Malekula Sand Tracings: A Case in Ethnomathematics

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Abstract

The Malekula sand tracing tradition is discussed as an exemplary case in ethnomathematics. The tradition evidences graph theoretic, geometric, and topological ideas. The sand tracings are placed within Malekula culture and the systematic procedures used to trace the figures are elaborated. Bridges to art and religion are noted.

Ethnomathematics

In this paper I discuss the sand tracing tradition of the Malekula of Vanuatu. But first I have several preliminary comments in order to put this into the contexts of ethnomathematics and of mathematics.

The basic tenet of ethnomathematics is that the expression of mathematical ideas is intimately related to culture; that ideas arise within cultural contexts and which ideas are emphasized and how they are expressed vary depending on the culture. My studies in ethnomathematics focus on the mathematical ideas of peoples in traditional or small-scale cultures.

Among mathematical ideas, I include those ideas involving number, logic, and spatial configuration, and, more significantly, their combination or organization into systems and structures. The need to clarify what is meant by mathematical ideas is an important issue raised by ethnomathematics. It is now generally recognized that what we refer to as *modern mathematics* is, in itself, the confluence of ideas from many cultures eventually merged through translation, media, and standardization of expression. But the term *mathematics* has no clear and agreed upon definition. More important, however, is that in most cultures, mathematics is not set apart as a separate, explicit category. Mathematical ideas, however, do exist with or without that explicit category and whether or not the ideas fed into or effected the mathematical main stream. As a result, mathematical ideas are found in contexts appropriate to the cultures in which they arise. These contexts could be, for example, what we might categorize as navigation, art, record keeping, religion, kinship, games, decoration, divination, construction, or calendrics.

By including the mathematical ideas of cultures previously ignored, we introduce considerable diversity and geographic breadth. The number of different cultures, using the criterion of mutually exclusive speech communities, that is, having different languages, has remained at about 5000 to 6000 during the past 600 years. (It is the number of people in the culture and the area they dominate that has changed considerably.) Although today there is an overlay of a few dominant cultures, traditional cultures still exist, even if in some cases blended with or blurred within the dominant culture as subcultures, part cultures, or composite cultures. The special contribution of ethnomathematics is elaborating the mathematical ideas of those traditional cultures while recognizing that the ideas are an integral part of the intricate web of language, beliefs, and life-ways that make up the culture. It is this focus that should make ethnomathematics of particular importance and relevance to people concerned with bridges that link mathematics to other cultural expressions. In ethnomathematics, we emphasize that mathematical ideas are embedded in cultural contexts and that, as we discuss the mathematical ideas, we *must* retain these bridges in order to properly and fully see the ideas for what they are.

Another important note is that until quite recently, over 90% of traditional cultures had no writing as we generally use the term. To learn about the mathematical ideas of cultures that had no writing systems and whose traditions are no longer extant, we must depend on information that can be extracted from artifacts or from the reports of observations left by others. Even where the ideas are recent or current, they may be part of an oral tradition and so must often be gleaned from observations and from the interpretation of material things. Thus, the study of ethnomathematics often interacts with or draws upon fields such as archeology, ethnology, linguistics, and culture history. This feature of ethnomathematics involves another set of bridges--bridges to disciplines and perspectives that are not the usual sources for mathematicians or mathematical investigations.

During the past 80 years, there have been vast changes in knowledge, understanding and theories about culture, language, and cognitive processes. We have come to understand that there is no single, universal path which all cultures or mathematical ideas must follow. When we learn about the varied and often quite substantial mathematical ideas of traditional cultures, we are not learning about some early phase in humankind's past. We are learning about pieces of a global mosaic. By incorporating expressions of different peoples, at different times, and in different places, we are enlarging our understanding of the variety of human expressions and human usages associated with the same basic ideas. (Recognizing that there is a plurality of paths, does not, of course, preclude that there was interaction, sharing or borrowing, but that would have to be specifically shown.)

Now, as we turn to the Malekula sand tracing tradition, we move beyond generalities and give more substance to many of the comments above about ethnomathematics. Within ethnomathematics, I am most interested in those cases for which analysis of structure can be combined with evidence that the people themselves were concerned with the structure. The Malekula sand tracings are one such case.

