A Ukrainian Easter Egg Monument Stands for Thirty Years

Robert McDermott
Center for High Performance Computing
University of Utah
Salt Lake City, Utah, 84112, USA
Email: mcdermott@chpc.utah.edu

Abstract

Modeling a physical structure with a computer was a considerable challenge in the early 1970's, since software for the modeling needed to be written from scratch. A smooth continuous surface for the structure needed to be defined and designed to look like a chicken egg. Corner-connected equilateral triangles needed to be positioned using this surface. Six of these corner-connected equilateral triangles determined a four triangle pieces for the monument. Each piece of the monument was engraved with an encoding for its position in the monument and its coloring. Pieces were transported to Vegreville, Alberta, Canada and the monument was assembled. The monument has been standing for over thirty years.

1. Introduction

In 1972 I enrolled in the Ph.D. program at the University of Utah, Department of Computer Science. I had previously met Ron Resch, an experimental structures designer and a faculty who I was very much interested in working with him. Resch subsequently was awarded a contract to design one of his patented folded plate structures in the form of a giant Easter Egg. This project had a multiplicity of problems to which I contributed mathematically sound software solutions. I wrote software to interactively model and display a smooth, continuous surface that was shaped like an egg and software to interactively position and display equilateral triangles on that surface. I wrote software that used these triangles to produce control tapes for a plotter, which cut the pieces used in the construction of an egg-shaped monument. This unique monument, built in 1974, has continued to attract considerable attention ever since because of its geometric and artistic significance.

2. Ron Resch, Commissioned to Build a Monument

Resch was commissioned to build a Ukranian Easter egg monument for the town of Vegreville, Alberta, Canada. The first six months of this commission Resch sepnt working with Hank Christiansen of the Civil Engineering Department of Brigham Young University. They attempted to use software to position a structure of Resch's design. This effort proved unsuccessful. At that time I was studying with numerical analysist, Robert Barnhill, who researched surface patches, and with computer scientist, Richard Riesenfeld, who wrote a Ph D dissertation on B-spline curves and surfaces. B-splines are parametric functions that use piece-wise polynomials to approximate a curve near an ordered set of control points in the design of curves and surfaces. I wrote software for B-spline curves and a resulting surface of revolution. I also wrote software to position corner-connected equilateral triangles on this surface. This project led me to a PhD dissertation in computer aided geometric design [1].

3. Resch's Design System of Corner-Connected Equilateral Triangles and *Three-Pointer Star* Patterns

Resch had designed a folded plate structure based on a single-sized equilateral triangle that he could draw on paper and fold into a physical paper model. This model exhibited flexibility for a chicken egg shape. Six equilateral triangles from Resch's system were joined together as in **Figure 1a**. Their void formed an equilateral hexagon that was filled with a four triangle *three-pointed star* pattern which has a central triangle surrounded by three isosceles triangles as in **Figure 1b**. The combination of the equilateral triangles and the *star* patterns produced a continuous facetted surface.

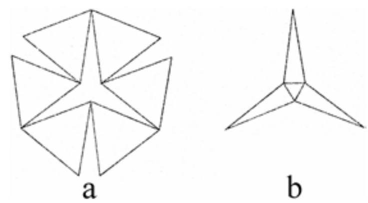

Figure 1: Equilateral triangles (a) and a *star* patterns for void of six triangles (b).

4. An Underlying Egg Shaped Surface for an Egg Shaped Monument

A search at the University of Utah's Marriot Library produced a United States Department of Agriculture, Egg Grading Manual. This manual had a picture captioned, "*Ideal egg shape*, usually found in AA or A quality" as in **Figure 2**. An 8 ½" by 11" photographic print of this *ideal egg shape* was produced from this picture. I overlaid the photograph with graph paper and aligned a vertical line for a symmetry axis with a line on the graph paper. I positioned a coordinate axis at this vertical axis of symmetry and distributed points along half of the silhouette in **Figure 3a**. These points were used to compute a B-spline curve as in **Figure 3b**.

