Some Surprising New Properties of the Spidrons

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Abstract

I shall present the complex surface that I have named the spidron on account of its S-shape and spider-web appearance, and which, in its various transformations, deformations and combinations, has yielded extremely interesting spatial forms and tessellations. The so-called spidron nests, composed of 4, 6, 8 (generally an even number) of spidron arms, form reliefs that can be deformed in an accordion-like fashion. Such nests cut several of the regular and semiregular solids into two mirror-symmetrical halves. With the assistance of professor Emil Molnár, we are attempting to determine and classify all the space-filling shapes using the "D-symbol" named after the three mathematicians Dress, Delaney and Delone. The first examples of that endeavour are shown using the cube cut in half by a spidron nest. We have discovered several sub-species of the spidron family, constructed from a sequence of a single type (45°, 90°, 45°) or alternating sequences of two different types of isosceles triangles.

1. Basic Elements

1.1 Semispidron. A semispidron is a surface consisting of an alternating sequence of equilateral and isosceles (30°, 120°, 30°) triangles. The figure is constructed by taking an isosceles triangle and raising an equilateral triangle on one of its legs. A smaller, similar isosceles triangle is raised on another edge of the equilateral triangle. Then a smaller equilateral triangle is raised on a leg of the second isosceles triangle, and the process is repeated ad infinitum, as shown in *figure 1,* creating a spiral shape. Of course, the sequence can also be continued in the opposite direction, with ever larger triangles. (The plane version is used for presentation.)

1.2 Spidron. The spidron is obtained by centrally projecting a semispidron through the mid-point of the base of the largest isosceles triangle.

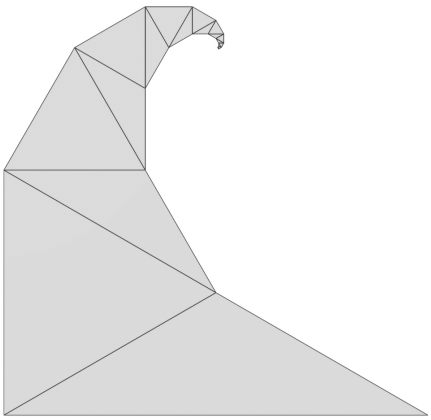

Figure 1: *Semispidron*

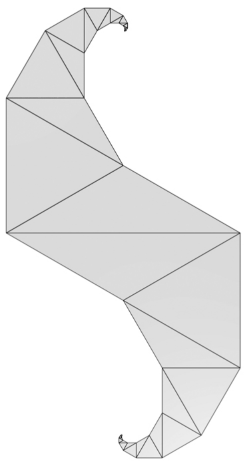

Figure 2: *Spidron and its component parts*

The spatial version of the semispidron is an open surface whose sides are all at convex angles on one side and at concave angles on the other side. Spidron arms can have either direction of spin, they can be clockwise or counterclockwise.

