

## **The Shape and History of The Ellipse in Washington, D.C.**

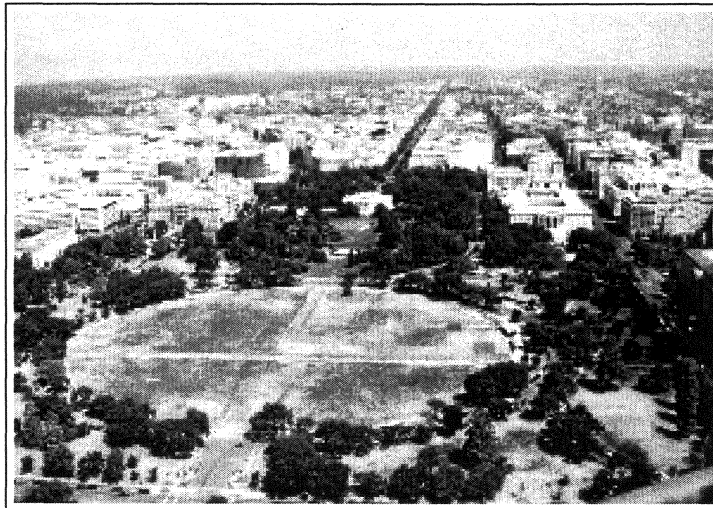
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### **1 Introduction**

When conic sections are introduced to mathematics classes, certain real-world examples are often cited. Favorites include lamp-shade shadows for hyperbolas, paths of baseballs for parabolas, and planetary orbits for ellipses. There is, however, another outstanding example of an ellipse.

Known simply as the Ellipse, it is a gathering place for thousands of Americans every year, and it is probably the world's largest noncircular ellipse. Situated just south of the White House in President's Park, the Ellipse has an interesting shape and an interesting history.



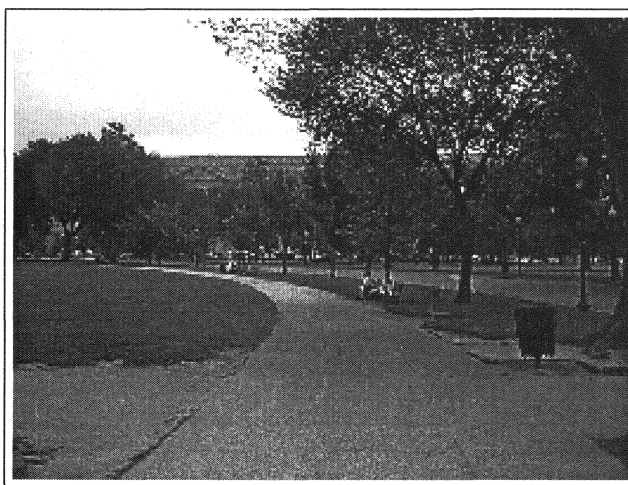
**Figure 1.** Looking north from the top of Washington Monument:  
the Ellipse and the White House [19]

When Charles L'Enfant submitted his Plan for the American capital city to President George Washington, he included many squares, circles, and triangles, and today, well known shapes in or near the capital include the Federal Triangle, McPherson Square, the Pentagon, the Octagon House Museum, Washington Circle, and, of course, the Ellipse.

Regarding the Ellipse in its present size and shape, a map dated September 29, 1877 (Figure 7) was probably used for the layout. The common gardener's method would not have been practical for so large an ellipse – nearly 17 acres – so that the question, "How was the Ellipse laid out?" is of considerable interest. (The gardener's method uses three stakes and a rope. Drive two stakes

into the ground, and let  $2c$  be the distance between them. Each end of the rope, which must be longer than  $2c$ , is attached at the bottom of a fixed stake and then the third stake is used to pull the rope tight. The third stake is then moved so that the rope remains tight and the stake stays perpendicular to the ground, with the bottom of the stake pressed against the ground. The path traced by the bottom of the moving stake is an ellipse. Its foci are marked by the stationary stakes. The significance of the number  $c$  is described in Section 3.)

