Beauty in Art and Mathematics: A Common Neural Substrate or the Limits of Language?

Daniel J. Goldstein
Departamento de Fisiología, Biología Molecular y Celular
Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires
Argentina

Abstract

Mathematicians often refer to the aesthetic qualities of mathematical works using the same terms and expressions employed by artists and art critics to evaluate visually apprehensible objects. Does this reflect the limits of human language, or is it a subtle indication that vision is somehow connected with the process of understanding and inventing/discovering mathematics?

Gian-Carlo Rota and Freeman Dyson have written on the beauty of mathematics and refer to non-visual mathematics in visual terms. Rota discusses the phenomenology of the beauty of mathematics using words and arguments that are strongly reminiscent of those which are used to analyze the issues of beauty and aesthetic relevance in art. Do Rota's and Dyson's writings reflect the existence of a connection between mathematics and vision, or do they just reflect the limitations of the human language?

Dyson and the Visualization of the Invisible

Visual mathematics has become a very popular subject in contemporary mathematics. Ian Stewart's *Nature Numbers* deals with mathematical objects derived from many different fields of mathematics–e.g. non-linear dynamics, chaos theory, and complexity theory–which can be visually represented by means of computers [1]. In his review of Stewart's book, Freeman Dyson contrasts the "new visual style of mathematical thinking" with the "old fashioned non-visual mathematics, the mathematics of equations and exact solutions"[2]. Yet Dyson uses visual metaphors for criticizing Stewart's mathematical preferences, as well as for addressing his own perception of mathematics.

> The beauty of Maxwell's equations becomes *visible* only when you abandon mechanical models, and the beauty of quantum mechanics becomes *visible* only when you abandon classical thinking [...] Quantum mechanics runs counter to the two cardinal principles of the new wave of mathematics. (Italics mine)[3].

Dyson also asserts that the Van der Pol equation "illustrates vividly the *blindness* of mathematicians to discoveries in unfashionable fields," and finds "examples of mathematical description...that [he would not] consider *deep*" (Italics mine) [4]. Thus, the lack of appreciation of the "deep" meaning of mathematics that is impossible to visualize is attributed to "blindness."

These kinds of oxymorons and extreme metaphors are also frequent in critical inquiries, where language is often stretched beyond its literal meaning. For the poet Simonides of Ceos (c. 556-468 BC), the founder of the *ut pictura poesis* tradition, painting is mute poetry [5][6]. Leonardo considers poetry a speaking picture, a blind painting [7]. Wölfflin makes the metaphorical distinction between classical painting (tactile, sculpturesque, symmetrical, and closed) and the baroque painting (visual, “painterly,” asymmetric, and open) [8].

Rota and the Phenomenology of Mathematical Beauty

Rota tries to “uncover the sense of the term ‘beauty’ as it is used by mathematicians.” The issue, he argues, is relevant because beauty becomes a matter of contention in an intellectual discipline in which its practitioners “are fond of passing judgments on the beauty of their favored pieces of mathematics” [9]. Beauty is a fuzzy expression to denote a fuzzy sensation of pleasure that itself is the result of the combinations of other fuzzy appreciations such as “elegance,” “surprise,” “potency,” and “opportunity,” all of them conditioned by the ideological context–the historical stage–of the artist and the beholder. Mathematical beauty and artistic beauty are both cultural constructs and active societal work is required to impose the new cannons. “Beauty” and “elegance” are institutional facts, and change according the predominant ideology. There is not a uniform canon of beauty–e.g. the beauty of Picasso's cubist and surrealist portraits is different to the beauty of Filippo Lippi’s *La Vergine*. The acceptance of new mathematical perspectives and new artistic perspectives is not immediate, and it is well known that mathematical theories and art styles became “objects of beauty” only after new generations are educated in them. Moreover, a given theory may induce pleasure in some mathematicians, and boredom and irritation in others.

Rota suggests that mathematicians use the word “beauty” to denote “the phenomenon of enlightenment”–i.e. the capture of the *sense* of a statement–“while avoiding acknowledgment of the fuzziness of this phenomenon” [10]. To appreciate mathematical beauty it is necessary to understand the mathematics involved–to force the brain to do mathematical work. Grasping the meaning of mathematical texts, not the aesthetic appreciation of its symbols or the sound of its phonetic reading, is what elicits the sensation of pleasure.

