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Supercircles:
Expanding Buckminster Fuller's Foldable Circle Models

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What would you call a circle with more than 360°? Supercircles are our solution to a folded circle problem that involves building modules to form great circle polyhedra.

Bucky Fuller demonstrated 7 folded circle models in his book Synergetics [1]. He was fascinated by the great circles of spherical polyhedra, probably from his sailing experiences, where navigation on the globe finds the shortest distance between two points is a circle whose plane cuts through the center. Land based geometrical thinking by contrast defines the shortest distance between two points as a straight line. Fuller's folded circle models show sets of great circles from the icosahedron and cuboctahedron (Fuller's Vector Equilibrium or VE). The great circles are the equators given by diametrically opposite poles. Each set of great circles is given by either pairs of opposite vertices as poles (6 great circles for both the icosahedron and VE), or centers of faces (10 for the icosahedron, 7 for the VE: 4 for triangular faces and 3 for squares), or centers of edges (15 for the icosahedron and 12 for the VE). Fuller's modules are made by folding a circle along scored lines into a conical form. Join the modules together to build the whole sphere. In Fuller's models it is important to note that the number of folded circle modules exactly equals the number of great circles in the set. The spherical surface of each conical module (except the 3 great circle model of the VE) forms a path known in graph theory as an eulerian closed circuit (each edge is traversed once and only once). The bowtie motif predominates which Fuller likens to wave interference, energy along an arc bounces or is shunted at the great circle intersections.

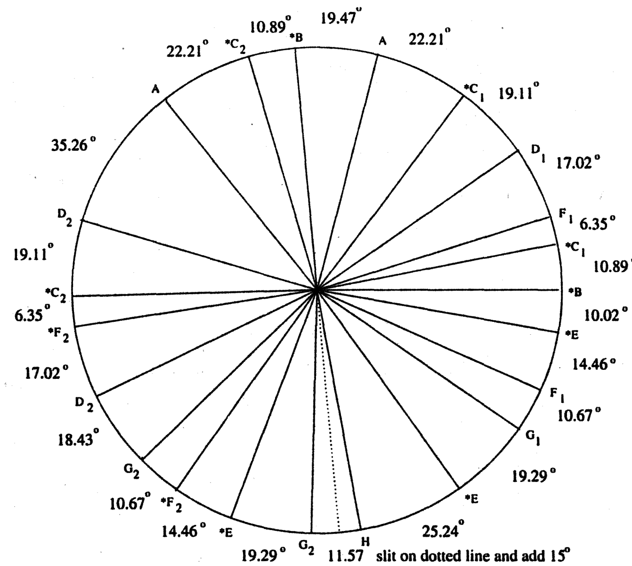

***Figure 1** shows one of 24 supercircle modules for the spherical cuboctahedron. This supercircle contains 375° when a 15° sector is inserted at the dotted line. Score asterisk (*) lines below and fold outward. Score non* lines on the topside and fold inward. Attach A to A, $*C_1$ to $*C_1$, and so on for like labeled vertices.*

Although Fuller examined the spherical polyhedron that results from combining the 4 sets of the VE ($6 + 4 + 3 + 12 = 25$), he did not demonstrate a folded circle module for its 25 great circles (totalling 6000°). Since each unique arc in Fuller's 48 Lowest Common Denominator (LCD) triangles of the VE [1] occurs exactly 24 or 48 times, there is no way to partition these arcs into 25 identical modules (pigeonhole principle). Could we fold 24 modules formed from an LCD triangle and its image by reflection in an edge? By cutting a circle along a radius and splicing in a 15° sector (one possibility is shown in figure 1), we discovered a *supercircle* of 375° which was foldable into the shape in figure 2 (24 of which make the 25 great circle model). A whole new realm is opened for exploration when the supercircle breaks away from the limitation of 360°. For example, the $6+10+15=31$ great circles of the icosahedron can be built with 30 supercircles of 372°. The $3+4+6=13$ great circles of the octahedron (or cube) can be built with 12 supercircles of 390°. Surprisingly the number of great circles in an aggregate set seems to have exactly one more great circle than the model's symmetry can accommodate.