The Malekula and Graph Theoretic Ideas

The Malekula live in the South Pacific in the Republic of Vanuatu, which was formerly known as the New Hebrides. A particular idea evidenced by their sand tracing tradition falls within what Western mathematicians call *Graph Theory* and associated with it are other topological and geometric ideas. So, first let us collect some of <u>our</u> ideas on graph theory. Described geometrically, graph theory is concerned with arrays of points (we call them *vertices*) interconnected by lines (which we call *edges*).

A classical question in graph theory is "For a graph, can a continuous path be found that covers every edge once and only once? And, if such a path exists, can the path end at the point it started?" This is the question that is said to have inspired the founding of graph theory by the mathematician Euler. According to the story, there were seven bridges in Königsberg where Euler lived. The townspeople were interested in knowing if, on their Sunday walks, they could start from home, cross each bridge once and only once and end at home. Between Euler in the 1730s and Hierholzer about 130 years later, a complete answer was found. Before stating the result, I have to introduce the *degree* of a vertex. The degree of a vertex is the number of edges that emanate from it. A vertex is odd if its degree is odd and even if its degree is even. First of all, not all graphs *can* be traced continuously covering every edge once and only once. If such a path can be found, we call it, in honor of Euler, an Eulerian path. Such a path exists if the graph has one pair of odd vertices, provided you start at one of them and end at the other. And, if all the vertices are even, such a path can be traced starting anywhere and ending where you began. The cases in which there cannot be such paths are when the graphs have more than one pair of odd vertices.

Some examples of graphs are in Figure 1. In example (d), every vertex is of degree four--you can start anywhere and end where you started. In (b) (which some of you might recognize as a children's game if you grew up on the streets of New York City or London or Berlin), there are three vertices of degree four and two vertices of degree three. An Eulerian path can be found provided you start at one of the odd vertices and end at the other. Example (a) has one vertex of degree four and *four* vertices of degree three and so it cannot be done. Finally, example (c), a 19^{th} -century Danish party puzzle, has eight vertices of degree three so it cannot be done. [(a) would require two lines or backtracking and (c) would require four lines or backtracking.] The philosopher Wittgenstein in his *Remarks on the Foundations of Mathematics* used this example of tracing a figure--with a figure very similar to (c)--as one that captures the essence of mathematics writing that "it is recognizable at once as a mathematical problem"[3].

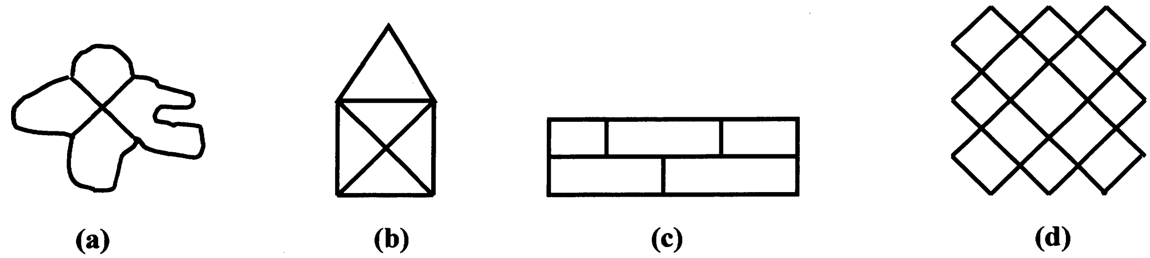

Figure 1: *Examples of graphs.*

Now back to the Malekula-----

In the 1920s, Bernard Deacon, a graduate student in ethnology at Cambridge University, studied among the Malekula. Deacon observed a tradition of tracing figures in the sand. Believing there was something quite important about it, he was meticulous in copying 95 figures and numbering the order in which *every* line on *every* figure was traced. While waiting for the boat home upon completion of his fieldwork, Deacon died of Blackwater fever. His doctoral mentor and a fellow graduate student published much of his work including his very detailed field drawings [2]. I think, perhaps, that his bad fortune led to the publication of this information (essentially raw data) that he might only have published in summarized form had he lived. Deacon, however, does seem to have had unusual insight combined with respect for the capabilities of the Malekula.