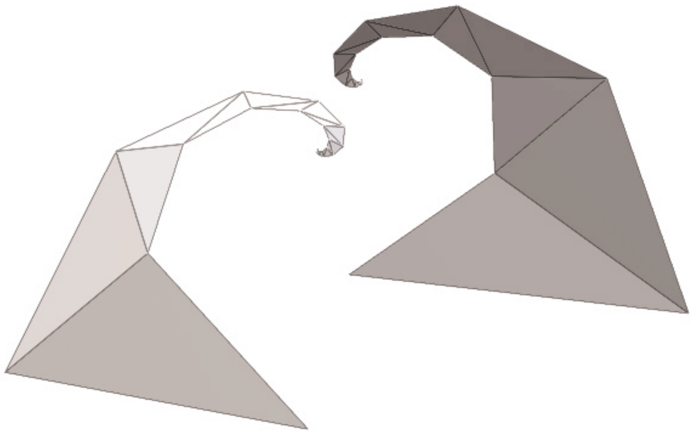

Figure 3: *Two views of a semispidron*

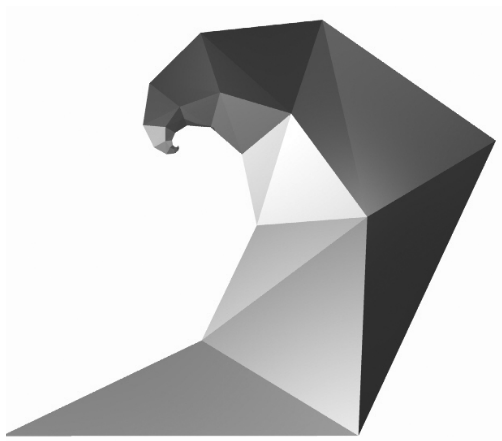

Figure 4: *Two joined semispidrons of opposite spin*

1.3 Why this shape? It seems to me that this composition, or to use the term that is more customary in mathematics, this "triangle complex", or the half of it I call a semispidron is the foundation for all the other shapes with their interesting spatial and almost "spatially flexing" properties which – according to the comments and the feedback I get – promise novel developments in the fields of geometry, physics and other branches of science.

1.4 The spidron belt (or spidron ring). A spidron belt is an open surface bounded by two spatial hexagons, consisting of six regular and six isosceles triangles. The surface may be deformed. Spidron belts can be nested. For our purposes, the highly symmetrical cases in which the mid-points of the edges of the spatial hexagons form regular planar hexagons (there are always two such positions) are of particular importance: in those cases, the angles between the faces of adjacent spidron belts can always be calculated from the angles between the faces of the individual spidron belts themselves, and the spatial figure maintains a threefold symmetry.

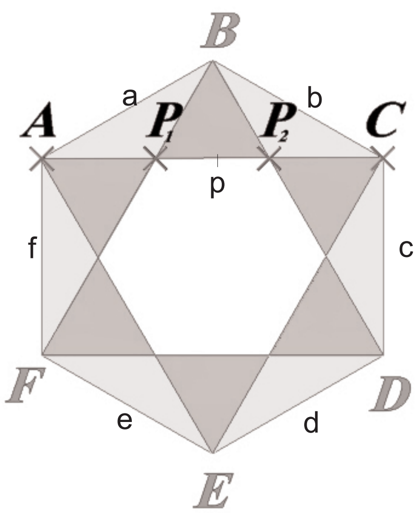

Figure 5: *Spidron belt*

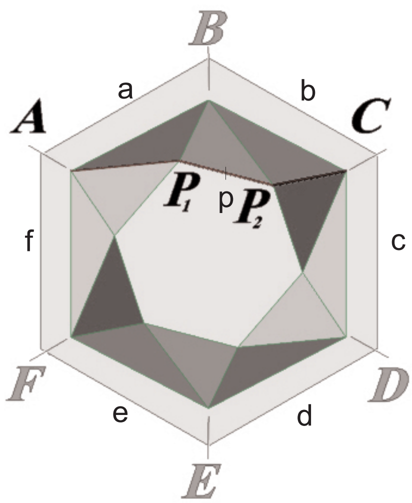

Figure 6: *The possible position of the vertices and "p"*

As the structure is deformed with that constraint, the edges that are perpendicular to sections connecting their midpoints to the centre of the figure (a, b, c, d, e and f) perform the following three kinds of movement:

1. The mid-points of those edges (p) rotate around the centre of the figure.

2. The edges a, b, c, d, e and f rotate relative to the base plane through the centre of the figure, while their mid-points remain on the base plane.

3. During the deformation, the edges a, b, c, d, e and f move closer to the centre of the figure.

It is an interesting property of spidron belts that their "direction of rotation" can be switched arbitrarily. Naturally, this is also true for sequences of spidron belts, and it was on account of this property that it was raised that a spidron nest is capable of representing any natural number in binary notation. According to some convention, the spidron belts, starting with the outermost one, are assigned the value 0 or 1 depending on whether it rotates clockwise or counterclockwise.

1.5 Spidron nest. A spidron nest is a surface made of nested regular hexagons [1] and six-pointed stars that can be deformed into a spatial formation consisting of 6 semispidrons, but we can also view it as a nested sequence of spidron rings or spidron belts.

