Looking at Math: Using Art to Teach Mathematics

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0. Introduction

Many works of art can be seen from a mathematical viewpoint, and they can be useful in illustrating mathematical concepts and sparking interest in students. A prominent example of this is the discovery of perspective by the Renaissance painters. One can use art occasionally as a pedagogical tool to introduce mathematical concepts. The process unfolds somewhat as follows: the art works inspire and heighten a student's interest immediately; the teacher uses the art to help the student draw out her/his own range of questions; and then, the teaching of mathematics begins.

To explain mathematics is, in a way, to give answers. I believe that only after a student has developed the questions herself is she able to seek and deeply understand an answer. Otherwise, the answers appear as only a series of ideas lacking sense. Understanding comes from the intense work on appropriate problems. Much as Ortega y Gasset suggested in metaphysics, we decide that we have found truth when we discover a thought that satisfies an intellectual necessity that we have previously felt ourselves.

Part one of this paper shows a few examples of art works that I have incorporated into the teaching of different traditional undergraduate mathematics courses. Part two includes an overview of a pilot experimental course first offered in Spring 2000 at Smith entitled Mathematical Sculptures.

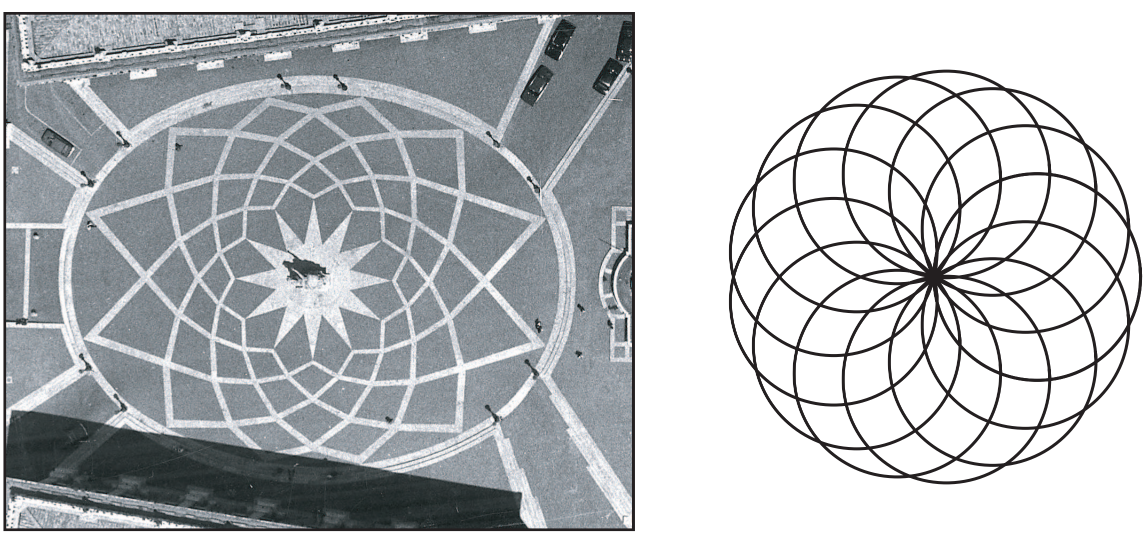

Figure 1: *Aerial view of Michaelangelo's Piazza del Campidoglio (detail of photograph © J.H.Aronson 1979), and a circular rosette of 13 circles.*

1. Part One

1) In a geometry course, I have used Michelangelo's beautiful geometric design for the pavement of the Piazza del Campidoglio in Rome to start the course and introduce circular rosettes (Fig. 1). After an overview of Michelangelo's work, the class instruction proceeded onto classical geometry by pointing out to the students that the small circle segments in the rosette that make the sides of the rhombic–shaped regions all have the same length (except for the exterior ones at the edge of the diagram, which are exactly twice as long as the interior ones). Students immediately began trying to figure out a proof of this fact. The mathematical problem was clear and the art work gave it an unusual context.

2) A design by Giordano Bruno (1548-1600) (Fig. 2) has inspired first year students to try to come up with the exact location of the different geometric components in their attempts to reproduce it on a computer. One could also ask the students to reproduce it with pencil, compass and ruler.