Two circles isometrical lived in sweet harmony
Began to brawl, could not agree on whose was whose degree
The poor sad cone was left alone quite unequivocal
And on a lark with extra arc ran selfish supercircle.

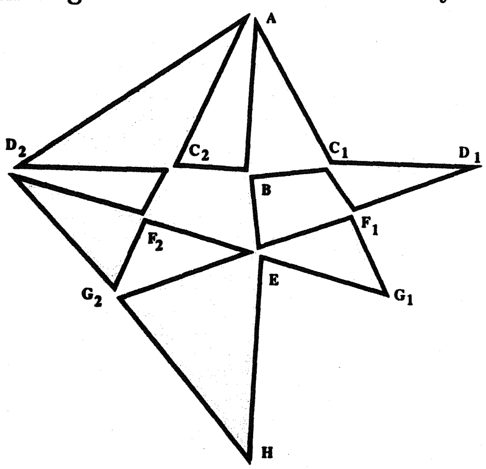

Figure 2 shows the result of folding the supercircle in figure 1. Connect inwardly within the module B (midpoint of edge), C_1, C_2, E and F_2. Connect F_1 outwardly within the module. Connect A (center of triangle), D (vertex), G_1, G_2, and H (center of square) to adjoining modules. Notice how in this symmetrical module spaces on one side are inside (shaded) and outside (unshaded) on the other, positive and negative space. 24 of these folded supercircle modules composes the foldable 25 great circle model of the spherical cuboctahedron (or VE).

Models of spherical polyhedra built from folded circle-like modules, besides being beautiful, offer some interesting details of stable 3D construction from a 2D medium such as paper or tyvek. One aspect is partitioning the sphere into bowties by joining two points of the edge of the supercircle to form the intersections of great circles. The 31 great circles of the icosahedron are totally triangulated, but the 25 great circles of the VE have a quadrilateral shape amid many various triangles. When a supercircle module forms an eulerian closed circuit, the whole model will use minimal material. This means that some triangulated (or quadrilateral) shapes will be enclosed by the module, positive spaces. Others will be open with some walls provided by an adjoining module, negative spaces. A folded module creates inside and outside space at the same time.

In spherical geometry arc lengths are measured in degrees, 360° on every great circle, 360° around each vertex. Napier's rule helps calculate all arcs and angles, given two knowns in a right spherical triangle. Spherical triangles with no 90° angles require three knowns. Calculations are often aided by identifying symmetries. Four equal angles at a vertex must be 90°. An angle at a pole is equal to the arc at the equator cut by the angle [2]. After calculating angles and arcs using symmetry and arithmetic, the rest are determined using the laws of sines and cosines [3].

Supercircles as a tool to build complex great circle models may have applications to spherical trigonometry education, graph theory, geodesics, electron orbits [4] and cell growth.

Now can you guess, what is a super-duper circle? A supercircle with a large number of degrees added will approach a sphere.

[1] Buckminster Fuller, Synergetics, Macmillan, Vol. 1, pp. 164-189, 1975. The on-line account is at http://www.rwgrayprojects.com/synergetics/s04/p5000.html.

[2] Kaj Nielsen and John Vanlonkhuyzen, Plane and Spherical Trigonometry, Barnes & Noble, pp. 103-151, 1946.

[3] Robert W. Gray, Notes to R. B. Fuller's Synergetics: Appendices, 1993. Additional notes at http://www.rgrayprojects.com/rbfnotes/greatc/greatc1.html.

[4] Edward Suzuki Hoerdt, Atommetrics: Another View of Atomic Structure Based on Electron Orbital Geometry, Forma, Vol 17, No 4, pp. 275–350, 2002.