This B-spline curve in **Figure 3b** did not look exactly like the *ideal egg shape*. The first curve **Figures 3b** and **4a** seemed a little flat on the top, so we moved the top point on the axis of symmetry up a little. This curve looked a little better as in **Figure 4b**. Next, the width of this curve looked a little wide, so we scaled the points in the width direction to produce the curve in **Figure 4c**. We felt that this was an improvement closer to *ideal*. To push our sense of satisfaction, we scaled the width coordinates in a little more for **Figure 4d**. This result seemed too narrow. However, we thought that these four curves in **Figure 4** provided worthy candidates for an egg-shaped silhouette of the monument.

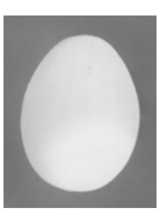

Figure 2: *Ideal egg shape*.

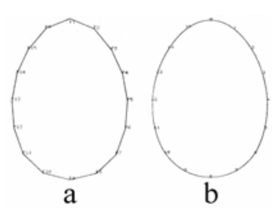

Figure 3: Polygon (a), B-spline (b).

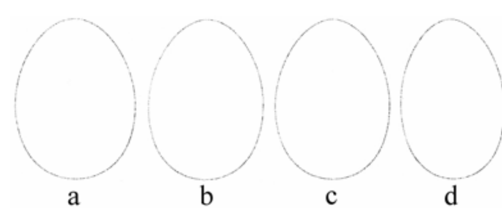

Figure 4: Initial (a), pointed (b), narrow (c), narrower (d).

A five foot by six foot Gerber flat bed plotter was used to draw three foot high drawings of the four curves in **Figure 4**, which we taped to the wall of Resch's Design Studio/Laboratory. Everyone who passed the open door gave us their opinion of the drawings, from the Dean of the

College of Engineering to the person who emptied the waste baskets. Even though there was only the most minute differences in shape everyone said that the curve of **Figure 4c** looked like an egg. In a short time we realized that the shape of an egg was much more of a mental icon than anything analytic. We were offered very strong opinions, whether they were solicited or not.

The curve of **Figure 4c** was the generating curve for a surface of revolution in **Figure 5a**. This became the monuments underlying surface and was displayed as a smooth-shaded surface defined from polygons and normal vectors at the corners of the polygons in **Figure 5b.**

5. Vertices of Equilateral Triangles Positioned on an Egg-Shaped Surface

Resch proposed a solution for positioning vertices of equilateral triangles on the egg-shaped surface. First he used two symmetry planes to cut the surface into a 60° portion of the full surface. Two points were located on the surface with each point coincident to a symmetry plane as seen in the triangle at the top of **Figure 6**. These two points served as the centers of two spheres where their radii were equal to the edge length of the equilateral triangle to be used in the structure. These two spheres intersected in a circle that intersected the B-spline surface in two points. A single solution point could be determined by ordering the two initial points so that the resulting three points were in a clockwise order when looking from the outside of the surface. In order to obtain a solution point computationally a two-variable Newton Mathod for a nonlinear equation was employed [2]. The success of Resch's solution was that two vertices of an equilateral triangle were always known with only the third vertex needing to find a solution.

The equilateral triangles were organized into rows as in **Figure 6**, having vertices coincident with symmetry planes and coincident with vertices of other equilateral triangles in the previous rows. **Figure 6** shows four rows of triangles that span the two symmetry planes.

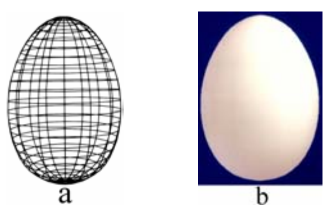

Figure 5: Surface of revolution (a), and smooth shaded surface (b).

Figure 6: Four rows of equilateral triangles.

For each row of triangles, an initial guess was made for locating an initial vertex along the B-Spline curve and coincident with one symmetry plane. This guess was a parameter for the B-Spline curve, it was adjusted so that each row of triangles would reach the far symmetry plane. This process attempted to achieve an even spatial distribution of triangles in a row of triangles. The triangle edge length and the curvature of the B-spline surface of revolution determined how many rows of triangles could be used in each 60° portion of a dome. Resch used four rows of triangles for each of the two dome sections as in **Figure 7 a** and **7b**.