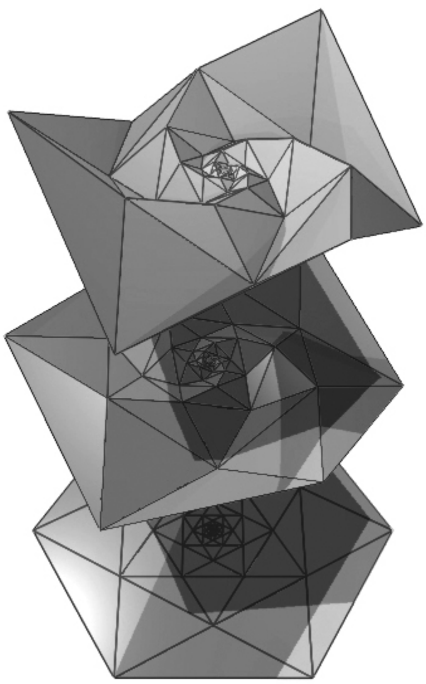

Figure 7: *Spidron nests*

1.6 Spidron subnest. A subnest is a formation consisting of several adjacent spidron belts. Individual subnests are only similar if they are all in a single plane. Any particular subnest can always be circumscribed by additional, larger and larger spidron belts. Otherwise, towards the centre of the nest we find an infinite sequence of smaller and smaller and also flatter and flatter spidron belts. Interestingly, the angle between the faces of the outermost spidron belt has an upper bound, but as the spatial deformation of a particular spidron belt can be arbitrarily small, the number of additional outward belts that can be added is the function of that deformation. The greater that deformation (plasticity), the smaller the entire nest.

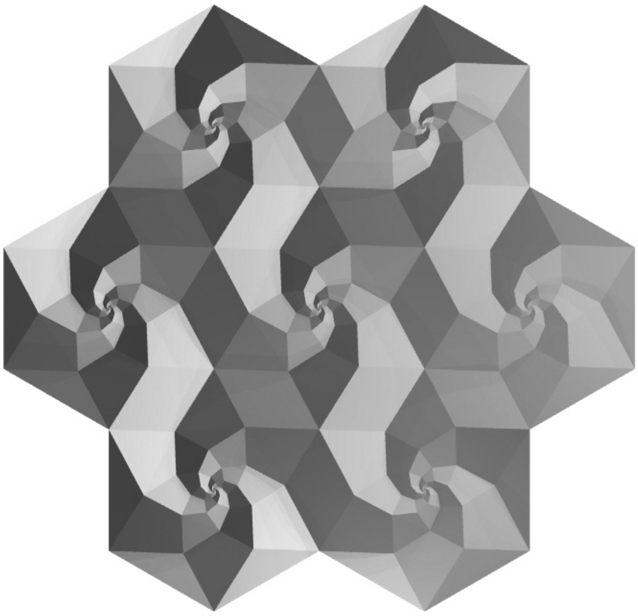

Figure 8: *Spidron relief*

1.7 Spidron relief. If several spidron nests are connected to each other, we get a special accordion shape that deforms in vertices around centre points arranged in a hexagonal grid, which can be described in all its transitional states as well as in motion using a mathematical algorithm. The system of equations indicates that with respect to the edges, the movement is, technically speaking, simultaneously recursive, which means that on the one hand, the angle between the edges of a specific spidron ring and the base plane can be calculated recursively from the angle between a corresponding edge on a neighbouring spidron ring and the base plane, and on the other hand, the rotation of the same edges with respect to the centre of the spidron nest can also be calculated the same way.

1.8 What is all this good for? In addition to the fact that the movements of the edges and faces of the spidron formations are interesting, this stable deformation may furnish ideas for many practical uses. It has been raised repeatedly that several layers of spidron reliefs could be used as shock dampers or crumple zones in vehicles. Its space-filling properties make it suitable for the construction of building blocks or toys. The surface could be used to create an adjustable acoustic wall or a system of solar cells that follow the sun in a simple manner. Various folding buildings and static structures could also be developed on the basis of my geometric investigation which may have utility in space travel.

2. Space-filling

The degree of plasticity of the spidron nest is continuously "adjustable". What I mean by this is that the angles of the individual faces to the base plane – that the figure occupies when the edges are not rotated – and to the neighbouring faces can be changed continuously during deformation. That is why the spidron nest generates a rich variety of forms with threefold symmetry. In some positions of the outside edges the nests form matching shapes. If the holes in the middle of such nests are "stopped up" using suitable triangles [2], we get several space-filling shapes which fill space in a uniform manner, without any overlaps or gaps, with their own copies. The naming and categorisation of those shapes is a task yet to be completed.