According to the Malekula, when a man dies, in order to get to the Land of the Dead, his ghost must pass a spider-like ogre who challenges him to trace a figure in the sand. The stipulation is that he must trace the entire figure without lifting his finger, without backtracking, and, if possible ending where he started. (These stipulations should be familiar--the Malekula are specifying what we discussed above as Eulerian paths.) If he does not meet the challenge, he cannot proceed to the Land of the Dead. They also have a myth about the origin of Death that involves figure tracing. The myth centers around two brothers Barkulkul and Marekul who have come to earth from the sky world. When Barkulkul leaves his wife to go on a trip, he places a vine in a certain configuration on the closed door of their house. When Barkulkul returns he sees that the vine has been disturbed. He goes to the men's house and challenges all the men gathered there to trace a figure in the ashes on the floor. Because Marekul cannot trace the figure *properly* (that is, with the stipulations previously stated), Barkulkul knows it was his brother who visited while he was away. The story goes on, but the important point here is that knowing the figures and tracing them *properly* is a serious matter. It is not a game and not just the concern of a few people. It is, however, restricted to men.

Tracing the Figures

The 95 figures range from simple closed curves to having more than 100 vertices and many having vertices of degree 10 or 12. Not only do we have the Malekula statement of an interest in tracing these figures continuously, covering every edge once and only once, and if possible, ending at the starting point, but we have the exact tracing paths that upon analysis bear it out. The Malekula refer to the figures as *nitus*. Figure 2 shows a few of them. In actuality the nitus measure about a square meter or more. We will now discuss some of these figures in greater detail.

