

Lengthening, Expansion and Torsion: Three Geometric Operations for Aesthetics Creation

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Abstract

From an hexagonal prism and with a series of control points we have been changing the configuration of that series until a sequence of shapes have been modelled to be used in architecture and civil engineering.

The process that provides the point nets allows us to generate some models where we can choose the desired length, expansion and torsion of the original prismatic shape.

By using convenient modelling parameters we can emulate all these geometric operations.

The examples shown in this presentation have been generated by using the Lagrange interpolation, and the algorithms have been written with Mathematica.

Introduction

Modelling a shape from a block of any material includes a series of operations that deform the original shape and sequentially the desired shape is being obtained.

Basically, the operations used to transform the aspect of a shape can be classified in four main operations: lengthening, expansion, flexion and torsion. We will use these operations in a double sense (e.g.: to length or to shorten) .

The four operations of lengthening, expansion, flexion and torsion provide important aesthetic possibilities to shape modelling.

In this presentation we have worked with the lengthening, expansion, and torsion operations, because they are the most convenient in the design of vertical shapes, as used in architecture and civil engineering. We start with a net of quadrilateral which are deformed until the proposed designs have been obtained.

With the net of interpolation points, used in lagrangian interpolation [1], is possible to use these basic operations [7] with facility and efficiency.

Therefore we can develop a very efficient modelling process for shapes[2] [3]. To do this, we will set a series of modelling parameters for the net of passing points of the surface in such a way that, with the lengthening, expansion and torsion operations, the deformation of the net will be possible.

Next, using this method, two examples are shown. In the first one a glass roof of umbrella type is designed by using the above mentioned geometric operations on a regular hexagonal prism. In the second example, the same prism has been deformed to get an hexagonal roof [2].

Analytical Definition of a Quadrilateral

The curve $P(t)$ (Figure 1) goes through the points P_0 and P_1 and its vector equation is given by :

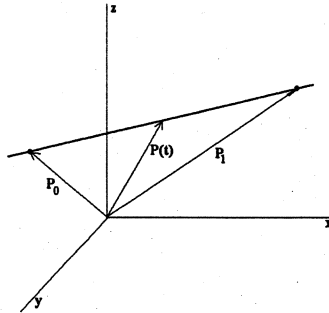


Figure 1

$$P(t) = f_0(t) P_0 + f_1(t) P_1 \quad (1)$$

where

$$\begin{aligned} [f_i(t)]_{t=j} &= 1 & \text{if } i = j \\ [f_i(t)]_{t=j} &= 0 & \text{if } i \neq j \end{aligned} \quad (2)$$

are the Lagrange polynomials.

The tensor product of equation (1) by itself gives:

$$P(t,u) = acP_{00} + bcP_{10} + adP_{01} + bdP_{11} \quad (3)$$

where:

$$a = -t+1 ; b = t ; c = -u+1 ; d = u \quad (4)$$

are the functions called previous functions.

The expression (3) is the vector equation of the surface (Figure 2) passing through the four points [5] P_{00} , P_{10} , P_{01} , P_{11} . This surface is a warped quadrilateral [4] (hyperbolic paraboloid) whose vertices are those points P_{ij} .

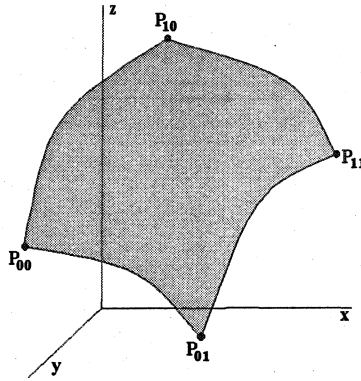


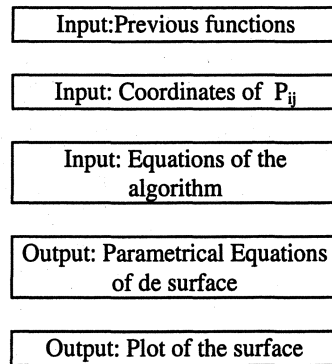
Figure 2

The parametric equations of the surface are:

$$\begin{aligned} x &= ax_{00} + bx_{10} + adx_{01} + bdx_{11} \\ y &= ay_{00} + by_{10} + ady_{01} + bdy_{11} \\ z &= az_{00} + bz_{10} + adz_{01} + bdz_{11} \end{aligned}$$

Computing Resolution with Mathematica

The scheme will be [6] :



Mathematica Codes Usados En El Proceso De Diseño

The main Mathematica codes used in the process would be the following:

