Mathematical Sculpture Classification

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Abstract

The introduction of mathematical sculpture in advanced education needs a taxonomy to classify all the different types of sculpture. From our point of view, this classification has never been arranged deeply. This paper is a first attempt to that classification. We expect to receive suggestions from the Art and Mathematics community in order to start a work that we will take away during the next two years and whose first step is given with this paper. As a preliminary starting point we have suggested the following nine categories for mathematical sculpture: I. Polyhedral and classic geometry, II. Non-oriented surfaces, III. Topological knots, IV. Quadrics and ruled surfaces, V. Symmetric and modular structures, VI. Boolean operations, VII. Minimal surfaces, VIII. Transformations IX. Others.

1. What is mathematical sculpture?

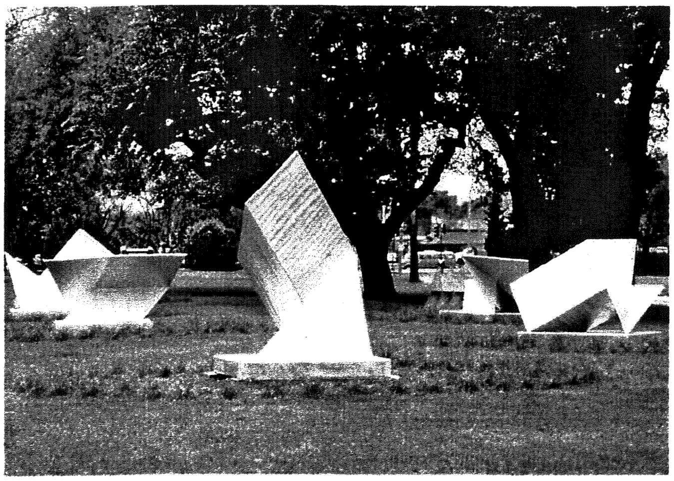
It is surprising that after so many Art & Math, ISAMA and Bridges Conferences, only a reduced number of participants has previously discussed a classification for mathematical sculpture and, as far as we know, never this task has been made with enough accuracy so as be taken as a guide for educators or people deeply interested on the topic. From our point of view, the introduction of mathematical sculpture in schools or universities requires a classification to frame the sculptures in different categories according to the mathematical concepts that they represent. Without that classification, mathematical sculpture can only be introduced in the classroom as a mere slideshow session that, although can be interesting, is not appropriate to teach this topic into high-level courses.

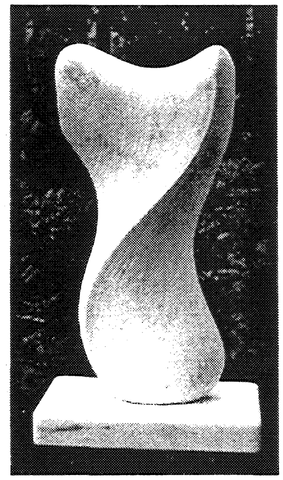
It is not an easy task to establish a set of categories for mathematical sculpture. In fact the first question we should make is: what is mathematical sculpture? The answer is simple: the sculpture that uses mathematics for its conception, development or execution, understanding mathematics in its wider sense, from the simplest geometry to the most sophisticated calculus theorems.

But even with a simple definition like this, its application in not as easy. There is no problem when we consider sculptures showing explicit mathematics, but there is also implicit mathematical sculpture that does not strictly shows mathematic elements, but uses mathematical ideas, for example Arthur Silverman's tetrahedral sculptures. Even understanding that this implicit sculpture needs to be considered for classification, it is difficult to trace a frontier between what is going to be to accepted, or not, as mathematical sculpture.

2. A proposal for a mathematical sculpture classification

There are several ways to design a mathematical sculpture classification. One interesting way is to define categories according to the construction materials. For instance, carved wood and stone sculpture usually emphasizes curvature (see the work of Brent Collins or Helaman Ferguson); meanwhile welded steel sculpture tends to produce polyhedral like sculpture with vertex and aristae, -for example Bruce Beasley's work. In contrast, the use of concrete produces sculptures with architectural properties; the best example is Eduardo Chillida.





Figures 1 & 2: *Two examples of implicit mathematic sculpture that do not strictly show mathematical elements, but mathematical ideas: "Attitudes" by Arthur Silverman and "Torso" by Nat Friedman.*

Many characteristics may be used to classify sculpture, but the classification we are proposing is based on different areas of mathematics. Note that although some of the areas represent a very small portion of mathematics, for example non-oriented surfaces, they are one of the most prolific ways to produce mathematical sculpture.