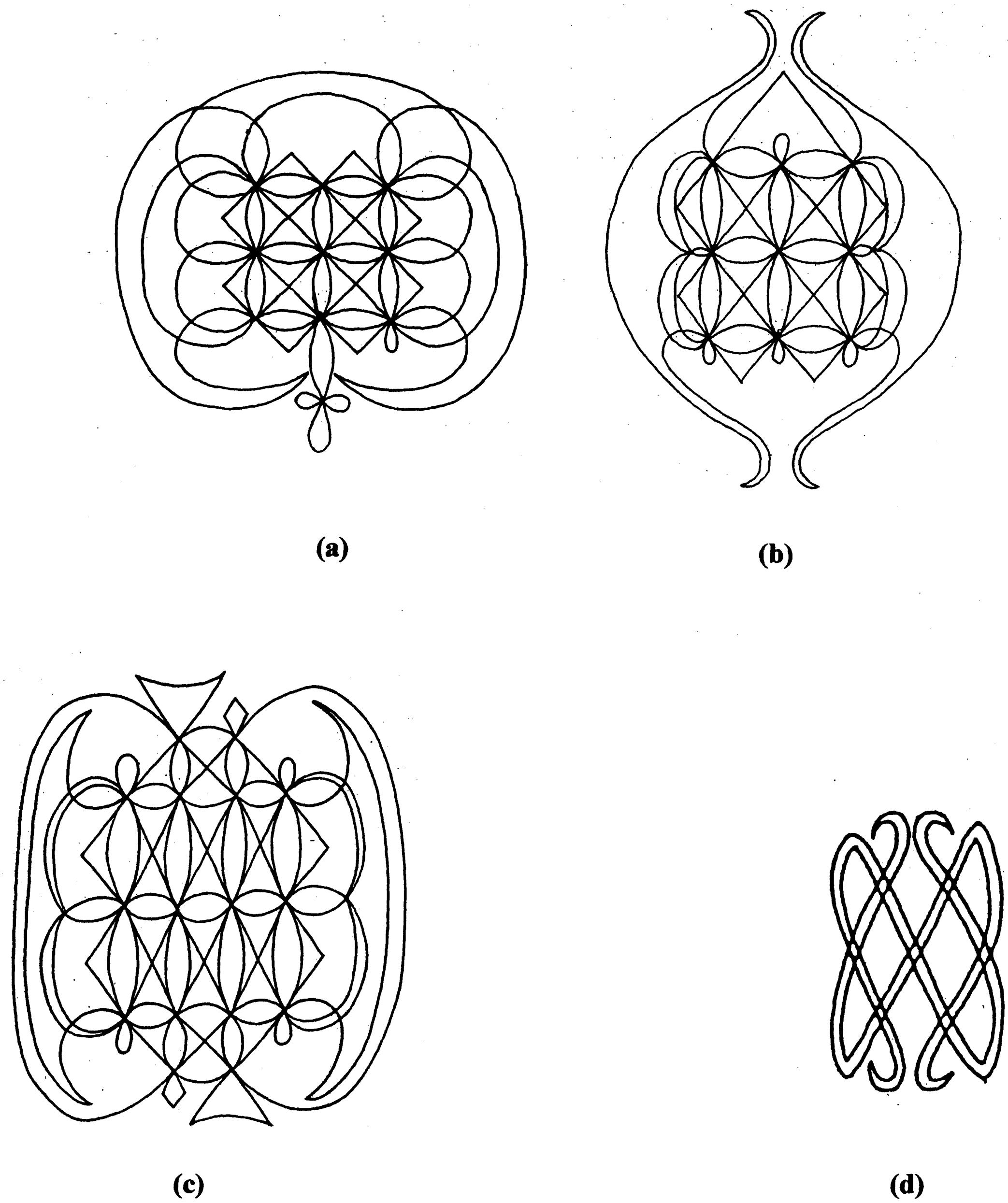

Figure 2: *Malekula sand tracings (nitus).*

In examining the tracing paths, I found more than just this graph theoretic concern. When an Eulierian path is possible, there can be many different ways to trace it. The Malekula tracing processes are very systematic within each figure but, even more important, the systems extend to groups of figures. There are three or four of these extended systems, one of which we'll look at in detail. First, for a large group of figures the system is what I call a *process algebra.* Namely, in the tracing of each figure there is some initial procedure (that is, an ordered set of motions) followed by a formal transformation of that procedure. And, for the group of figures, only a particular set of transformations is used. I'll expand on this using as an example a small, made-up initial procedure. Let us say that the initial procedure is the ordered set of motions A shown in Figure 3. It can be followed by itself--AA . It can be followed by each motion rotated through 90° --AA_{90}. There can be a reflection of each motion across a vertical axis (right and left interchange, up and down remain the same) --AA_V. The order for performing the motions can be inverted-- AA'. Notice that the transformed procedures do not yield the visual effects one usually associates with these words because, since the tracing is continuous, each procedure picks up where the last leaves off. In all, the set of transformations used by the Malekula are I (identity), rotate 90°, rotate 180°, rotate 270 °, reflect vertical, reflect horizontal, each with or without inversion.

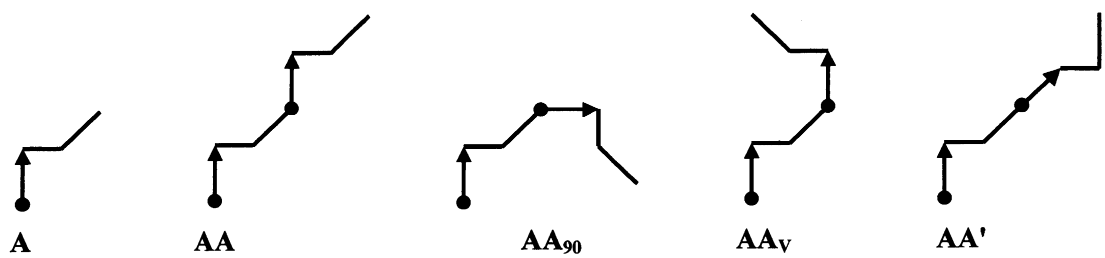

Figure 3: *Initial procedure A followed by some of its transformations.*

Now, let us look at some of the nitus to see how the Malekula traced them. Figure 4 shows a nitus and its initial procedure A. In terms of A, the complete tracing path can be described as $AA_{90}A_{180}A_{270}$. The visual fourfold symmetry results from successive rotations of the procedure. (The basic unit is not what I would have visualized from the end result.)

Figure 5 shows a tracing in three stages. Each stage is a procedure followed by its rotation through 180°. The visual effect is horizontal and vertical symmetry but it was created by rotation of the procedures. Usually discussions of symmetry rely only on after completion static effects. Here we have both that and the dynamic symmetry of construction.

Another nitus traced in three stages is one previously shown in Figure 2c. Here, too, each stage is a procedure followed by its transformation by a 180° rotation. In this case the procedures, the visual effect, and the nitus description all reiterate the 180° rotation. The Malekula description of the nitus is two of the same kind of fishes placed head to tail.