This article establishes that the Ellipse, practically speaking, really is an ellipse. Features, such as eccentricity and location of foci, are considered, along with the striking but rarely recognized placement of the Ellipse relative to a special axis determined by Charles L'Enfant and Thomas Jefferson. The Ellipse (boundary) is defined in [26] by the face of the inner curb of the driveway shown in Figure 2. (Thus, the world's *largest* ellipse may be the one determined by the *outer* curb of the driveway.)



**Figure 2.** The walkway inside the Ellipse. The boundary of the Ellipse is the inner curb of the driveway behind the trees.

Photo by John E. Brown [4].

The Ellipse was laid out during 1877-1880 by the Army Corps of Engineers, under the direction of Thomas Lincoln Casey. In 1933, responsibility for the Ellipse passed to the National Park Service. This article was made possible by NPS, and thanks are here given to Ann Bowman Smith and David R. Krause, both of the NPS Office of White House Liaison. Special thanks also goes to David A. Doyle of the National Geodetic Survey and John E. Brown, who visited the Ellipse, confirmed several features indicated on the NPS survey [21], and took photographs [4], as in Figure 2.

## 2 Area and Perimeter

We begin with measurements of the Ellipse. Quoting from [13], "This information was taken from the most recent survey of the Ellipse and provided...by a staff civil engineer.

major axis	1058.26 feet
minor axis	902.85 feet
area	751071.67 square feet
perimeter	3086.87 feet

All measurements were taken in microstation from face to face of the curb line." Thus, with reference to the standard form of equation for an ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

the semimajor and semiminor axes of the Ellipse are given by  $a = 529.13$  and  $b = 451.42$ . The formula  $A = \pi ab$  for the area of an ellipse yields a calculated area of 750400 square feet. Using the measured values of  $a$ ,  $b$ , and  $A$ , the difference in areas is given by

$$D \approx 751071.67 - 750400 \approx 672, \text{ so that } \frac{100D}{750400} \approx 0.09,$$

an error of only 0.09%. This does not prove that the Ellipse is an ellipse, but that a first quick-and-easy test has been passed.

Exact formulas for an elliptic perimeter are not in closed form, but in practice, formulas that give a close approximation suffice. The most common such formula gives

$$\text{perimeter} \approx \pi \sqrt{2(a^2 + b^2)} \approx 3090,$$

so that the difference between measured and calculated values is given by

$$d = 3086.87 - 3090 \approx -3.1, \text{ so that } \frac{100d}{3086.16} \approx -0.1,$$

again, an error of less than one part per thousand. The Ellipse has passed a second test. Further consideration is given in Section 5, but before that, we shall treat the Ellipse as sufficiently close to a mathematical ellipse that we may speak of its foci, directrices, and other features.

### 3 Foci of the Ellipse

The foci of the ellipse  $(x/a)^2 + (y/b)^2 = 1$ , where  $a \geq b$ , are the points  $(-c, 0)$  and  $(c, 0)$  given by  $c = \sqrt{a^2 - b^2}$ . For the Ellipse,

$$c = \sqrt{(529.13)^2 - (451.42)^2} \approx 276,$$

so that the distance between foci is  $2c \approx 552$ , and the eccentricity is

$$e = c/a \approx 0.52.$$

It is the eccentricity that tells the "shape" of an ellipse. If  $e = 0$ , the ellipse is a circle, and if  $e = .99$ , the ellipse is much longer than wide. When  $e = 1$  the curve is no longer an ellipse, but a parabola, and when  $e > 1$ , the curve is a hyperbola. The cases  $e = 0$  and  $e = 1$  can be regarded as extremes for ellipses, so that we may say loosely that the shape of the Ellipse is half way between the extremes.

Every ellipse has directrices, one associated with each focus. Let  $F$  denote the west focus of the Ellipse, and let  $D$  denote the associated directrix. For any  $P$  on the Ellipse, the point-to-point distance  $|PF|$  and the point-to-line distance  $|PD|$  maintain a constant ratio as  $P$  moves around the Ellipse. That constant is the eccentricity:

$$\frac{|PF|}{|PD|} = e = 0.52.$$

Taking  $P$  to be the west vertex of the Ellipse, and using values already calculated,

$$|PD| = 0.52|PF| = 0.52(a - c) \approx 0.52(529 - 276) \approx 131.5.$$

That is, the west directrix passes through President's Park about 130 feet west of the west vertex, and about half way to 17th Street. On the other side, the east directrix passes about half way from the east vertex to 15th Street.