The categories are the following:

I. **Polyhedral and classic geometry**

II. **Non-oriented surfaces**

III. **Topological knots**

IV. **Quadrics and ruled surfaces**

V. **Symmetric and modular structures**

VI. **Boolean operations**

VII. **Minimal surfaces**

VIII. **Transformations**

IX. **Others**

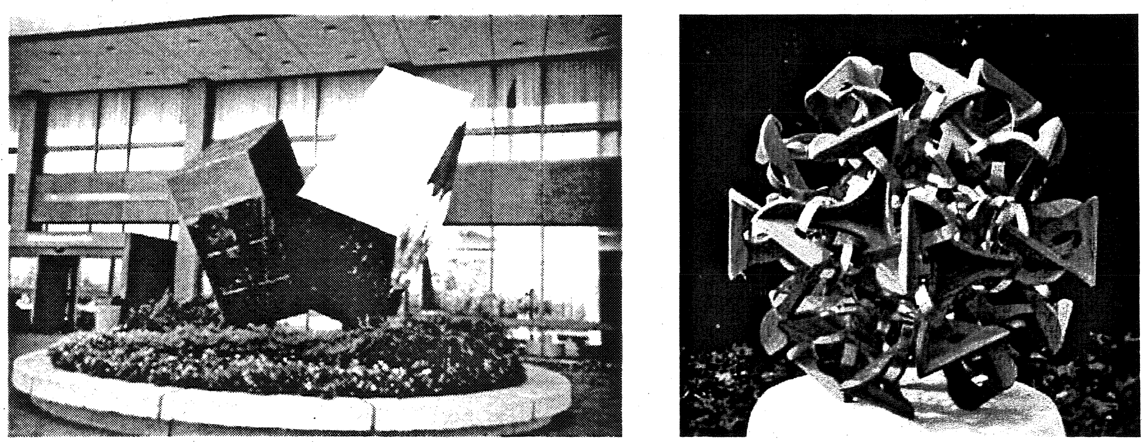
Of course we do not pretend to create a definitive classification with all categories of mathematical sculpture, just a starting approach that we expect could be modified according to the suggestions of mathematicians and artists. We must also consider that the main objective of this classification is educational, to allow the introduction of mathematical sculpture in education.

3. Polyhedral and Classic Geometry

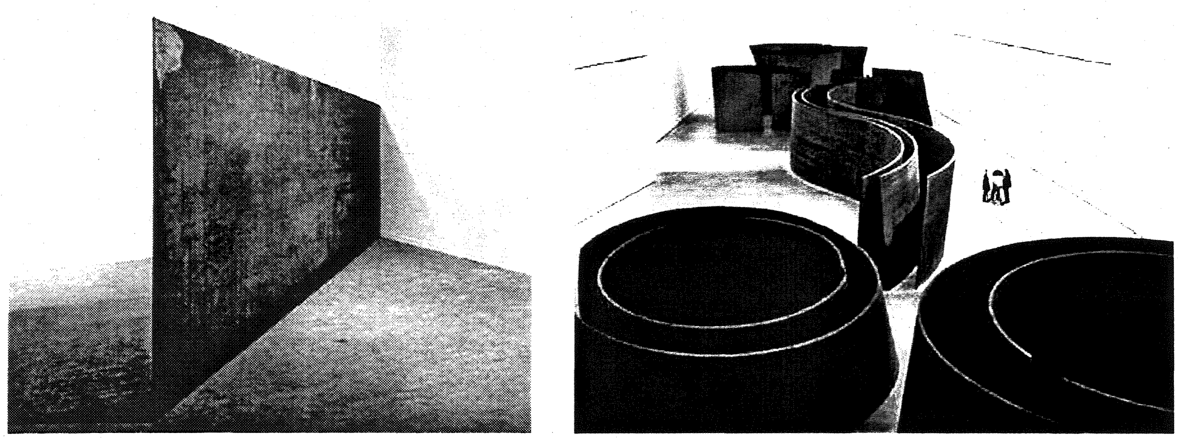
Several polyhedra, like dodecahedra or icosahedra, have been widely used in mathematical sculpture. The work of George W. Hart is the best example of the huge number of variations that polyhedra allow both on its conception and its material execution by combining different materials.