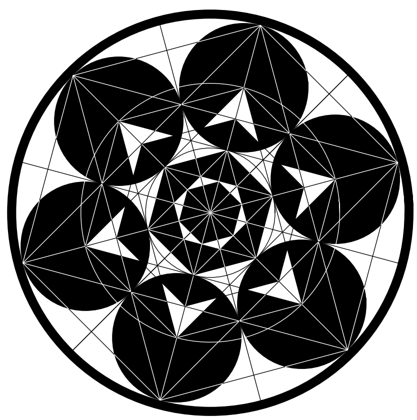

Figure 2: *A computer–generated diagram after a design by Giordano Bruno.*

3) Ken Hiratsuka's sculptures involve one-line carvings on the surface of large stones (see Figs. 3 and 4). His beautiful pieces, when seen from a mathematical point of view, can be used to give a hint of Peano curves, index theorems, the Jordan curve theorem, and the "flavor" of typical mathematical arguments, such as counting the number of intersections that a straight line segment has with the carved curve to determine whether the two endpoints are on the same side of the curve or not.

2. Part Two

In Spring 2000, I ran a new experimental course (for majors and non-majors) at Smith College with 14 students called Mathematical Sculptures. The stated goal of the course was to construct a large wire sculpture representing an important mathematical surface. The course resulted in the construction of a nearly seven–foot tall steel–wire sculpture illustrating the Boy surface.

The Boy surface (Fig. 5) is an immersion of the projective plane in euclidean 3-space (no singularities, only self-intersections). That such an immersion is at all possible was discovered around 1901 by Werner Boy, then a student of David Hilbert [1].

The course had a mathematical component including the theory of topological surfaces, a computer 3D visualization component where the students were introduced to specialized 3D visualization software (Geomview), and a construction component where they learned hands–on in the machine shop about different options for materials, the use of various tools and welding techniques.

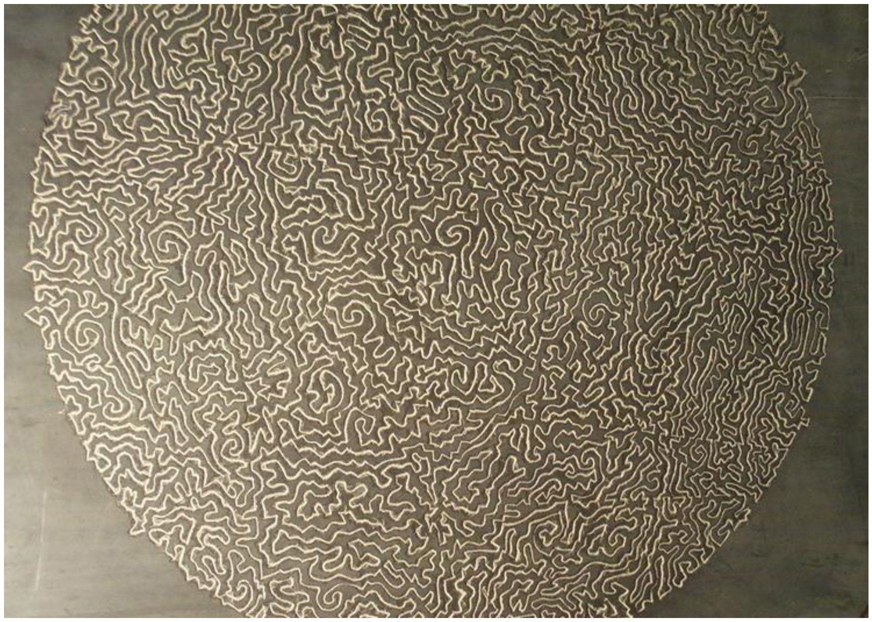

Figure 3: *A carved stone by Ken Hiratsuka.* (Photo courtesy of the artist.)

We decided as a group that we would construct the surface out of 1/8–inch thick steel wire. Greg Young, the Smith College machine shop director, taught us about both spot and torch welding techniques. Using the Bryant–Kusner ([2,3]) parameterization of the surface (see Fig. 5) we first made several numerical calculations on the computer and made marks on the wires where the welding would take place. During the construction there was a constant need to make new calculations and to compare results with what we were creating. I witnessed how students were constantly surprised and highly stimulated by the dynamic interaction among mathematics, computer calculations and the construction of the sculpture. The resulting object is currently hanging in Smith College's McConnell Hall (see Fig. 6).

Two artists whose works have complemented the mathematical sculptures course in its current offering include Tobias Putrih and Vince Roark.