Next, Figure 6 shows a nitus traced in four stages--each stage is a procedure followed by its inversion.

In the last illustration, Figure 7, the complete tracing can be described as $AA_{180}A_VA_H$ which brings us to the need for a bit of algebra and which raises an important point about this use of modern symbolism.

In each description, such as $AA_{90}A_{180}A_{270}$ for the nitus in Figure 4, I described each subsequent procedure with reference to the initial procedure. These could be described differently. For example, if each procedure is referred to the one just before, it would be $AA_{90}(A_{90})_{90}(A_{180})_{90}$. Or, referring the last pair of procedures to the first pair, the result is $(AA_{90})\ (AA_{90})_{180}$. All of theses versions show successive rotations but with different emphases. For the last nitus (Figure 7), however, the different versions involve different transformations. Referring each procedure to the initial procedure, we had $AA_{180}A_VA_H$. Instead we'll refer each procedure to the one before it [$AA_{180}\ (A_{180})_?\ (A_V)_{??}$] or we'll refer the last pair to the first pair if the pairing is possible [$(AA_{180})\ (AA_{180})_?$]. To solve this we need a product table for the transformations; that is, a table showing the result of one transformation followed by another. (See Table 1). Notice that the X and Y in the table are not in the set of transformations used by the Malekula and so the pairs that lead to them could not have occurred. Specifically, there could be no 90(V) or H (270). The transformations X and Y are, in fact, reflections across the diagonals and these were not present. Using the table, we have three different, but equivalent, symbolic representations:

$$AA_{180}A_VA_H = AA_{180}(A_{180})_H(A_V)_{180} = AA_{180}(AA_{180})_V.$$

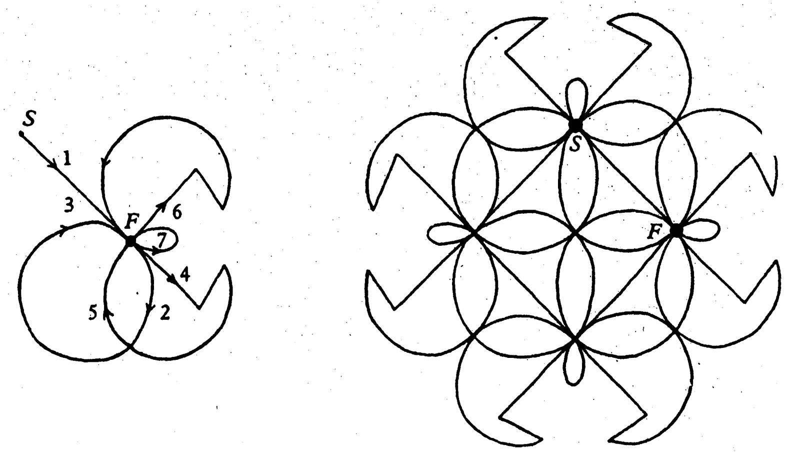

Figure 4: *The initial procedure A and the final figure* $AA_{90}A_{180}A_{270}$.