Simpler elements from classic geometry, like spheres, torus or cubes, amongst others, are also used in mathematical sculpture. In this case the simplicity of the geometry is complemented with added values for the sculpture, like impacting colors, textures, illumination, positioning, for example Carl Zollo's Split Cube.

During the ISAMA 99 visit to the Bilbao Guggenheim Museum, we had the privilege to admire Richard Serra's exhibition composed by very simple surfaces, from a steel plane to his torqued ellipses, (composed by conic sections by rotation of an ellipse). One of his most famous sculptures, "Snake", is a defiance of gravity based on a set of three third degree polynomial functions. Although Serra's work abolishes the minimalist sense of scale, it could be considered as an example of mathematical sculpture minimalism.



Figures 3 & 4: *"Split cube" by Carl Zollo and "Fire and Ice" by George W. Hart.*

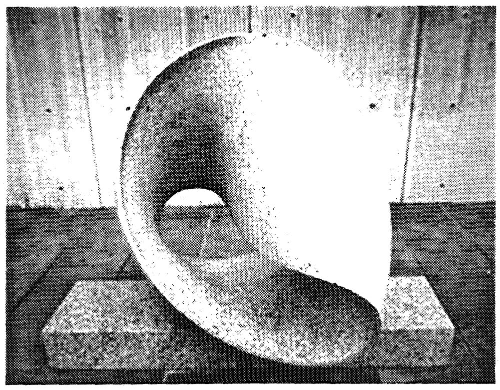
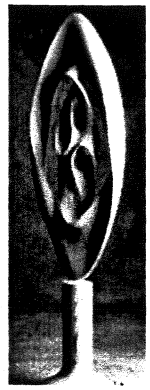
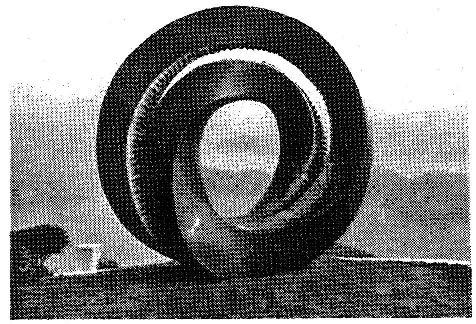


Figures 5 & 6: *Richard Serra's work in the Bilbao Guggenheim Museum.*
An impressive example of mathematical sculpture minimalism.

4. Non-oriented Surfaces

The Moebius strip is one of the first non Euclidean objects to be represented in sculpture. The Swiss sculptor Max Bill made several of these pieces fascinated by the geometry generated by changing orientation, this is, the outside surface comes inside and then outside, being impossible to determine which part of the surface lies inside or outside the sculpture.

Brent Collins has also developed several models showing that characteristic. The Japanese sculptor Keizo Ushio has performed dozens of non-oriented surfaces based mainly on single and double Moebius strips.

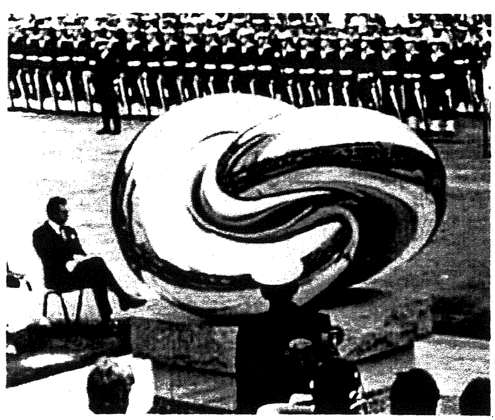




Figures 7, 8 & 9: *Non-oriented sculptures by Max Bill, Brent Collins and Keizo Ushio.*

5. Topological Knots

Mathematicians have studied knots extensively for the last hundred years. Recently the study of knots has proved to be of great interest to theoretical physicists, molecular biologists and also artists! One of the most peculiar things which emerges as you study knots is how a category of objects simple as a knot could be so rich in mathematics and art connections.

Almost all the important mathematician sculptors have used knot topology in their works sometimes. One of the sculptors who have made an extensive use of knot theory is John Robinson.