While I was writing the software to positioning equilateral triangles on the egg-shaped surface, Jim Blinn joined our effort. He contributed his exceptional knowledge of computer graphics and his extraordinary programming skills to this positioning of equilateral triangles for this modeling. His contributions were invaluable to the success of the project [3].

6. Two Dome Sections, One Barrel Section and Two Transition Sections

Resch decided to use three major sections of his system to cover the majority of the egg-shaped structure. There would be a dome-shaped section at the top and another at the bottom. There would be a barrel-shaped section between the two dome sections. Each dome section would be connected to the barrel section by a set of triangles using points from the dome and barrel sections for a transition section to complete the facetted egg-shaped structure.

7. Symmetry Planes Subdivide Sections

With Resch's decision to model the egg in sections, symmetry helped to subdivide the problem of positioning equilateral triangles. This use of symmetry would increase the repetition of modules with Resch's design system. An axis of revolution for the surface ran from top to bottom. This axis was a rotational axis for symmetry planes, so that these planes subdivided the triangle positioning problem into smaller regions.

Resch used two 60° symmetry planes for positioning triangles. A 60° portion of a dome in **Figure 7a** was reflected through a symmetry plane for a second portion in **Figure 7b**. After a 60° rotation this portion as in **Figure 7c**, was united with the portion in **Figure 7a** for a 120° portion as in **Figure 7d**. This 120° portion could be rotated twice for a full 360° dome section in **Figure 7e**. This dome was for the more pointied end of the egg-shaped surface.

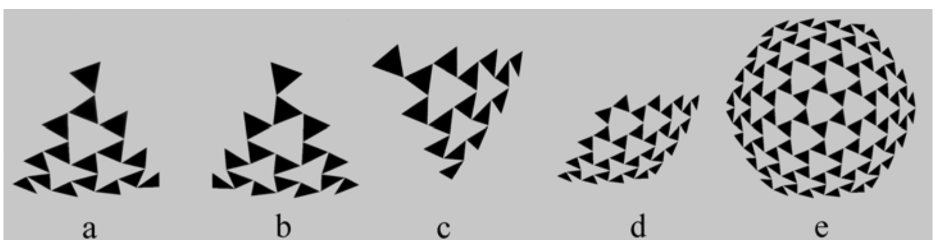

Figure 7: Triangles for a portion of a dome (a), these triangles reflected (b), then rotated (c), initial triangles plus reflected and rotated triangles for a 1/3 portion (d), full dome section (e).

Resch decided to divide the barrel section with two 10° symmetry planes for positioning equilateral triangles in **Figure 8a**. These equilateral triangles were reflected through each adjacent symmetry plane for a 20° portion or a single stave for the barrel section in **Figure 8b**. This stave section was rotated to form a full 360° barrel section in **Figure 8c**.

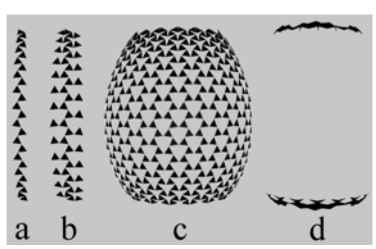

Figure 8: Triangles for a portion of barrel (a), these triangles were reflected (b), then rotated for a full barrel section (c), triangles of top and bottom transition sections (d).

Two transitional sections in **Figure 8d** were defined**,** one toward the top or pointed end of the egg-shaped structure, and one toward the bottom or rounded end. For the transition sections, Resch decided to use two symmetry planes that met at 120°. The 120° transition sections connected two 60° portions of an adjacent dome section and six 20° portions of the barrel section. These 120° transitional portions would be rotated twice for each of the full 360° transition sections. These two transition sections, together with the barrel section, and the two dome sections would complete a continuous, triangulated surface to be used for physical pieces in constructing the egg-shaped monument.