#### 4 Center of the Ellipse

In Figure 3, you can see the Ellipse and, at its center, the notation **BM $\Delta$ 18**. This image is copied from one of those wonderful topographic maps published by the United States Geological Survey. The USGS website has a contact link that was used to ask what **BM $\Delta$ 18** means. The surprising answer was that *it's a benchmark but not one of ours...at first we didn't know what it was...turns out to belong to the National Geodetic Survey, and it has a name: the Meridian Stone.*

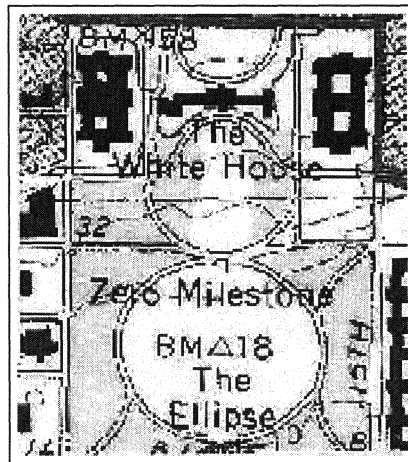


Figure 3. Portion of a USGS map [29]

To say that the location of the stone is well-documented is an understatement. Indeed, the center of the little brass conical hole embedded in the granite has the following location:

Latitude  $38^{\circ} 53' 38.17002''$  North of the Equator  
 Longitude  $77^{\circ} 02' 11.55845''$  West of Greenwich Meridian  
 Elevation 5.205 meters above sea level.

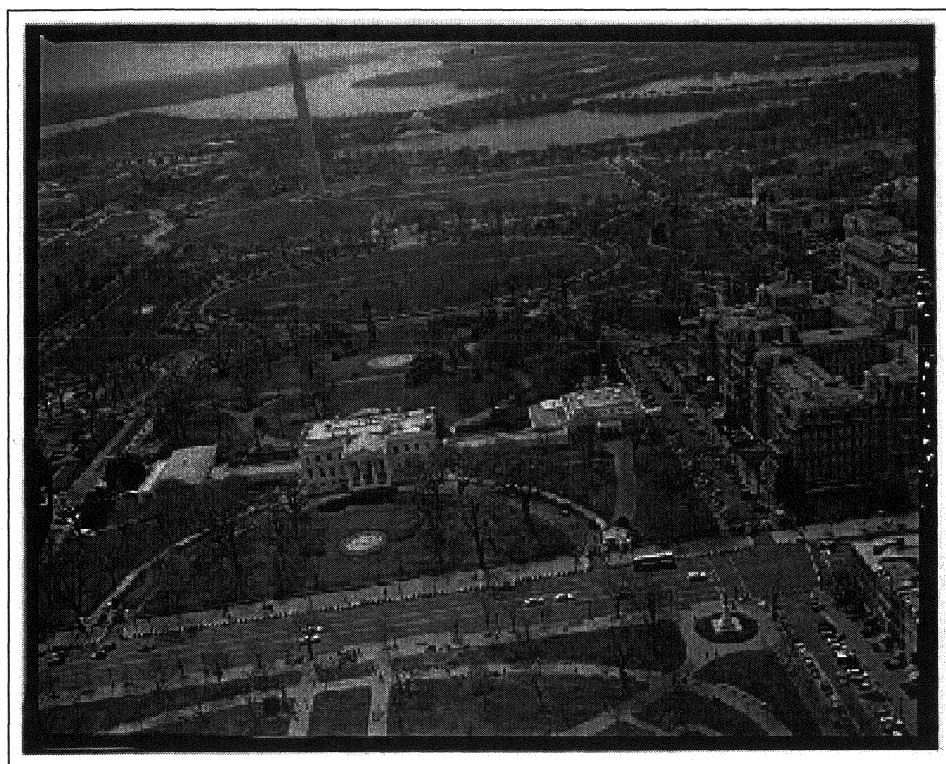
Relative to the center (of gravity) of the earth, this location is given in meters as a point in cartesian coordinates:

$$(x, y, z) = (1115076.648, -4844021.259, 3983142.714).$$

To confirm these data and find explanations, start by visiting the NGS (National Geodetic Survey) website [16] for data sheets. That site explains various options for viewing data sheets and offers links to specific stations; including the station [17] designated Meridian Stone. Internet images [18] and [19] of the Meridian Stone are easily found. However, despite its internet presence, the Meridian Stone seems to be little known to NPS. For example, one of my former students spent an hour at the Ellipse in August, 2003, and tried unsuccessfully to find the stone. Moreover, the stone is not mentioned in lengthy reports ([22], [31]) written for NPS. Indeed, an NGS surveyor wrote to me that in 1990, he and a surveyor friend "rediscovered" the Meridian Stone...