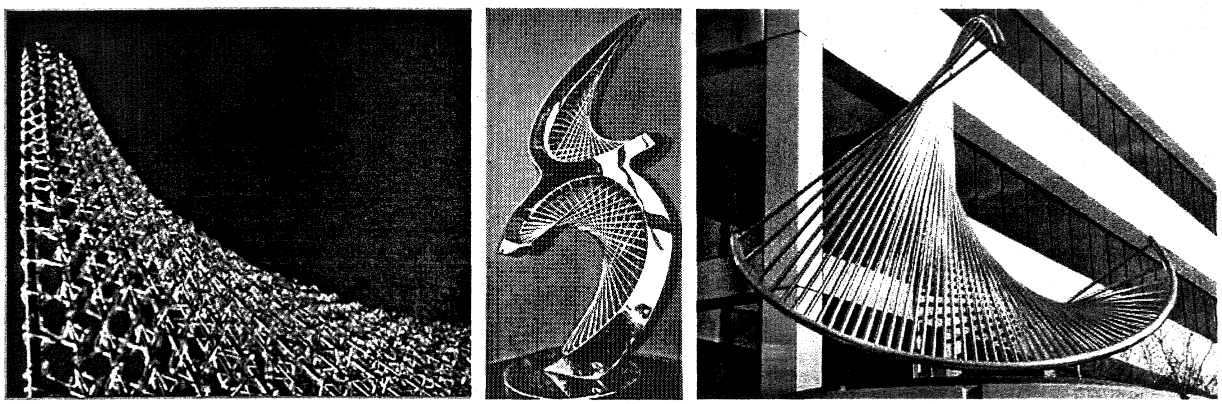



Figures 10 & 11: *"Bothers of Friendship" and "Immortality" by John Robinson.*

6. Quadrics and Ruled Surfaces

A surface defined by an algebraic equation of degree two is called a quadric. Spheres, cones, cylinders, hyperboloids (of one and two sheets), paraboloids and hyperbolic paraboloids are quadric surfaces. Ruled surfaces are surfaces generated by straight lines. These surfaces have been used as inspiration for many artists and architects.

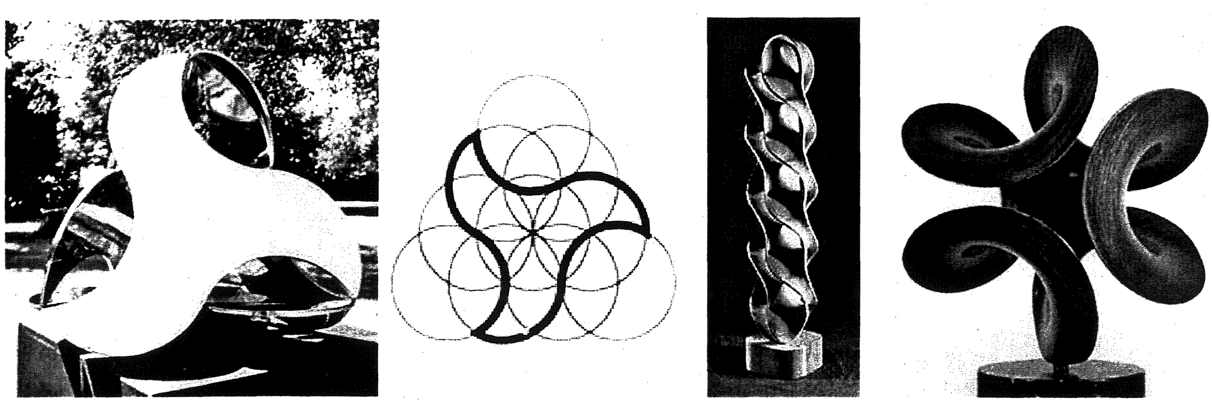
The hyperbolic paraboloid, also known as saddle, is both a quadric and a ruled surface. Due to its interesting geometric properties it is probably the most represented surface from both families.



Figures 12 & 13: *"Transformer" by Aaron Fein, "Hyperbolic Paraboloid" by Jerry Sanders, and "Hyperbolic Maze" by Charles Perry.*

7. Symmetric and Modular Structures

This category refers to the design of any sculpture composed of separated components that can be connected together. The beauty of modular sculpture is that you can be replace or add any component (module) without affecting the rest of the system. The opposite of a modular architecture is an integrated sculpture, in which no clear divisions exist between components. Sculptures based on tilings, hyperbolic tessellations or symmetry groups are also included in this category.

