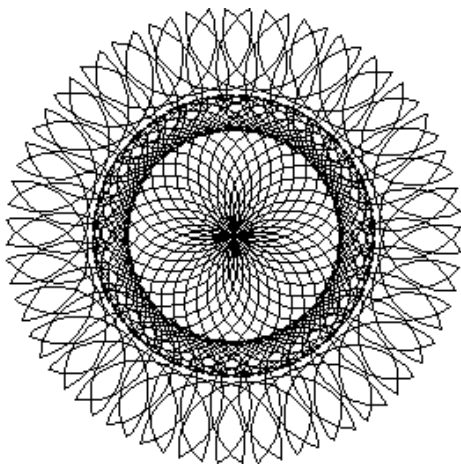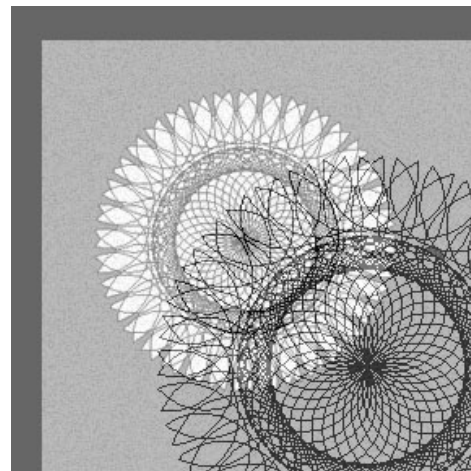

Figure 6: *n = 36*

Figure 7: *PhotoShop Enhancement*

3. Square Ball Series

A second set of designs resembles offset squares with raised hemi-spherical interiors. In this case the basic structure is the ellipse. A simple command in Logo, ellipse :a :b, will draw an ellipse with width a and height b about the center point. Drawing ten ellipses on top of each other with decreasing widths and increasing heights produces Figure 8. The following subroutine does the drawing automatically, with an initial width and height specified by the user.

```
to ells :a :b
repeat 10 [ellipse :a :b   make "a :a-10   make "b :b+5]
end
```
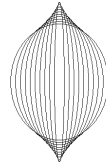
Repeating the figure four times and rotating it about the center leads to the following "square ball" designs.

Figure 8

ells2 100 120   ells2 90 100   ells2 100 100
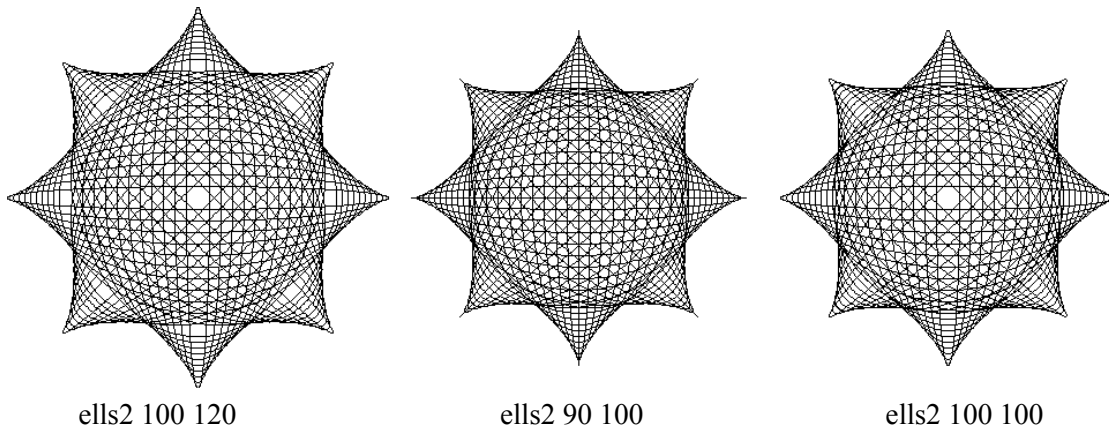
Figure 9: *Square Ball designs*

The subroutine for this combination is

```
to ells2 :a :b
repeat 4 [ells :a :b   right 45]
seth 0  ;sets the pointer to face up, toward the top
end
```

It is interesting to note the subtle differences among the designs. In the first one there is some open space around the interior and the center is open. The middle design is filled throughout and the points come exactly to a line at the peak. The center is nicely filled in as well. The right design combines these qualities to include some openness within a solid "ball". These designs of course can then be enhanced in PhotoShop or other program of choice such as Paint to add color, backgrounds, borders, etc.

Shown here are just two of the limitless possibilities for design development. It is hoped that these examples will encourage exploration into the wonderful worlds of math appreciation and computer programming.

References

[1] Susan McBurney, *A Collection of Curves*, Joint Mathematics Meeting 2006, San Antonio, TX, USA
[2] Robert Williams, *The Geometrical Foundation of Natural Structure*, Dover, 1979
[3] Robert J. Krawczyk, *Workshop*, ISAMA-CIT conference, Chicago, IL, USA, 2004