> Appreciation of mathematical beauty requires familiarity with a mathematical theory, and such familiarity is arrived at the cost of time, effort, exercise, and *Sitzfleisch* rather than by training in beauty appreciation. [11]

For Rota, “classical Euclidean geometry is often proposed by non-mathematicians as a paradigm of a beautiful mathematical theory [but not] by professional mathematicians”[12]. The fact that non-mathematicians often find Euclidean geometry pleasant proves Rota's argument. Euclidean geometry is probably the only tiny segment of the vast and expanding mathematical universe that (a minority) of high school students invest time and *Sitzfleisch* to understand.

Art appreciation is also based on understanding, and requires intellectual work and *Sitzfleisch*. Vision uninformed by previous knowledge and context means blindness. Bernard Berenson observed that “many see paintings without knowing what to look at” [13]. Mitchell, Gombrich and Nelson Goodman agree in that “the innocent eye” is blind [14]. An *oeuvre* is deemed “beautiful” and not merely elegant or technically accomplished when it makes sense–i.e. when it is enlightening. Moreover, significant images and icons–from prehistoric cave painting to cubism and concrete (abstract) art–are useful because they explain things, open new perspectives, and are symbolic resources. Picasso tried to understand the work of Matisse, his most important competitor [15].Yet the understanding of a picture does not mean deriving pleasure from it. Matisse understood Cubism and could “read” Picasso and Braque, although he disliked its aesthetics [16].

For Rota, mathematical beauty is associated with shortness and compactness [17]. The grasping of the representative essentials is one of the characteristics that are associated with beauty in the fine arts–e.g. the images depicting animals in the prehistoric caves, the drawings of Picasso and Matisse [18]. The notions of shortness and compactness are complemented with the appreciation of the beauty of “streamlined proofs.” “Hilbert's original axioms were clumsy and heavy-handed, and required streamlining” [19]. Rota’s use of the concept of streamlining underlines the historicity of the concept of beauty in mathematics. Streamlining began as a visual concept (the course of water and air currents as inferred by visual cues), which did not exist as an aesthetic concept until its invention by automobile and aeronautical designers in 20th Century USA [20]. The aesthetic canons of modernity, therefore, influence the aesthetic cannons of beauty in mathematics.

Mathematical beauty is often *partial*, in the sense that the mathematician often detects beauty in a small component of a much larger structure that is not perceived as beautiful as a whole. The portion that is deemed beautiful is “a brilliant step in an otherwise undistinguished proof” [21]. This is also well known in the fine arts, where isolated segments of an otherwise irrelevant picture (or sculpture) can impress as beautiful when considered in isolation. The photographic enlargements of portions of an *oeuvre* often disclose a hidden beauty that is lost in the whole. Different sectors of a painting have different meaning for different beholders. Jackson Pollock identified small areas in Picasso’s paintings that justified his claim that the Spanish master was the direct precursor of abstract expressionism. These same areas are considered utterly irrelevant by other observers, which see them as technical accidents resulting from Picasso’s sloppiness and speed.

G. H. Hardy and the Element of Surprise

G. H. Hardy believed that the beauty of a mathematical proof depends on the element of *surprise.* Rota concedes that “the beauty of a piece of mathematics is often perceived with a feeling of pleasant surprise [and] instances [can be found] of surprising results which no one has ever thought as classifying as beautiful.” Still, Rota recognizes “instances of theorems that are both beautiful and surprising abound” and attributes the beauty of Galois theory of equations to “the once improbable notion of a group of permutations” [22]. Surprise is a central ingredient in the appreciation of a work of art, where the unexpected derives from leaps over conventional limits of theme, subject, and style, and humor. The works of Hyeronimus Bosch, Picasso, Dalí, and Magritte are a source of surprises, and the pleasure that they elicit is often related to their capacity of surprising the beholder. *La blague d' atelier* always lurks, with a touch of depravity and/or the inversion of the so called “natural hierarchies.” In general, this is achieved by depicting conventional objects in new contexts, and things and circumstances that previously were confined to the fringes of artistic representation (where they were mostly unseen), or were not represented at all, suddenly became central and are in the spotlight.

In Courbet’s *The Burial* and *The Vagina*, the artist jumps over all the conventions of representation of his time. The visual depiction of a group of minor landowners in an obscure French village and the scrupulous anatomical rendition of a body part that is normally hidden suddenly acquire aesthetic relevance. Until Courbet, small landowners and rural personages were not the adequate subject for high art, and the representation of the woman sexual organs unthinkable. Manet’s *Olympia*, the rendition of an inexpensive prostitute, shocked a French public used to an art that represented haughty courtesans. Nan Goldin’s photographs were revolutionary because they exposed domestic violence and physical degradation in all its magnitude and horror. Duchamp’s urinal shocked and surprised because the functionality of the object (the collection of urine) had so far determined its automatic exclusion from the realm of aesthetics. Courbet, Manet, Goldin, and Duchamp produced abrupt discontinuities in the realm of visual narratives by bringing things peripheral into the central point of attention.