8. Six Equilateral Triangles Fully Define a Four Triangle *Star* Pattern

Sixteen equilateral triangles were positioned between two 60° symmetry planes in **Figure 6**, **Figure 7a**, and **Figure 9a**. Six equilateral triangles completely surround a four triangle *star* pattern as in the central part of **Figure 9b.** The sixteen equilateral triangles and their associated symmetry planes were used for ten *star* patterns in **Figure 9a**. Three *star* patterns were in the interior of the reflected equilateral triangles in **Figure 9b**. These two regions were rotated for a full 360° dome section of *star* patterns as in **Figure 9c**. The dome section in **Figure 9c** covered the pointed end of the monument and a similar approach produced a slightly different dome for the rounded end of the monument at the bottom of **Figure 10** and in **Figure 11c**.

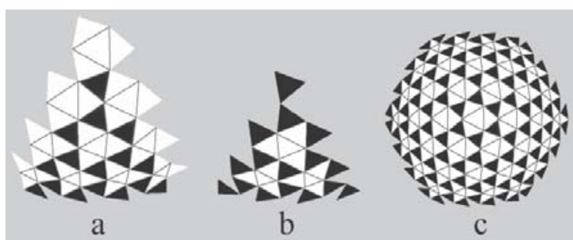

Figure 9: *Star* patterns overlapping symmetry planes (a), interior (b), full dome (c).

When the two complete dome sections were computed for the egg-shaped monument, they were positioned for the top and bottom of that monument in **Figure 10**. A stave section with associated *star* patterns can be seen in the middle of **Figure 10**. This stave section was used to complete the barrel section, as in **Figure 11b**. These three major sections combined with the two transition sections, that fully covered the egg-shaped surface to produce a complete facetted egg-shaped monument as in **Figure 11**.

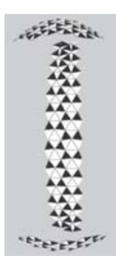

Figure 10: Top, stave, and bottom.

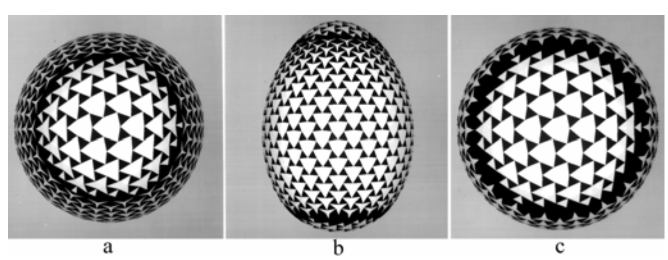

Figure 11: Top (a), side (b), bottom (c)

9. Markers and Line Drawings were Used to Develop a Color Pattern for the Monument

Resch, Paul Sembaliuk, and Elaine Kruelski worked together to develop a color patter. Sembaliuk is an authority on Ukrainian culture from Alberta, where Kruelski was an art student at the University of Utah of Ukrainian ancestry, who was experienced with the traditional craft of batiking chicken egg shells. They used magic markers and line drawings as in **Figure 12** to develop a color pattern for the egg-shaped monument as in **Figure 13**.

Resch decided that the pattern would use three basic colors. The colors were determined by the process of anodizing the flat sheet aluminum to produce pieces with permanent color. The anodizing process is a timed process. If the time is short a white color is produced. If the time is extended a gold color is produced, and if the time is extended even further a bronze color results.

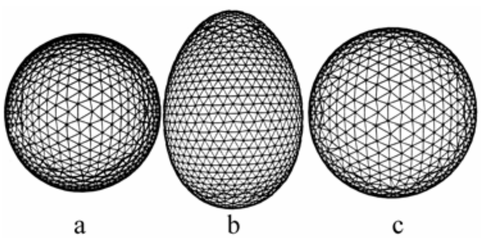

Figure 12: Top (a), side (b), bottom (c).

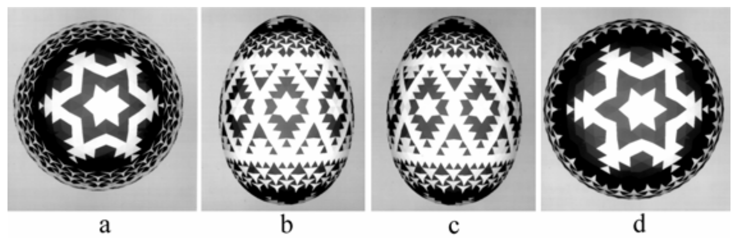

Figure 13: Top (a), left (b), right (c), bottom (d).