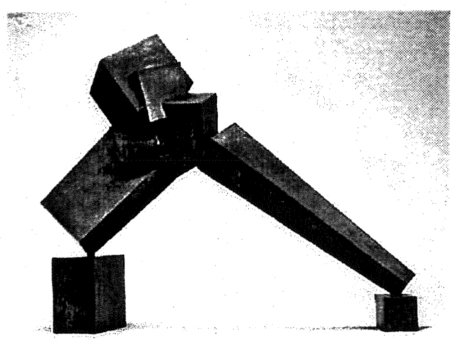
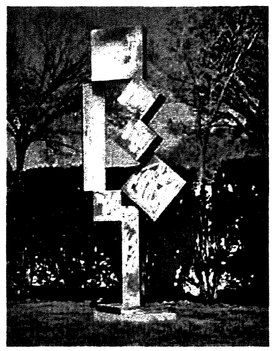
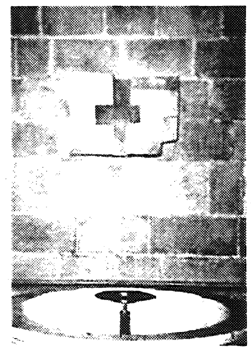


Figures 14, 15 & 16: *"Prometheus' Heart", a sculpture by John Robinson based on the tiles of the Alhambra, fitted together as tetrahedral, a modular spiral by Brent Collins and "Arabesque" by Robert Longhurst.*

8. Boolean Operations

A Boolean operation is an operation that follows the rules of Boolean algebra and produces one of two values: true or false. Applied to sculpture, Boolean algebra can describe how objects ensemble together their volumes or one carves into the other producing emptiness. Boolean operation OR represents the union and the AND operation, represents the intersection. There are other variants for this rules, based on inversion and exclusion of both rules than are also used.

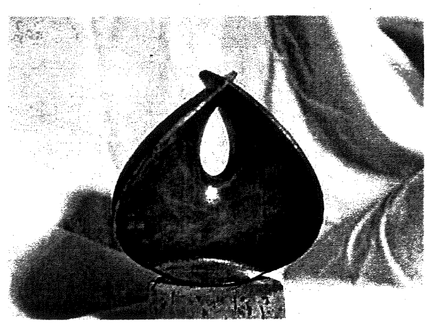
Sculptors like Bruce Beasley or David Smith use the OR rule in different ways. Although in both cases their work is non-modular, sometimes almost random, Bruce Beasley intersects the volumes of his sculpture, meanwhile David Smith simply join them, without intersection. Other sculptors like Eduardo Chillida, make a wide use of the Boolean exclusion in his sculpture. Two examples of Chillida's technique are shown below, featuring two crosses, the first one in the Church of Santa Maria and the second one in the Cathedral of the Buen Pastor, both in San Sebastian, Spain.


Figures 17, 18, 19 & 20: *"Intersections II" by Bruce Beasley and "Cubi XII" by David Smith. On the right, two crosses made by Eduardo Chillida using Boolean intersection and exclusion.*

9. Minimal Surfaces

A minimal surface is a surface that is locally area minimizing, that is, a small piece has the smallest possible area for a surface spanning the boundary of that piece. Minimal surfaces necessarily have zero mean curvature, like soap films. Helaman Ferguson has several examples of this kind of sculpture. His most famous works are based on the Costa Surface, that takes its name from the Brazilian mathematician Celsoe Costa, who, inspired by the twirling of a street dancer's skirt, formulated the equations describing a minimal surface with holes.