Putrih constructs large-scale sculptures using simple materials. In one particular piece the sculpture illustrates what in mathematics is called a triangulation of the sphere. The beautiful

Figure 4: *Detail of one of Ken Hiratsuka's stones.*

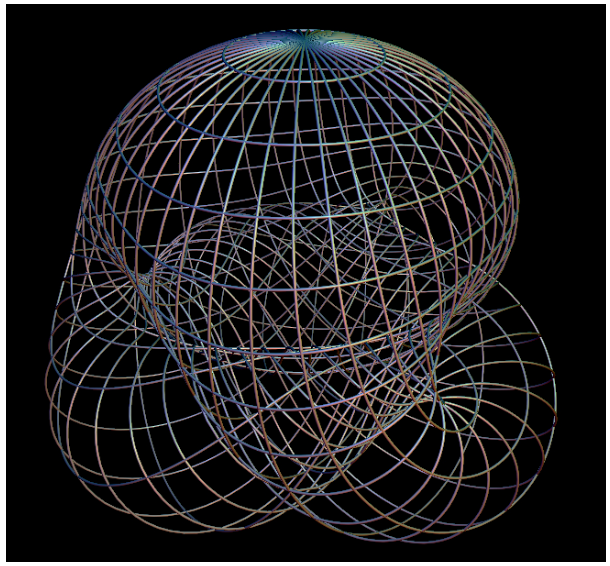

Figure 5: *The Boy surface using the Bryant–Kusner parametrization.*

sculpture is made out of thin wooden dowels held together by plastic tubular joints (Fig. 7). The sticks are the edges of the triangulation, while the plastic joints are the vertices. In attempting to recreate Putrih's sculptures, students become fascinated by the restrictions they feel in trying to construct a sphere. Their struggle serves as the right moment for me to present Euler's polyhedral formula $V - E + F = 2$. One can then go on to explore consequences of the formula, the differences between five-edge, six-edge, seven-edge vertices and so on. Some of Putrih's intricate pencil drawings comprised of only triangles further enhances this (Fig. 8). In the future, we will integrate the Smith College Art Museum collection and resources into the course and lectures by the artists.

Roark's intriguing work consists of two– and three–dimensional renditions of high–dimensional objects rendered on paper with pencil and ink, and in 3D using wooden dowels and other materials. I use his work to stimulate the study of high dimensions. Students are absorbed instantly by the complexity and allure of his drawings (Fig. 9). Almost all of the students wanted to participate in the suggested project: to build a series of 3D projections of a hypercube, representing its rotation in high–dimensional space.

This paper shows ways I have taught some undergraduates using the engaging works of Renaissance and contemporary artists by developing computer simulations, complex computations and hands-on model construction projects for the classroom. Whether or not these artists specifically or intentionally intended to portray mathematical concepts, looking at their work from a mathematical perspective allows me to spark and initiate students interest in mathematical questions. The end result for a student is a more experiential way of looking at and learning mathematics.

References

[1] W. Boy, *Über die Curvatura Integra und die Topologie Geschlossener Flächen*, Dissertation, Göttingen, 1901. Math. Ann 57, pp. 151–184. 1903.
[2] R. Bryant *A Duality Theorem for Willmore Surfaces*, J. Diff. Geom. 20, pp. 23–53. 1984.
[3] R. Kusner,*Conformal Geometry and Complete Minimal Surfaces*, Bull. Amer. Math. Soc. 17, pp. 291–295. 1987.

Figure 6: *Three images of the Boy Surface. A 7–feet tall steel wire sculpture.* (Photos courtesy of Stan Sherer.)