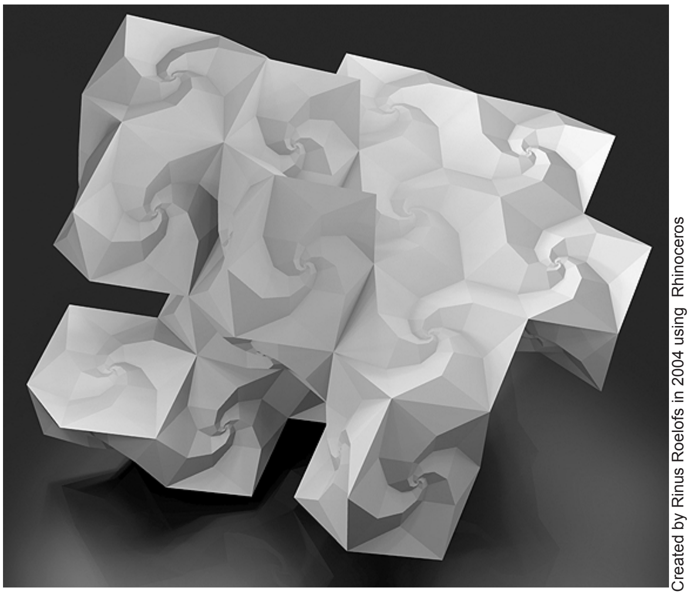

Figure 9: *Space filling with tetrahedral spidorn solids*

3. Particularly interesting aspects

3.1 When deformed in space, a spidron nest produces a complex shape that cuts various regular and semiregular solids into two symmetrical halves. If the various solids have identical edges, the resulting surfaces will have equal areas. This spatial hexagon, made from a regular hexagon by deforming it in space, or as we call it, spidron nest, cuts the regular icosahedron into two halves of equal volume when its longest (outside) edges are at an angle of 108°, while it cuts a cube into two symmetrical halves when the same angle is 90°.

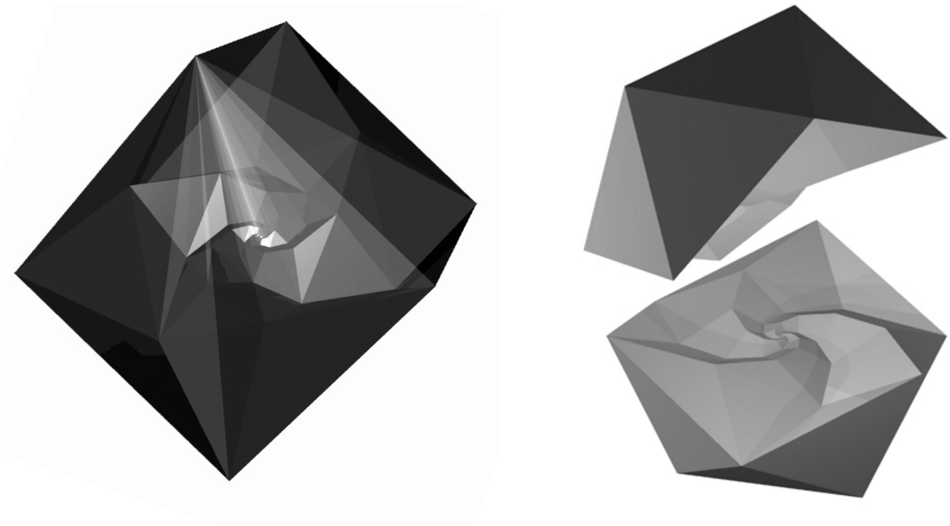

Figure 10: *Cubic and icosahedral dissection*

3.2 Several of the 3D bodies composed of identical spidron nests are space-fillers: they include the spidrotetron, bounded by four spidron nests and the spidrohedron, bounded by eight spidron nests, of course only those that are bounded by an even number of such nests. Half of the nests composing such bodies are left-handed, the other half are right-handed. Or, as we have already seen that individual spidron belts can have opposite spins, each face has to be accompanied by its mirror-symmetrical counterpart.

3.3 Spidron complexes can be built from spidron arms made of a single type of isosceles triangle (45°, 90°, 45°) or from two types of isosceles triangles. We haven't even mapped all the 2D spidron forms. The spatial possibilities of various 2D forms can produce many further surprising results.