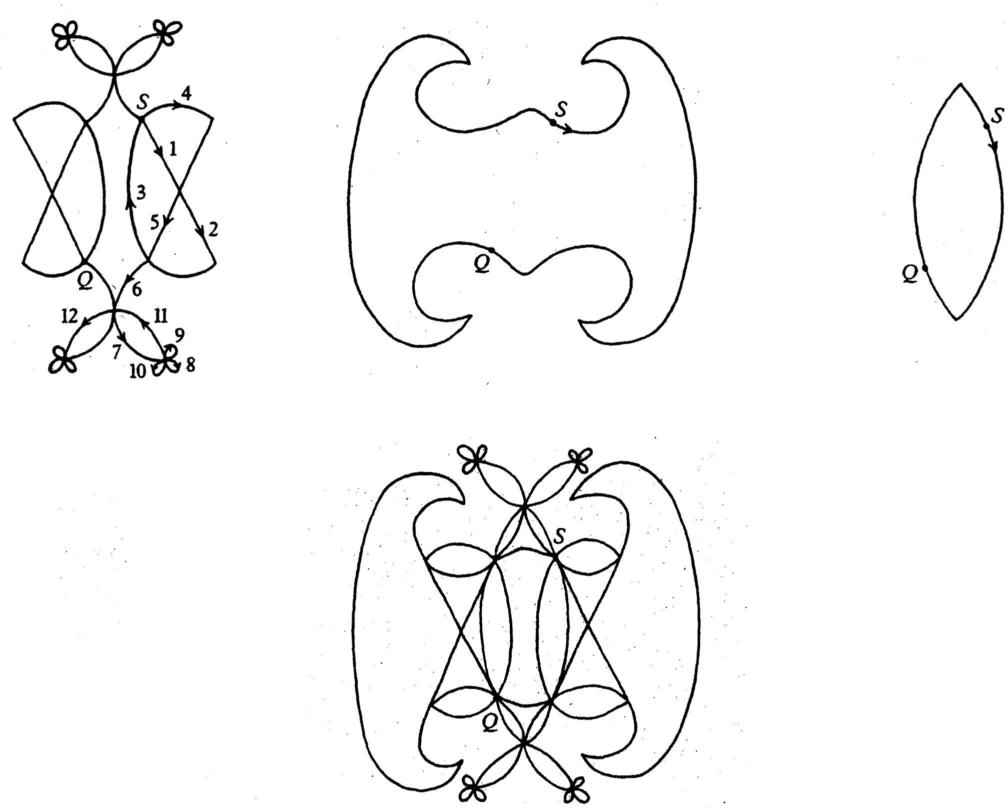

Figure 5: *A nitus traced in three stages. In each stage, the initial procedure (A, B, and C) starts at S, ends at Q, and is followed by the procedure rotated 180°. The final figure is* $AA_{180}BB_{180}CC_{180}$.

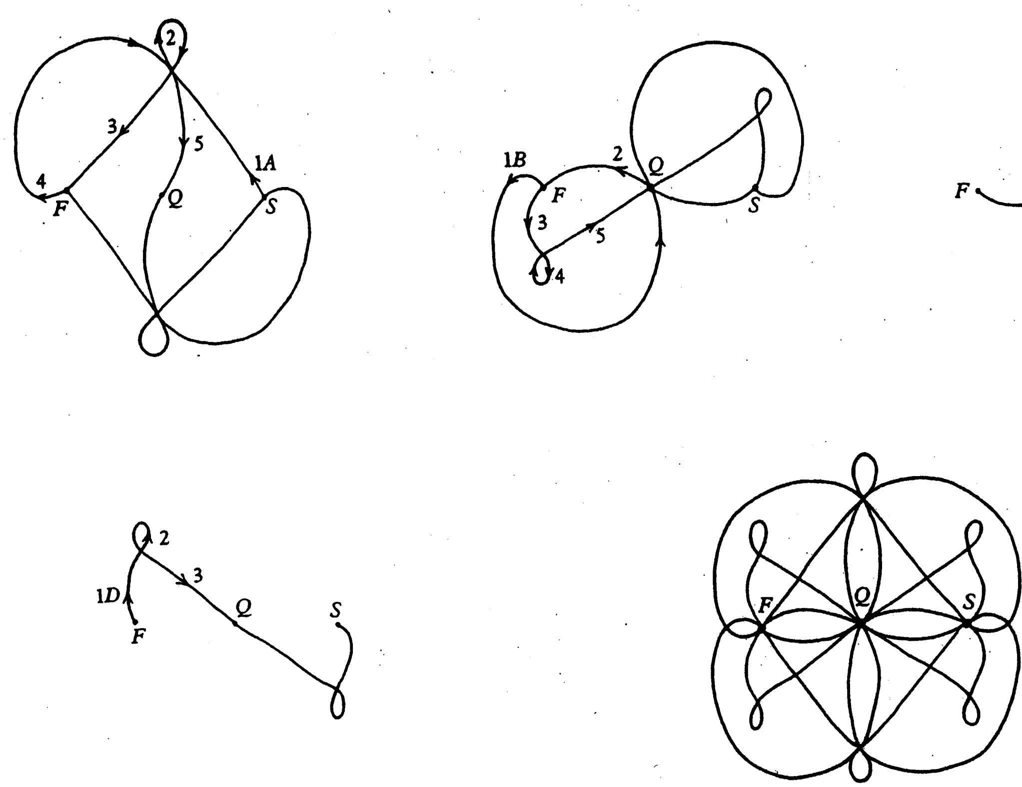

Figure 6: *A nitus traced in four stages. In the first and third stages, the initial procedures (A and C) start at S, end at Q, and are followed by the procedure transformed by inversion. For the second and fourth stages, the initial procedures (B and D) start instead at F. The final figure is AA'BB'CC'DD'.*