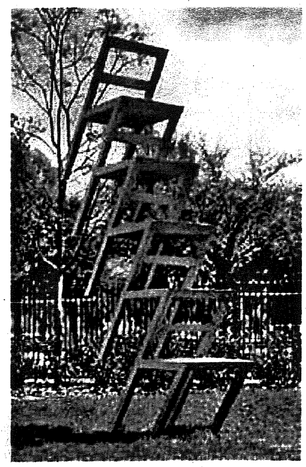



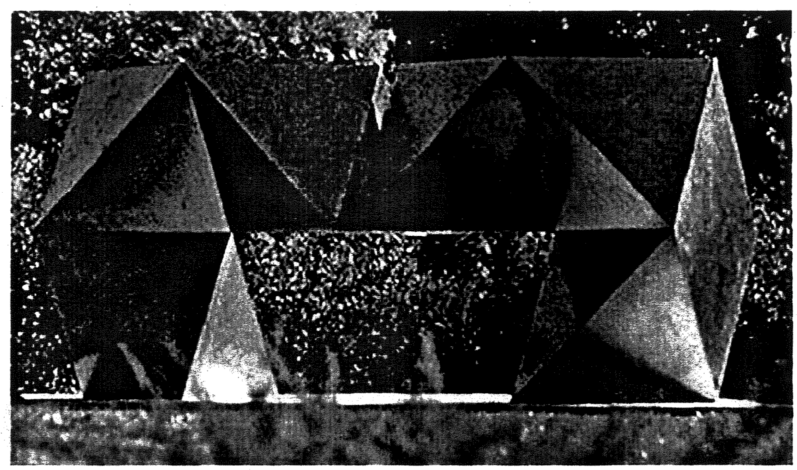
Figures 21 & 22: *Costa Surface by Helaman Ferguson and Enneper's minimal surface by Stewart Dickinson.*

10. Transformations

Many times the mathematics and the beauty behind a sculpture do not lie in the object itself, but in the transformations that are applied to it. Simple mathematical transformations like homothety, decomposition or rotation applied one or several times may produce very interesting results.

"Chair Transformation", by Lucas Samaras, is a good example of how a good sculpture can be produced using two transformations -rotation and translation- applied to an ordinary object, a chair, without any initial artistic value. In the same way, "Bisected Dodecahedron", by Charles Perry, uses a very clever decomposition of a dodecahedron into two pieces. This sculpture has another added value, as it can support following transformations by placing the resulting pieces in different positions.





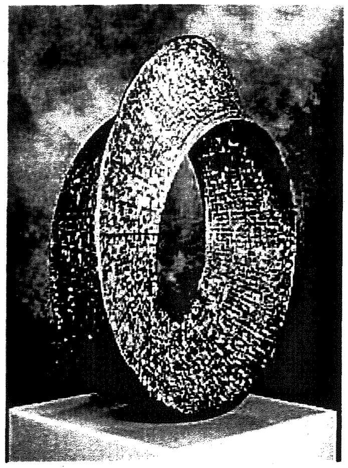
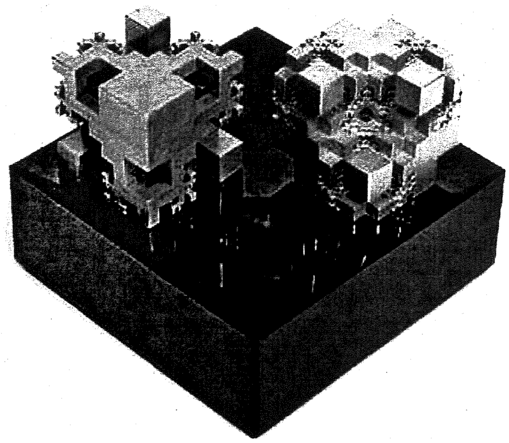
Figures 23 & 24: *"Chair Transformation" by Lucas Samaras and "Bisected Dodecahedron" by Charles Perry.*

11. Others

It is almost impossible to fit all the mathematical sculptures in such a reduced number of categories. Some sculptures do not have an adequate category to be classified. On the other hand it might also happens that a sculpture can be fitted in several different categories. For these cases, an extra group has been created in this classification.

An example of this difficulty of classifying sculptures can be understand analyzing Helaman Ferguson's fantastic sculpture "Umbilic Torus NC". Although at first sight it can be presented as a non-oriented surface, probably the most distinctive characteristic of this sculpture is its surface texture formed by a computer milling operation that uses the fifth stage of Hilbert's version of Peano's surface filling curve.

In the same way, Bathsheba Grossman has designed a perfect cube of silver. But its taxonomy is difficult, because it is divided into two complementary fractal sets, on a base of black marble. Ignoring the Pauli exclusion principle for a moment, the pair would lock together into a solid cube, but in this case the fractal nature of the piece results determinant for its classification.

Figures 25 & 26: *"Umbilic Torus NC" by Helaman Ferguson and "The Unit Cube" by Bathsheba Grossman.*

12. Conclusion

This taxonomy of mathematical sculpture does not pretend to be the final one, just a first attempt for a classification that may be successfully used in advanced education. We expect and encourage the delegates of the ISAMA BRIDGES conferences to bring suggestions and advices for this work.

For the next two years, members from the Universities of the Basque Country and Valencia Polytechnique will work together on this project whose first step has been given with this paper and hopefully will be presented to the art and mathematics community after its conclusion.