Initially, our discussions with the Park Service indicated that they had no idea of what we were talking about. They did agree however to assist us in a survey. On a very nice spring Sunday morning, we set a theodolite over the Zero Milestone, backsighted the Washington Monument, and using the computations from the positions we had in the National Spatial Reference System. I computed the direction and distance to the Meridian Stone. I had Alan keep me on line and using an electronic distance instrument, he put me at the computed distance. I inserted a probe at the computed point and about 2 inches down I hit the mark. The representative from the Park Service was amazed...

The Meridian Stone is located 0.607 meters (1.99 feet) from the north-south axis of the Ellipse. This axis, with longitude  $77^{\circ} 02' 11.58362$  W, is of historical interest, as President Thomas Jefferson once desired that there be an American prime meridian. He instructed Nicholas King, Surveyor of the Federal District, to establish this Meridian as the north-south axis through the front door of the President's House. You can view King's handwritten report to Jefferson by following these steps: first, access <http://memory.loc.gov/ammem/mdbquery.html>. Type **Nicholas King** in the Search Box; scroll to item 9, and click the underlined tag, dated Oct. 15, 1804. Details about this and other attempts to establish an American prime meridian are given by Silvio A. Bedini [1], Historian Emeritus of The Smithsonian Institution.



**Figure 4.** Aerial view [11] of The White House Grounds, The Ellipse, and Washington Monument Grounds, Looking South.

The north-south axis of the Ellipse could be called the Jefferson Axis, as it passes – at least, cartographically – through the Jefferson Memorial, although there seems to exist no official statement regarding the possible placement of the top of the dome of the Memorial right on the Jefferson Axis. Figure 4 shows the Jefferson Memorial in line with the White House and the center of the Ellipse.

Let's revisit those remarkably precise angles (latitude and longitude) and distance coordinates for the Meridian Stone. These can be confirmed using [20] to convert from one system to the other, and [28] to compute surface distance between two user-input points given by latitude and longitude.

As a final consideration regarding the center of the Ellipse, one may speculate that the method of layout—for which no description seems to have survived—may have been simply to start at the center,  $(0,0)$ , and to "send the chain man" out to the point  $(x,y)$  for many choice of  $x$ , with  $y$  determined by the equation  $x^2/a^2 + y^2/b^2 = 1$ ; or, equivalently, for many choices of angle  $\theta$ , to place a stake at the point  $(a \cos \theta, b \sin \theta)$ .

## 5 Quadrarcs

In the literature of landscape architecture, possibly the most often cited example of an oval that is often called an ellipse is the Piazza of St. Peter, or Piazza Obliqua, in front of St. Peter's Basilica in Rome. Designed by Gianlorenzo Bernini some 300 years ago and based on sixteenth-century oval constructions by Sebastiano Serlio, it is well known [10] that the Piazza employs four circular arcs, as suggested by Figure 5.



**Figure 5.** View [30] from the cupola of St. Peter's Basilica, Rome  
Major axis: 650 ft. Area: 254306 sq ft. Perimeter 1815 ft,  
of which  $1815/4$  ft measures each of the four circular arcs.

Gridgeman [10] calls such four-circle ovals *quadrarcs*. The defining property of a quadrarc is that at each point of intersection of the circles, the normals are identical. Equivalently, the circles have a single point of intersection, called a *joint*, at which, of course, they share a common tangent line. Now, two questions arise: (1) how closely can a quadrarc approximate an ellipse, and (2) was the Ellipse laid out as a quadrarc? Let  $E(a,b)$  denote the ellipse  $x^2/a^2 + y^2/b^2 = 1$ , where  $a > b$ . Following Gridgeman, let  $h$  and  $k$  be distances as shown in Figure 6.