The color pattern was developed with the governing rules that each triangle would be one of three colors and that symmetry would be used whenever possible. For example, Resch decided to use two staves of the barrel section as a rotational part of the color pattern. This decision resulted in nine identical patterns for the barrel section. This odd number resulted in each side of the monument with a different appearance. On one side as in **Figure 12b** there is a diamond shape and on the other side, as in **Figure 12c,** there is an X shape.

10. Flat Plans for a *Star* Pattern

Three-dimensional *Star* patterns were flattened to two dimensional *star* patterns piece for the monument, where additions were added as in **Figure 14**. One-third of an equilateral triangle with two bolt holes was added to each of the six edges to provide material to connect a *star* pattern with equilateral triangles. Each equilateral triangle in the structure had six bolts affixed to it. Nuts were threaded on these bolts to secure a *star* pattern and adjacent triangles. These star pattern pieces were engraved with color code **BAAG** for bronze, aluminum (white) and gold anodizing and location code **13B8** for a *star* pattern in the barrel section of the monument.

Figure 14: Star pattern with 1/3 triangles, position and color coding plus drill holes.

I produced plotter tapes for each *star* pattern in the monument. A student was hired to run the plotter to produce each of these pieces. Each *star* pattern had its interior lines engraved so that the pattern could be folded. This folding gave each *star* pattern the shape it would need in the final structure. I made exact tables of fold angles for *star* patterns. When I later viewed these elaborate tables, I noticed that the fold angles all varied between 2° and 8°.

All of the drilling, engraving, and cutting of aluminum were performed with a motorized engraving tool, designed by Resch, that fit on to our military strength plotter. Twelve holes were drilled in each *star* pattern to accommodate two bolts from each of the six equilateral triangles which surrounded the *star* pattern. Coloration coding and position coding were lightly engraved on each *star* pattern. The other interior lines were engraved for folding and the lines on the exterior of the pattern were passed over multiple times to cut through the flat sheet of aluminum. Thus, 524 *star* patterns were produced, in addition, 2208 equilateral triangles were produced for the monument.

11. Monument

Resch led a group of Vegreville citizens to assemble the monument. **Figure 15** is Resch and his son, Jon, near the end of this assembly in the summer of 1974.