Does something similar to this occur in mathematics? In the 18th Century, sines and cosines, although they were standard methods for analyzing waves–e.g. harmonics–did not belong to the advancing edge of the mathematical sciences of the time. When Fourier, half a century later, showed that any function could efficiently approximated by using the summation of a series of sines and cosines he suddenly put trigonometric functions in the center of mathematical inquiry. Fourier's discovery led Dirichlet to the precise definition of the concept of function, and Riemann to invent the Riemann integral to deal with situations that could not be tackled with the Cauchy integral. Fourier, Dirichlet, and Riemann brought to the center of mathematical inquiry objects and anecdotes that had been considered to be just useful algorithms or plain curiosities, and showed that they are endowed with extraordinarily interesting and useful mathematical attributes. These were mathematical surprises.

Rota also refers to *elegance*, although he recognizes that "mathematical elegance has to do with the presentation of mathematics, and only tangentially does it relate with its content" [23]. Like beauty, elegance is an age-and context-dependent attribute. Mathematics that was considered elegant in the past may be seen as dull now, in the same way that aesthetic preferences change with time. One hundred and fifty years ago, Botticelli's women, the paradigms of feminine beauty in the Twentieth Century, were described as swallow semi-skeletons stricken by consumption. "Heavy," "clumsy," and "massive" mathematics could also have been aesthetically appealing in the past, as once were *à la* Rubens nudes.

The Aesthetics of Authority

Canons of beauty in mathematics and in the fine arts are institutional facts, human-made constructs based on complex metaphysical assumptions than change with time and context [24]. Institutional facts are shaped by a power structure that defines those resources that conform a culture and its attributes in a given moment [25]. This power structure establishes an ideology, namely a systems of normative precepts that establishes the canons of value, and informs both the practitioner of a craft and the beholder[26][27]. Successive ideologies impose paradigmatic definitions of beauty that become the golden rule of the times, establishing which forms of discourse are substantive, beautiful, trivial, or merely decorative. Each one of the successive power structures (in mathematics and in the fine arts) determines the values and the content of the prevailing culture, defining its classicism–the language of authority–and controlling its social perpetuation.

In the realm of the fine arts, influential critics, patrons, *marchands*, museum directors and curators, impose the genres and styles that are worthy of exhibition at museums and galleries. Then, these genres and styles receive acclaim by the media, and are enthroned as the mainstream imagery of a generation. Many artists have painted apples, mountains, and men playing cards, yet Cezanne's apples, Mount Saint Victoire, and men playing cards have became the canonical representation of these objects. The Analytic Cubism of Picasso, Braque, and Gris is the exclusive paradigm of Cubist representation, while the Scientific Cubism of Lohte and Metzinger is deemed trite and uninteresting. Selection implies preferences and omissions; preferences lead to the establishment of paradigms, and omissions result in the suppression of alternative art forms. The preferred art is consecrated at canonical temples of high culture, is covered by the media, and becomes the object of the art market. The art omitted becomes an item in obscure specialized art encyclopedias.

Is there an equivalent power structure in mathematics, ruling which theorems are interesting and worthwhile, and which are not? Do the intellectual leaders of the mathematical community impose to the mathematicians of their generation their styles and preoccupations, and suppress alternative avenues of research? Cultural trend-setters in academia define the fashionable topics and methodologies of the day and displace from the limelight other subjects and trends. In every intellectual and scientific discipline, each generation of trend-setters determines what is mainstream, and what is "out." Research grants, publications, prizes, appointments, editorial boards of professional journals, and graduate students follow.

Yet usefulness, and not beauty, is what matters in mathematics. Beauty *per se* is not the aim of the mathematician, who is primarily interested in proving theorems and finding solutions to mathematical problems [28]. Mathematical truth–within its very limited and precisely defined context–is not a matter of opinion. Mathematicians prove theorems, and sometimes the mechanics of the intellectual process of asserting the logical truth of a proposition strikes other mathematicians as being "elegant" and "beautiful." Yet mathematical beauty is an epiphenomenon unrelated to the intrinsic value of a piece of mathematics, defined by its capability to solve mathematical problems.