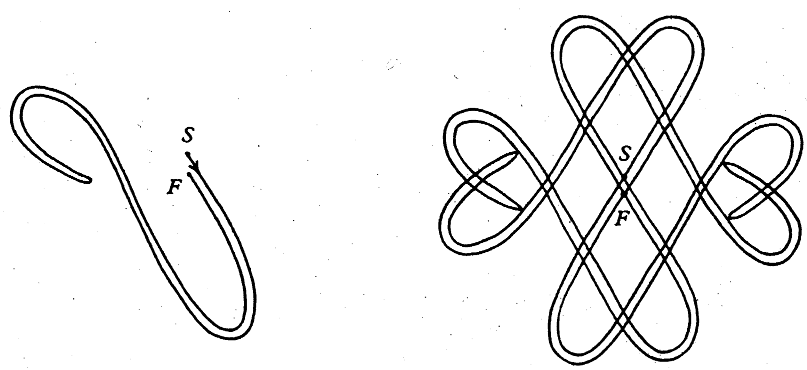

Figure 7: *The initial procedure A and the final figure* $AA_{180}A_VA_H$.

Each of these describes the structure of *what was actually done* but each version implies a slightly different conceptualization by the Malekula. That is, even though we know exactly what they did, we do not know exactly what they thought while they did it. But, being able to generate different versions can give us different insights. To me, the last version seems more in keeping with the overall procedures for the other figures, but that is just conjecture on my part.

	I	90	180	270	V	H
I	I	90	180	270	V	H
90	90	180	270	I	**Y**	**X**
180	180	270	I	90	H	V
270	270	I	90	180	**X**	**Y**
V	V	**X**	H	**Y**	I	180
H	H	**Y**	V	**X**	180	I

Table 1: *Transformation product table.*

These are just a few of the 95 tracings but in them you can see that in this sand drawing tradition there are:

1. A graph theoretic goal, which is carried out, of tracing figures continuously covering each edge once and only once and, where possible, beginning and ending at the same point.
2. There is the creation of visual symmetry in most of the figures.
3. Within these self-imposed constraints, the individual figures are traced quite systematically.

And 4. For different groups of figures, these systematic procedures are particular expressions of larger systems. For the system we have looked at, selections from a specific set of transformations are applied to different basic procedures.

Taken by themselves, simply as figures, the tracings are quite intricate and quite attractive. In fact, they have been compared to well-known works of Western art. In 1944, the eminent art historian, Ananda Coomaraswamy, who was curator of the Boston Museum of Fine Arts for some 30 years, wrote an article about Dürer's engraving entitled "Knots" and da Vinci's engraving "Concatenation". In his view, these are part of a worldwide tradition of single-line drawings. "But...", he says and shows one of the Malekula figures, "But it is, perhaps, in the New Hebrides that the one-line technique attains its fullest development"[1]. From their place in Malekula culture, the figures represent another, perhaps more important bridge, that is, they are clearly religious expressions, particularly related to Malekula myths about death. But above all, for those of us interested in mathematical ideas, as we consider the tracing goals, follow the tracing procedures, and view the tracing outcomes, they are a bridge to some understanding of an intellectual endeavor of the Malekula. In 2003, UNESCO included the Malekula sand tracing tradition on the World Heritage list proclaiming it 'A Masterpiece of the Oral and Intangible Heritage of Humanity'.

References

[1] A. Coomaraswamy, *The Icononograpy of Dürer's 'Knots' and Leonardo's 'Concatenation,* The Art Quarterly, Vol. 7, pp.109-128. 1944.

[2] M. Ascher, *Graphs in Cultures: A Study in Ethnomathematics*, Historia Mathematica. Vol. 15, pp.201-227. 1988. See this article for a more extensive discussion of these tracings, more illustrations of them, and numerous references.

[3] L. Wittgenstein, Remarks on the Foundations of Mathematics, G.H. von Wright, R. Rhees, and G.E.M. Anscombe, eds., Blackwell, Oxford, p. 174e. 1956.