4. Results of cooperative efforts

4.1 In 2003, Lajos Szilassi succeeded in capturing the form and interesting movements of spidron nests in mathematical alogrithms. This was one important contribution to the 3-dimensional modelling of several hundred spidron models, which was completed during 2004 with the help of the excellent Dutch sculptor, Rinus Roelofs.

4.2 With the help of another Dutch colleague, Walt van Ballegooijen and using the algorithms developed by Dr. Lajos Szilassi we have produced an interactive Excel table which will calculate other parameters from any particular angle between external edges along with the coordinates of the vertices of the spidron nest and the angles between the faces of all the spidron belts.

4.3 During the last semester, I participated in a seminar run at the Faculty of Geometry of the Budapest University of Technology and Economics by Professor Emil Molnár, on the subject of crystal groups. In private consultations we tried to identify an already described crystal group that is similar to a cube cut in two by a spidron nest. We found that one half of the cube can be divided further into identical pieces. We found three of those. Therefore the cube can be divided into 6 so-called fundamental domains, of which 3 and 3 are mirror images of each other. The group of crystals is related to the group of pyramids that cut the cube into six parts along four of its spatial diagonals, with the difference that

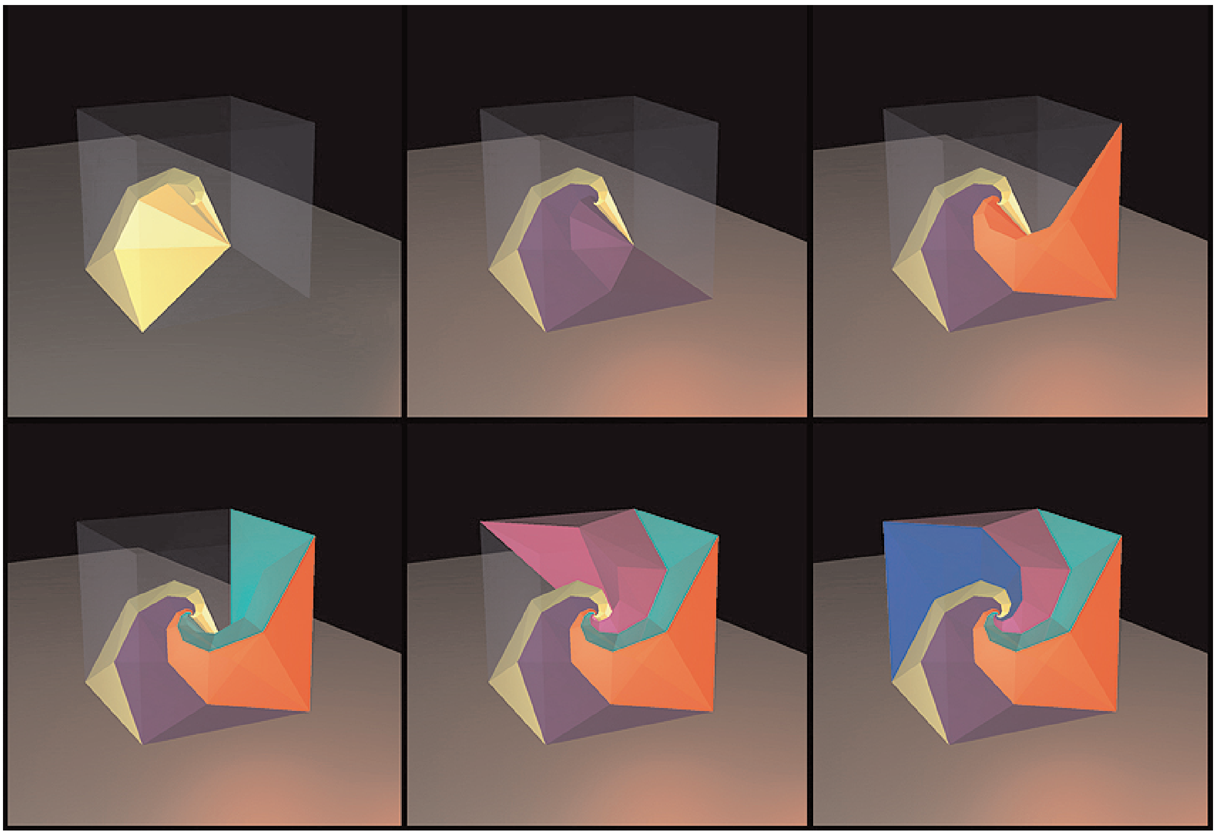

Figure 11: *Fundamental domains bisected*

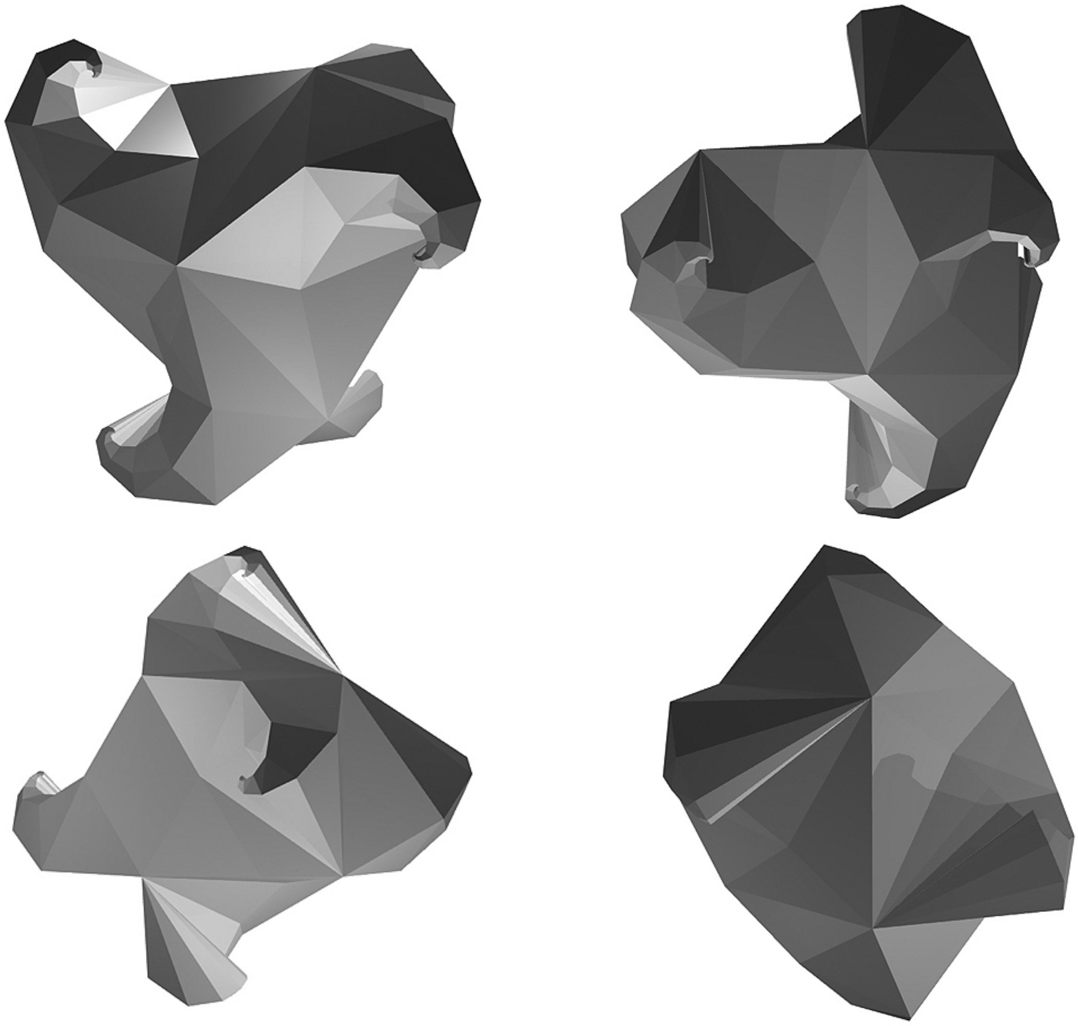

Figure 12: *Four views of a cube cut up into fundamental domains and turned inside out*