For tens of thousands of years, pictorial beauty was an epiphenomenon of visual representation– the very concept of "visual art" is a very recent acquisition in human cultural evolution. Visually perceived objects impress and can be understood without the need of language, and this makes iconic representation an extraordinarily effective medium of communication. This is why images and visually apprehended objects, imbued with strong formulaic and symbolic content, were developed as tools for ideological propaganda and psychological warfare thousands of years before the invention of written languages. The control of images and imagery is still today one of the most tightly regulated elements in politics [29].

Yet Rota warns that "the beauty of a mathematical theory is independent of the aesthetic qualities, or the lack of them, of the theory's rigorous expositions" [30]. Something analogous happens in art. For the great masters of the modernist revolution in the visual arts, beauty was also an epiphenomenon of their search for "realism" in representation. Courbet search for "realism" led him to bypass the canons of beauty of the 19th Century. Picasso and Braque tried to grasp visual "reality" through Cubism. For them, conventional beauty was contingent, something that was not deliberately looked upon. Picasso's masterpiece *Les demoiselles d'Avignon*, the most revolutionary picture of modernity, is a frontal assault against conventional taste– childishly rude, spiked with silly depravity, and a smatter of elemental sexual symbolism [31].

Conclusion

The fact that mathematicians can *see* the beauty of the visually unrepresentable, unseen and unseeable, suggests at least three different interpretations.

1. ***All mathematics is visual, in the sense that is elaborated by some module of the visual brain*** [32]**.** 1Between 80 to 90 per cent of the cortex of the brain and numerous subcortical neuronal structures are involved in processing visual stimuli. We are a very visual species and can absorb very fast–in the microsecond range–a vast amount of visual information. This extraordinarily complex and rapid visual equipment endows humans with the almost instantaneous capacity of apprehension by sight. Michael Atiyah suggests that "instantaneous visual action" provides a wealth of spatial information that geometry perfects. He considers that geometry, concerned with *space*, and algebra, concerned with *time*, provide us with two orthogonal perceptions of the world. For Atiyah, the comparative easiness with which we grasp visible structures could explain the tendency of mathematicians to "geometrize" algebraic problems [33].

Yet the sensorial inputs from the retina that recruit the brain visual system are not needed for the understanding and the invention/discovery of mathematics—as demonstrated by the fact that there are blind mathematicians. Therefore, to understand and create mathematics other cues must be involved in the activation of the visual processing system. In this restricted sense, the creation of a mathematical inner world could be considered a type of visual hallucination. It is a fact that the brain images of visually detected regular solids, and those produced through mathematics seem to coincide. The mental image of visual perception is consistent with that generated by the manipulation of abstract elements according to an arbitrary set of operational rules. On the other hand, the assumption that the operational logic of the brain must necessarily be equivalent to that of human mathematics is highly debatable. The brain is a “brute” fact, the product of hundreds of millions of years of chaotic evolution through adaptive and non-adaptive selection [34][35][36]. Mathematics is an institutional fact, a human-designed system operating with a human-designed arbitrary logic. Even if mathematics can be used to model some brain phenomena, this does not mean that we are reproducing the way in which the brain functions [37][38][39]. However, if the same brain structures are involved in processing visual stimuli and understanding and inventing/discovering mathematics, it would be plausible to assume that understanding mathematics could generate the same sensations that we feel when *seeing* something that we deem pleasurable or rewarding.

2. ***The visual system is not involved in the creation and understanding of mathematics, but the activation of whatever brain regions are involved in that process gives raise to the same sensation of pleasure that is generally associated with visual perception.*** Anecdotal evidence suggests that the activation of sensorial systems can elicit paradoxical sensations–e.g. visual perception is sometimes translated in melodic impressions, and vice versa. The association between surprise and pleasure in arts and mathematics does not in itself prove that art and mathematics have anything in common, beyond the fact that innovations elicit sensations of pleasure.

3. ***The use of visual metaphors to discuss mathematical beauty indicates the limits of language***. Human language is distinctly limited in its capacity to convey sensations and affective tones—feelings themselves being non-transferable. Therefore, to describe what they feel about mathematics, mathematicians must convey these sensations through metaphors. The vision-related words they use are just “pieces of meanings” transferred from one descriptive discourse to another, used primarily as a literary devise to increase the power of conviction of their argument [40].There is nothing wrong with the use of metaphors in science, if one remembers that *they are* metaphors [41].

References

[1] I. Stewart, *Nature Numbers,* (London: Weidenfeld & Nicolson, 1995).