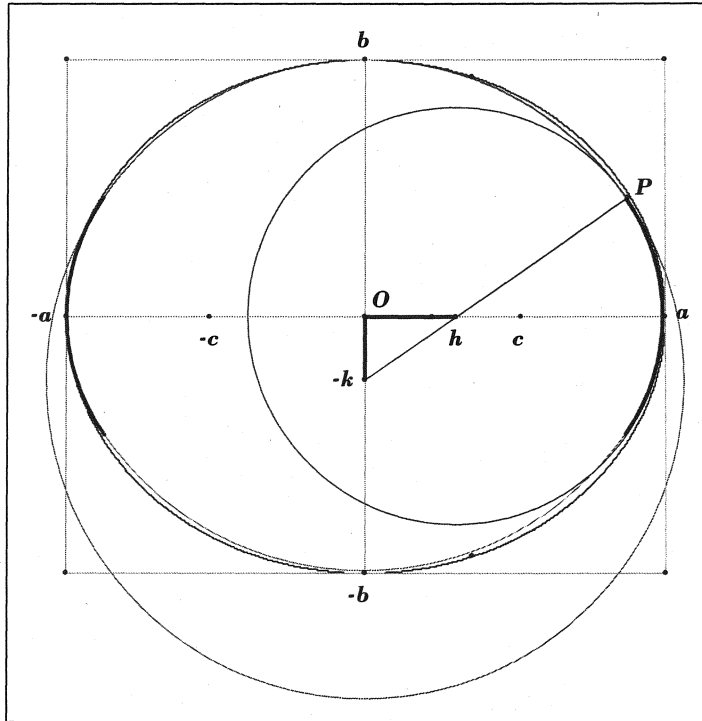


Figure 6. Ellipse and Quadrarc (drawn by *The Geometer's Sketchpad*)

The two circles are given by

$$x^2 + (y + k)^2 = (b + k)^2 \quad \text{and} \quad (x - h)^2 + y^2 = (a - h)^2.$$

Subtraction yields  $y = (2bk - 2hx + b^2 - a^2 + 2ah + 2bk)/2k$ . Using this to substitute for  $y$  in either circle-equation and then solving for  $x$  gives a discriminant,  $D$ . The meeting of the circles in a single point is equivalent to  $D = 0$ , which leads directly to the following dependence of  $k$  on  $h$ :

$$k = \frac{a - b}{2} \cdot \frac{2h + b - a}{h + b - a},$$

or, equivalently,  $(a - b)(h + k) - hk = (a - b)^2/2$ , showing that  $h$  and  $k$  are interchangeable. Moreover, the joint in the 1st quadrant is the point

$$P = (M, Mk/(h - k)), \quad \text{where } M = h + \frac{h(a - h)}{\sqrt{h^2 + k^2}}.$$

Table 1. Comparisons of quadrarcs  $Q(a, b, h)$  with ellipse  $E(a, b)$

$h(a, b)$	$k(a, b)$	area, % error	perimeter, % error
23.529	21.982	750400, 0	3071.3, -0.6
21.946	23.564	759660, 1.234	3090.1, 0

To interpret Table 1, note that in the first row of numbers, the percentage error in the area of  $Q(a, b, h)$  and  $E(a, b)$  is zero because this is the condition from which  $k(a, b)$  is calculated from  $h(a, b)$ . When the areas of the two shapes are equal, the perimeter of  $Q(a, b, h)$  is 3071.3 feet, which differs from the perimeter of  $E(a, b)$  by an error of -0.6%. The other row shows that if  $k$  is chosen to equalize the perimeters of  $Q(a, b, h)$  and  $E(a, b)$ , then the error in area is 1.234%.

In Table 2, the quadrarcs of Table 1 are compared with the Ellipse in Washington, D.C.