the fundamental domain itself has less symmetries than the square-based pyramid. In crystal geometry, the description that Dr. Molnár calls the "D-symbol" can be used to divide individual building blocks further in a barycentric fashion, which allows all the adjacencies belonging to the individual crystal groups to be described unequivocally. This means that the position, transformations and adjacencies of all identical components (i.e. the faces, edges and vertices along which the usually not regular tetrahedrons, simplex orbits forming the fundamental domains meet) can be described. The method is so elegant and general that it can be used to specify tessellations in hyperbolic and spherical spaces and it can even be used to describe crystal groups in higher than 3-dimensional Euclidean spaces. One of the keys to the process is the finding of fundamental domains. This is always performed by taking a component of the highest dimension (in 3D, for instance, the centre of a snub octahedron), connecting it to the centre of a component whose dimension number is one less (this may be the centre of one of the faces of the snub octahedron, a square), then to the mid-point of a component with one less dimension (in our example, the mid point of one of the edges of the square face) and so on, and finally to a vertex. Then we return to the centre of the space-filling shape in reverse sequence through the centres of components of increasing dimensions. The end-points of this connected sequence of segments are the vertices of the fundamental domain. The barycentric simplex orbits must be located within it. Using them, we obtain the corresponding D-diagram, which, along with 3 matrices, allows crystal groups to be determined and classified. We shall classify the rich world of spatial forms that can be built from spidrons using that modern and highly effective method in the near future

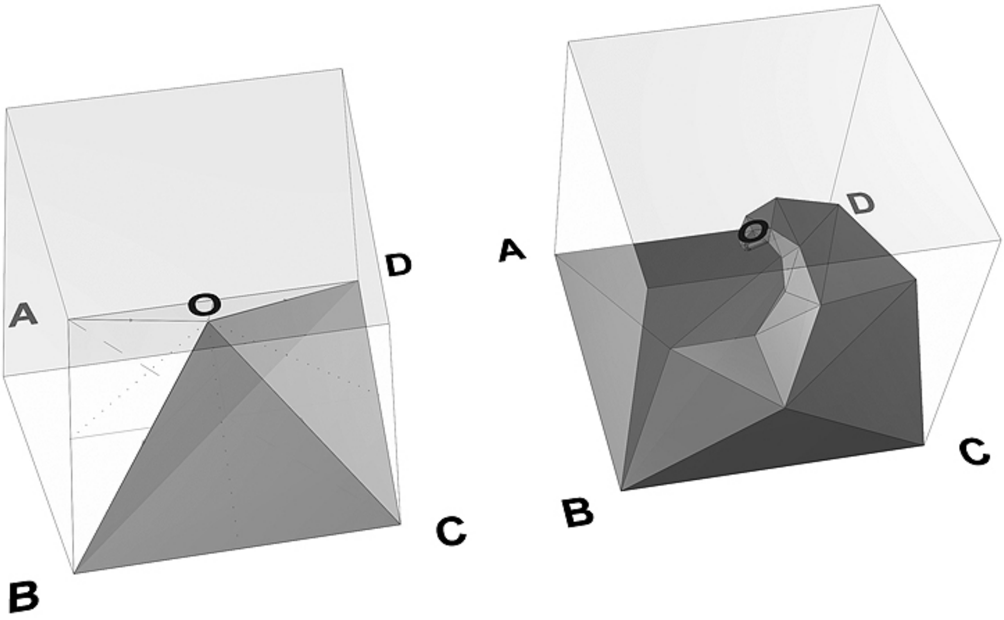

Figure 13: *A cube can be divided into six pyramids or into six shapes that are one-third of a cube cut in half by a spidron nest*

[1] It is possible to build a nest from four identical spidron arms as well, but that differs from the one made from six arms in that it can only be deformed to a more limited extent and it cannot be laid out in the plane. Six spidron nests with four spidron arms each can be assembled to form a body similar to a cube which has 3 left-handed and 3 right-handed spidron nests of this kind and as such it is also a space-filler.

[2] Towards the centre of spidron nests, subsequent rings are increasingly flat. The question is whether such a complex of triangles, with a singularity in the middle can be called a "closed" surface. If yes, then the bodies built from nests are also closed, i.e. they describe a specific volume. And hence they could be described as space-fillers. During the deformation of a nest - when it is compressed from the sides - the vertices of the spidron nest closer to the centre do not rotate at gradually increasing angular velocities but on the contrary, they slow down, though they "never" reach zero velocity. The centre itself is the only exception.

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Lajos Szilassi: The right of doubting - and the necessity of doubt
Thoughts with regard to the analysis of the Erdély's Spidron-System, presented with the author at the Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching Conference, May 21-22, 2004, Pécs

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