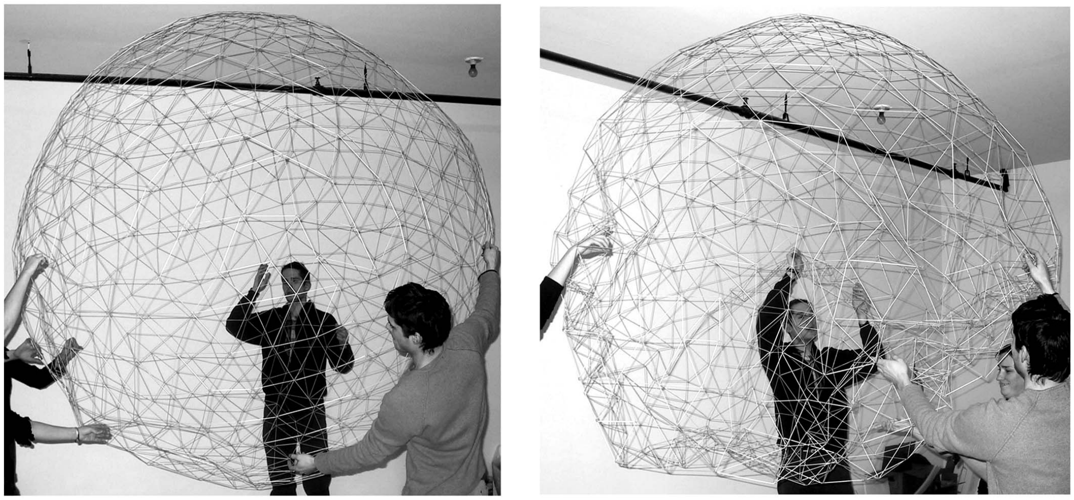

Figure 7: *Tobias Putrih (r) and his assistants installing "Quasi–Random," a study on Buckminster Fuller's Cloud Nine project.* (Photos courtesy of Max Protetch Gallery.)

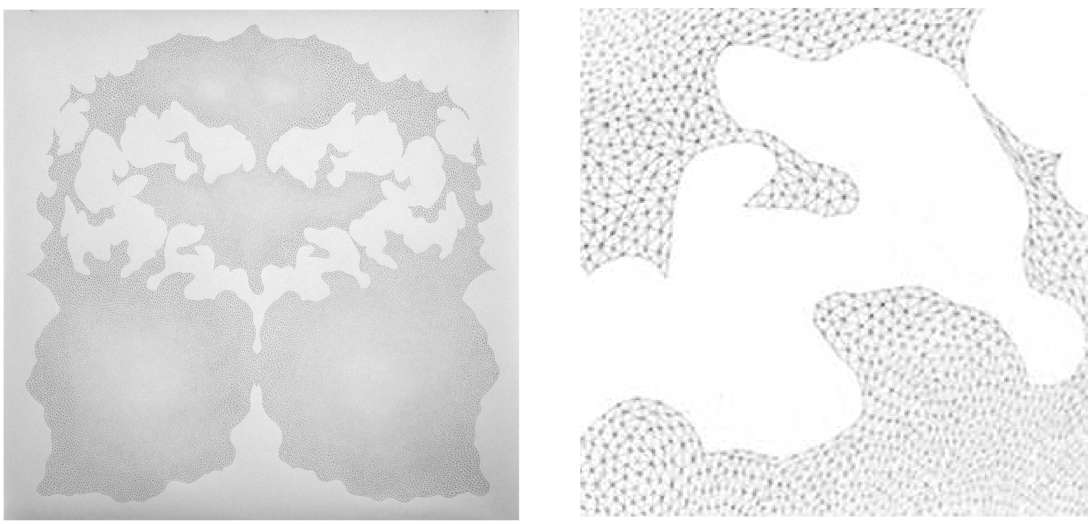

Figure 8: *"QRb-s." A pencil drawing and detail by Tobias Putrih (20x20 in). The triangulated region is homotopic to an annulus, so* $V - E + F = 0$. (Photo courtesy of Max Protetch Gallery.)

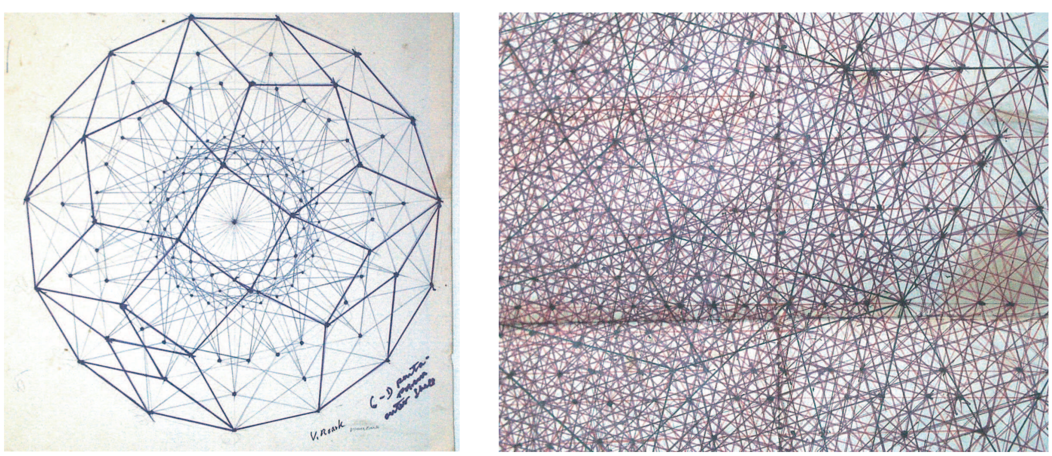

Figure 9: *Two drawings by Vince Roark.* (Photos by the author.)