A New Use of the Basic Mathematical Idea of Twelve-Tone Music

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Abstract

We here briefly describe a collection of pieces which we have written, and which have been performed for large audiences, in which mathematics is used. Specifically, every one of the 12 major chords, and every one of the 12 minor chords, appears in each of these pieces. We argue that this is a more pleasing use of the number 12 in music than the twelve-tone system of Schönberg.

The Twelve-Tone System

One of the most obvious connections between mathematics and music is that of twelve-tone music, pioneered by Schönberg and also used by Anton Webern, Alban Berg, and Hanns Eisler (see [1, 2, 3]). A piece of music is made up of tone rows, each of which consists of the 12 notes of the chromatic scale in some order. Mathematical transformations of a tone row may be used; for example, it may be transposed into a different key, or it may be played backwards. Also, it may be inverted, meaning that downward and upward intervals are interchanged. Any combination of these three transformations may be used. Also, adjacent notes in a tone row may be played together, as a chord. The number of possible variations of a tone row depends on that tone row; as a simple example, a chromatic scale (which is a tone row), inverted and then played backwards, gives the original scale, so it is not a different variation.

In academic music departments in the US, twelve-tone music was popular for a long time. Today, however, it seems to have fallen on hard times. Much of the blame for this has been placed on Schönberg's own initial insistence that the twelve-tone method, rather than the classical ideas of musical harmony, should be the basic principle behind composition: "As originally designed by Schönberg, the [twelve-tone] method was intended to preclude tonality" [4]. This was fascinating for a long time but has generally lost its appeal. Contrary to popular belief, however, not all twelve-tone music is atonal; both Berg and Schönberg wrote twelve-tone music in which some semblance of tonal harmony was preserved (also see [4]).

Our Alternate Use Of Twelve Tones

Our own interest in the mathematical ideas of twelve-tone music was an outgrowth of our interest in the composition of satirical songs. Not that we had any idea of satirizing the twelve-tone system itself (although Leonard Bernstein has done this). Rather, we were interested in expanding the range of tonal music, which normally uses only a tiny fraction of the available chord changes. Our interest was drawn, in this connection, to Bach's Well-Tempered Clavier, containing 24 preludes and fugues, one in each of the 12 major and 12 minor keys. However, we wanted to write, not 24 pieces, but a single piece. We were also fascinated by the Symphony in D Minor by Cesar Franck, an early exponent of chromaticism (later greatly expanded by Schönberg). Franck, although his music was unabashedly tonal, used sequences of chord changes which had never been heard before.

Could it be possible, we wondered, to write a long piece in the style of Cesar Franck, which used, at some point in the piece, every one of the 12 major chords and every one of the 12 minor chords? This would preserve the connection between mathematics and music that was inspired by Schönberg, without introducing atonality, which has little appeal to audiences for satirical songs. The result was a series of songs, written in this way, for the annual Hexagon Revue, which runs for over 15 performances each year in the city of Washington. These were called "Election Jeer" (about the election of 1996), an opener for the 2001 show (about the election of 2000), and "Wee Puns of Mass Distraction" (about the invasion of Iraq). Space here does not permit the reproduction of the lyrics of these songs. The following table, however, shows the measure numbers of typical measures in which these chords occur, for each song:

Chord	Election Jeer	Opener 2001	Wee Puns of Mass Distraction
C major	16, 66, 69	98, 99, 100	18, 20, 31
Db major	25, 28, 104	21, 22, 23	60, 136, 138
D major	55	63, 64, 105	67, 154
Eb major	7, 34, 38	25, 26, 115	12, 14, 24
E major	20, 91, 163	186, 187, 188	55, 56, 148
F major	32, 68	76, 95, 96	51, 58, 59
Gb major	30, 59, 102	7, 11, 39, 40	61, 62
G major	63, 65, 159	73, 74	10, 38
Ab major	1, 2, 4-6	27, 28, 44	145, 166
A major	12, 22, 24	161, 181	64, 134
Bb major	33, 48, 57	42, 67	52, 53, 83
B major	19, 31, 40	195	54, 148
C minor	115	25, 83, 231	4, 6
C# minor	138	213	43, 144, 147
D minor	18, 158	162	26, 150
Eb minor	58, 106, 107	43	85, 142, 159
E minor	62, 67, 169	249, 251	33, 101
F minor	53, 114	245	75, 140
F# minor	171	215, 253	160
G minor	54	55, 56, 57	46
G# minor	60, 89	5, 6, 8, 10	70, 92, 153
A minor	9, 10	250	29, 124
Bb minor	187	107	91
B minor	21	216	65

The full music and lyrics are available from the author.

References

[1] Perle, G., *Serial Composition and Atonality: An Introduction to the Music of Schoenberg, Berg, and Webern*, University of California Press, 1962.
[2] Schoenberg, A., *Fundamentals of Musical Composition*, Faber and Faber, London, 1967.
[3] Betz, A., *Hanns Eisler: Political Musician*, Cambridge University Press, Cambridge, 1983.
[4] Sadie, S., ed., *The Norton/Grove Concise Encyclopedia of Music*, W. W. Norton, New York, 1994.