Table 2. Comparisons of quadrarcs  $Q(a, b, h)$  with the Ellipse

$h(a, b)$	$k(a, b)$	area, % error	perimeter, % error
23.529	21.982	750400, 0.01	3071.3, 0.39
21.946	23.564	759660, -1.1	3090.1, -0.1

The two tables indicate that the ellipse  $E(a, b)$ , the Ellipse, and the two quadrarcs are so close to one another that two conclusions follow: (1) the shape of the Ellipse is an ellipse (architecturally speaking), and (2) measurements (and historical records, as we shall see in Section 6) are insufficient for deciding whether the Ellipse was laid out as a quadrarc. Regarding (1), it should be kept in mind that the Ellipse is not perfectly flat and horizontal (see Section 6), and that no physical object can be a true ellipse in the mathematical sense. This latter observation is a reminder that a mathematical ellipse consists of infinitely many non-physical stationary points, whereas a physical ellipse consists of a finite number of jiggling atoms. When we speak of a real-world "true ellipse," we mean a "curve" that qualifies as a "true ellipse" as these terms are understood in the appropriate nonaxiomatic discipline, such as landscape architecture or astronomy. For example, in landscape architecture, the Piazza Obliqua in Rome does not qualify as a "true ellipse".

Curves other than quadrarcs have been used to approximate ellipses. Such curves, as well as quadrarcs, are discussed by Rosin ([24], [25]).

## 6 Maps, Documents, and Historical Notes

Probably, the most detailed account of the history of the Ellipse is that given in the Zaitlevsky Report [31], especially pages 4-29 to 4-36. The Zaitlevsky account has served as a guide for the present account.

Prior to 1867, and for years thereafter, the elliptical area shown in [3], considerably smaller than at present, was known as the White Lot. This area, leading down to Tyber Creek, had served as a campground and infirmary during the Civil War. By March 1867, Congress had transferred responsibility for the White Lot to the Chief Engineer of the Army.

In 1877, Thomas Lincoln Casey (1831-1896), then a Lieutenant-Colonel, was placed in charge of the grounds. Casey [7] writes, in a paragraph on Grounds South of the Executive Mansion: "The improvement of the southern portion...progresses very slowly; and consists mainly in grading such soil, clay, and refuse material as may be deposited therein by contractors and others, who find this a convenient dumping ground. To properly complete the grading ...would require a large appropriation to purchase a sufficient quantity of soil..." A year later, under the same heading, Casey writes,

This reservation, known as the White Lot, has been filled to within about 3 feet of grade, over a large portion of its surface, the sunken parts being along Seventeenth street. A plan of improvement after the project of Downing and as approved by President Fillmore was commenced during the year. It consists substantially of a large field or parade some 17 acres in extent, and elliptical in shape, which will occupy the center of the reservation... . The construction of the parade will be continued the coming year.

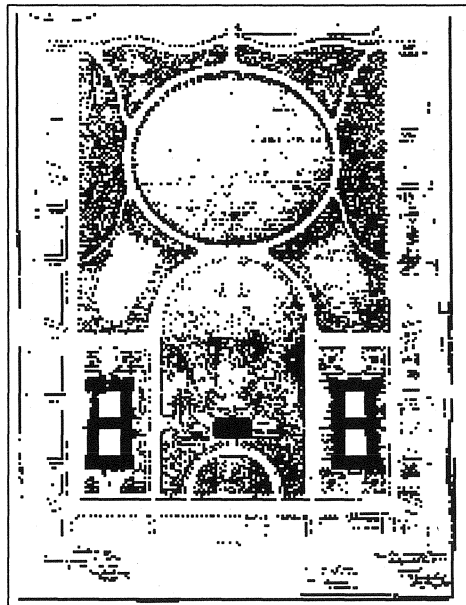
Zaitlevsky [31, pp. 4-30] notes that Casey's 1878 report [6] seems to be the first place where the word "ellipse" appears. In his 1880 report, Casey [7] continues:



These grounds are being laid out in accordance with a general project for the improvement of the contiguous reservations south of Pennsylvania avenue, as designed by A. T. Downing in 1851-'53. The main feature for this lot is an open elliptical field, covering some 17 acres in the center of the square, the borders of the square to be planted thickly with trees and shrubbery. During the past year very satisfactory progress has been made with this improvement, and the grading, soiling, and seeding of the central ellipse has been completed, excepting a small area in its center, left by reason of an incompleting sewer in charge of the District Commissioners. The large depression along Seventeenth street was filled to grade, mainly from the excavations of the cellar of the north wing of the State, War, and Navy Departments building...

