ISAMA
The International Society of the
Arts, Mathematics, and Architecture

BRIDGES
Mathematical Connections
in Art, Music, and Science

Cayley Tables as Quilt Designs

Gwen L. Fisher and Elsa Medina
Mathematics Department
California Polytechnic State University
San Luis Obispo, CA 93407, USA
E-mail: glfisher@calpoly.edu
emedina@calpoly.edu

1. Introduction

The algebraic structure of Cayley tables can be analyzed using color-coded tables. In our presentation we would like to discuss the use of colors to display characteristics of groups and monoids. The designs that are constructed by these color-coded tables provide the basis of several quilt designs.

2. Definitions

In order to talk about the structure of Cayley tables, we begin with some definitions.

2.1 Monoid. A set G together with a binary operation * is called a *monoid* if
(a) G is associative under * (For every a, b, and c in G, we have $a * (b * c) = (a * b) * c$.)
(b) G has an identity element (There exists an element e such that for any a in G, $a * e = e * a = a$.).

2.2 Group. A set G together with a binary operation * is called a *group* if
(a) G is a monoid
(b) Each element in G has an inverse (For each a in G, there exits an element b such that $a * b = b * a = e$.

2.3 Commutative Property. A binary operation * on a set G is said to be *commutative* if for every a and b in G, $a * b = b * a$.

3. Colored Cayley Tables

The Cayley tables describing monoids have many beautiful symmetries and patterns embedded within them. Using colors to represent individual elements in the set, instead of more traditional symbols such as letters or numbers, allows one to easily recognize the properties of the groups or monoids. For example, from the Cayley table of Z_6 under multiplication (see Figure 1) one can see that the table describes a monoid that is commutative since the table has reflection symmetry about the main diagonal.

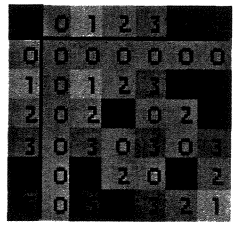

Figure 1: *The Cayley Table for Multiplication on Z_6*

The reflection along the main diagonal is also apparent in the multiplication table of Z_7 (see Figure 2).

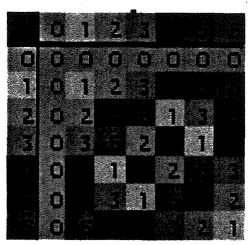

Figure 2: *The Cayley Table for Multiplication on Z_7*

In contrast to the commutative group of Z_7 under multiplication (see Figure 2), the group S_3 is not commutative. Therefore, the Cayley table of S_3 in Figure 3 does not have reflection symmetry about the main diagonal.

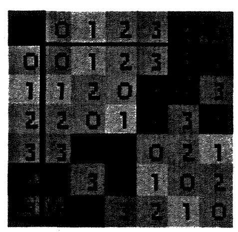

Figure 3: *The Cayley Table for S_3*

Colored Cayley tables can help show the difference between a monoid and a group. For example, the multiplication table of Z_7 shows that Z_7 is a group (see Figure 2). To see this, notice that every column and every row has every color in the table. Similarly, S_3 is also a group (see Figure 3). In contrast, in the algebraic structure in Figure 1, some elements do not have an inverse.

In our presentation we plan to discuss Cayley tables of other algebraic structures. We will discuss how properties of the algebraic structures are reflected in the properties of these tables, including the visual differences between tables made with even, odd, prime, and composite numbers of elements, as well as cyclic versus noncyclic groups. Additionally, we plan to present several of these tables as quilts made from fabric.