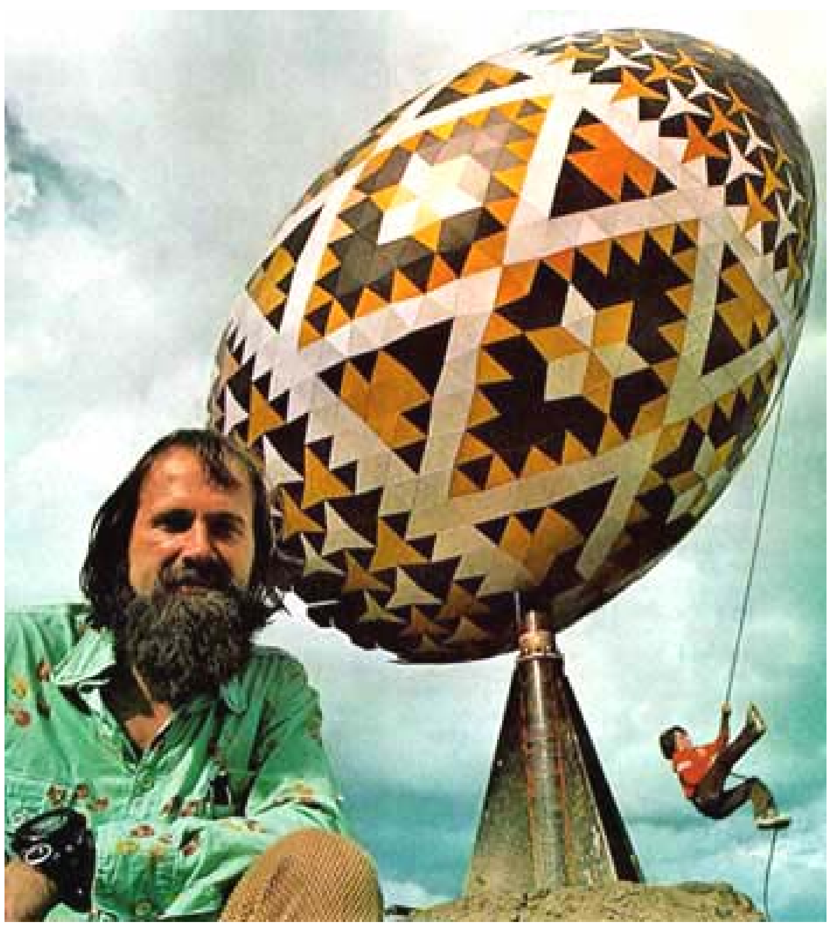

Figure 15: Resch and son during the construction in summer of 1974.

Conclusion

The monument has now stood for more than thirty years. This is a testimony to the multiplicity of design talents of Ron Resch. The monument has proved to be worthy of a community with a long tradition of batiking easter eggs and also worthy of the severe weather conditions of central Alberta, Canada. It has also become the focus of folklore and folk celebrations, as in **Figure 16,** attended by Queen Elizabeth of England in 1978.

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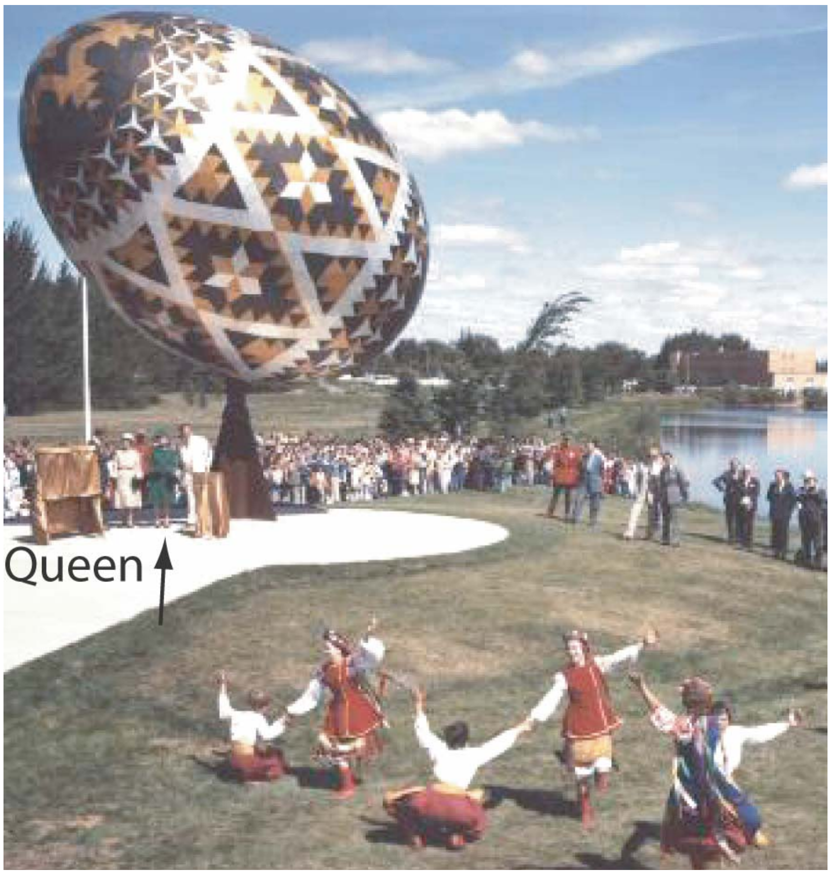

Figure 16: Queen Elizabeth visiting monument in 1978.

Acknowledgements

I would like to thank my spouse, Deborah, for proof reading drafts for this paper, both before and after the reviews.

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