The total quantities of materials deposited in the lot during the year were as follows: 56,980 cubic yards of earth; 9,053 cubic yards of soil; and 2,228 cubic yards of gravel. During the coming year it is expected to complete the grading, soiling, graveling, and planting of the eastern half of the lot, giving to that portion something of a park-like appearance.

The next annual report [23], written by Casey's successor, states that "The ellipse now presents an unbroken lawn surface. The total quantities of materials received and deposited during the year were 18,943 cubic yards of earth, 7,225 cubic yards of soil, and 3,637 cubic yards of gravel...During the coming year it is expected to complete the grading, soiling, graveling, and planting of the western half of the grounds..."



**Figure 7.** Map dated September 29, 1877, showing several signatures of approval

A map entitled "Ellipse at White Lot" is signed "J. S. 6/9/82". This map by John Stewart, employed under Casey's direction, includes measurements: margin, 3092' 0"; red line 4 feet out from margin, 3116' 4"; green in center of drive, 3260' 9". This map, with scale 80' = one inch, is kept in a vault in the Library of Congress.

Casey's Annual Reports to Congress, cited above, do not reveal the manner in which the Ellipse was laid out. In an effort to learn the manner of layout, two possible archival sources were consulted: the Office of History of the U. S. Army Corps of Engineers, and the Society for Preservation of New England Antiquities (SPNEA).

A historian at the Office of History wrote that the archival collection there does not have much about Casey or the Ellipse. However, the Office does have a biographical sketch [27] of Casey, who became Chief Engineer in 1888.

SPNEA has a 72 manuscript boxes of Thomas Lincoln Casey Papers, with Accession number MS 84-2160. These contain a great deal pertaining to Casey's leading roles in the completion of the Washington Monument and the design and building of the Library of Congress. However, a lengthy search did not locate anything pertaining to the method of layout of the Ellipse

Casey's father, General Silas Casey, was a graduate of the U. S. Military Academy at West Point, New York, which had been founded by Thomas Jefferson in 1802. Thomas Lincoln Casey graduated from West Point, first in the class of 1852. His son, also named Thomas Lincoln Casey, graduated from West Point and became a renowned coleopterist.

## 7 Conclusions

There are two aspects of our knowledge about the Ellipse that leave much to be desired. One is the absence of records indicating how the Ellipse was laid out, and whether this layout was undertaken after the extensive grading was completed. The other is the high level of precision of the four measurements stated in Section 2. Communications requesting clarification were subject to security screening, and the recent NPS Survey [21] was obtained only after considerable security checking and several months of waiting. The final communication states that the precise measurements "were derived using the measuring tool in MicroStation V8, from an electronic compilation of existing conditions computer generated survey files. Locations of the minor and major axes, as well as the perimeter delineation were estimated using these files as a base."

The measuring tool calculates area of a simple closed curve by approximating the curve as a polygon; thus, both area and perimeter are easily calculated from measured lengths of straight line segments, and the approximations are close if there are many well spaced points on the curve selected as vertices of the polygon. For the Ellipse, these points were on the face of the inner curb of the driveway. Unfortunately, the number of such points and information about their spacing was not available, so that we are left to trust, on the basis of the remarkable precision of the reported area (751071.67 square feet), that there were many such points and that the reported value really is a close approximation of the actual area of the Ellipse. Note that the method just described pertains to the plane curve determined by the curb, as suits our purposes in this article; in other words, the slight internal "hilliness" of the Ellipse, indicated by many internal elevation marks on Survey [21], was disregarded in the measurement of area.

One other feature of the Ellipse is that it is not horizontal, but lies on a slope from the White House down to the level of the Potomac River. The elevations at the top of the granite curb at the four vertices and near the center of the Ellipse are as shown here:

	(north) 24.60'	
(west) 18.79'	(center) 17.89'	(east) 14.99'
	(south) 10.80'	

It is hoped that appropriate measurements will someday enable a closer examination of this question: "How close is the Ellipse is to a mathematical ellipse?" A future researcher may wish to know that "the most recent survey" cited in Section 2 may differ from the 2001 Survey [21]. The latter consists of eleven  $2' \times 3'$  sheets (i.e., 66 square feet) with many details, but it does not show the four measurements cited in Section 2.

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