[2] F. Dyson, Book review of Ian Stewart’s Nature Numbers, *The Mathematical Intelligencer* Vol. 19, pp. 65-67. 1997.

[3] *Ibid*.

[4] *Ibid*.

[5] W.J.T. Mitchell, *Picture Theory: Essay on Verbal and Visual Representation*, (Chicago:University of Chicago Press, 1994), p 116.

[6] R. Debray, *Vie et Mort de l’Image: Une Histoire du Regard en Occident* (Paris, Gallimard, 1992).

[7] *Ibid*. p. 117.

[8] A. Sokal and J. Bricmont, Intellectual Impostures, (London: Profile Books, 1998).

[9] G.-C. Rota and F. Palombi (Editors), Indiscrete Thoughts, (Boston: Birkhäuser, 1997), p.121.

[10] *Ibid*. pp. 126, 36-37.
[11] *Ibid*. p. 128.

[12] *Ibid*. p. 122.

[13] B. Berenson, *Essays in Appreciation*, (London: Chapman & Hall, 1958).

[14] Mitchell, p. 118.

[15] J. Richardson. *A Life of Picasso*, (New York: Random House, 1991).

[16] *Ibid*.

[17] *Ibid*. pp. 121-122

[18] J.-P. Kahane and P. G. Lemarié-Rieusset, *Séries de Fourier and ondelettes* (Paris: Cassini, 1998).

[19] Rota p. 123.

[20] J. Heskett, *Industrial Design*, (New York: Oxford University Press, 1980).

[21] Rota p. 122.

[22] *Ibid.* p. 123.

[23] *Ibid.* p. 128.

[24] J.R. Searle, *The Construction of Social Reality* (New York: Free Press, 1995).

[25] W. Sewell, *A Theory of Structure: Duality, Agency and Transformation*, American Journal of Sociology, Vol. 98, pp. 1-29, 1992.

[26] 1See Henri Zerner, *L'Art de la Renaissance en France: L'Invention du Classicisme*, (Paris: Flammarion, 1996).

[27] Charles Rosen and Henri Zerner, *Romanticism and Realism: The Mythology of Nineteenth-Century Art*, (New York: Viking Press, 1984).

[28] Rota p. 126.

[29] C. S. Goldstein, *The Political Control of Images: Iconoclasm and Indoctrination in American Occupied Germany, 1945-194*9, Ph. D. Dissertation, University of Chicago, 2002.

[30] Rota p. 24.

[31] Richardson.

[32] D. J. Goldstein, “Do we do mathematics with our visual brain?” *The Mathematical Intelligencer* , Vol. 20, pp. 5-10, 1998.

[33] M. Atiyah, *Mathematics in the 20th Century*, The American Mathematical Monthly, Vol. 108, pp. 654-666, 2001.
[34] S.J. Gould, *The exaptive excellence of spandrels as a term and prototype*, Proceedings of the National Academy of Sciences, U.S.A., Vol 94, pp. 10750-10755, 1997.

[35] S.J. Gould, Darwinian Fundamentalism, New York Review of Books, 12 June 1997.

[36] S.J. Gould and R.C. Lewontin, *The Spandrels of San Marco and the Panglossian Paradigm: A Critique of the Adaptationist Programme*, Proceedings of the Royal Society, Series B, Vol. 205, pp. 581-598, 1979.

[37] Paul C. Bressloff, Jack D. Cowan, Martin Golubitsky, Peter J. Thomas, and Matthew C. Wiener, *Geometric visual hallucinations, Euclidean symmetry and the functional architecture of striate cortex*, Phil. Trans. R. Soc. London. B Series, Vil. 356, pp. 299-330, 2001.

[38] M.C. Reed, *Why is Mathematical Biology so Hard?* Notices of the American Mathematical Society, Vol. 51, pp. 338-342, 2004.

[39] D. J. Goldstein, *1If mathematicians do not do it, who will?*,The Mathematical Intelligencer Vol 23, pp 11-12, 2001.

[40] S. Maasen, E. Mendelsohn, and P. Weingart, *Biology as Society, Society as Biology: Metaphors*, (Boston: Kluver Academic Publishers, 1995), pp. 1-2.

[41] R. Lewontin, *The Triple Helix: Gene, Organism, and Environment*, (Cambridge: Harvard University Press, 2000).

Correspondence: Daniel J Goldstein, 1615 Q street NW. Apt T-6, Washington DC 20009, USA. E-mail: cs@scs.howard.edu