Input of the previous functions:

$$a=-t+1, b=t, c=-u+1; d=u;$$

Input of the control points of the surfaces' coordinates (fill with the numerical data)

$$x_{ij}=; \quad y_{ij}=; \quad z_{ij}=;$$

Output of the parametric equations of each one of the patches that compose the design:

```

xi= a*b*x00+b*c*x01+b*d*x11; Expand[xi];xi= Simplify[xi]
yi= a*b*y00+b*c*y01+b*d*y11; Expand[yi];yi= Simplify[yi]
zi= a*b*z00+b*c*z01+b*d*z11; Expand[zi];zi= Simplify[zi]

```

Output of the 3D graph of each one of the patches that compose the design:

```
si= ParametricPlot 3D[{xi,yi,zi},{t,0,1},{u,0,1}]
```

Output of the total 3D graph:

```
Show[ s1, s2,.....sn]
```

Design of a Regular Hexagonal Prism

The design of a regular hexagonal prism (Figure 3) of side length L and height H can be made from the design of each one of the faces by applying the above mentioned algorithm.

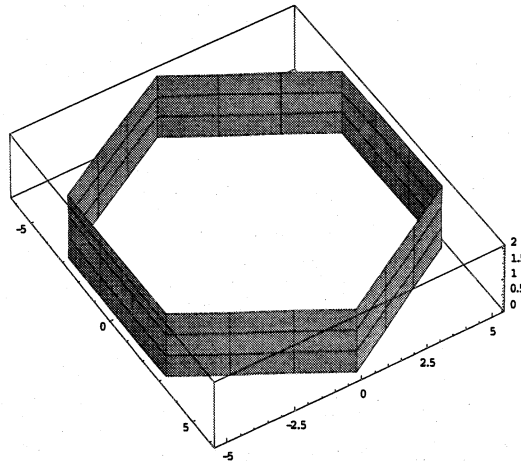


Figure 3. Regular hexagonal prism

The height of the prism can be adjusted with parameter H . To get an expansion or contraction of the prism we should increase or decrease the value L in the hexagon.

About the torsion, we can get it by rotating the upper face around an orthogonal axis and passing through its center. The rectangles that form the sided faces are transformed into quadrilaterals defined by the vertices in the inferior face and the other two vertices in the upper face.

Example of Transformation of a Regular Hexagonal Prism into a Glass Roof of Umbrella Type

We start from a regular hexagonal prism, which we divide in three regions, each one being characterized by two parameters: length and side. By playing with both parameters and with the rotation between basis in each one of these regions of the prism we got the glass roof (Figures 4 and 5).

The process starts from the algorithm above defined. This has been implemented by using Mathematica, in order to obtain a program that generates virtual models in three dimensions that can be manipulated to obtain different configurations of this glass roof.

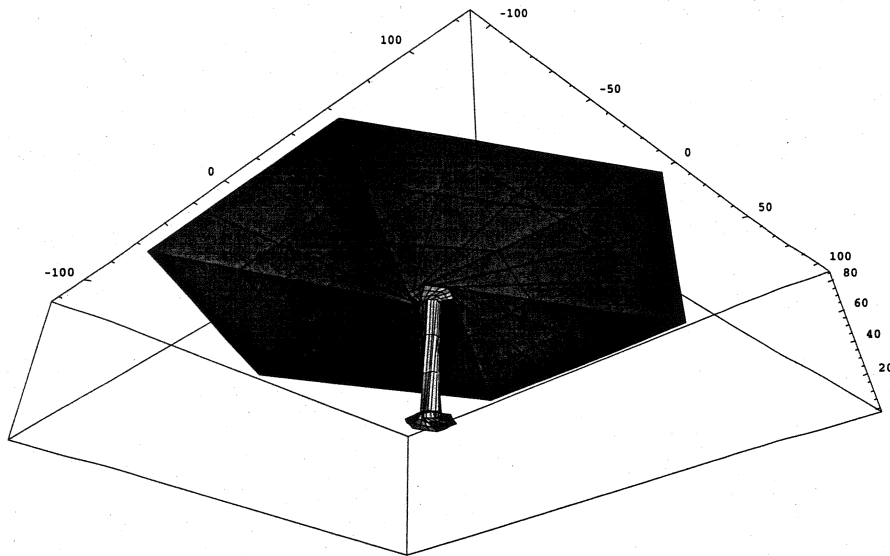


Figure 4. Glass roof of umbrella type. Perspective

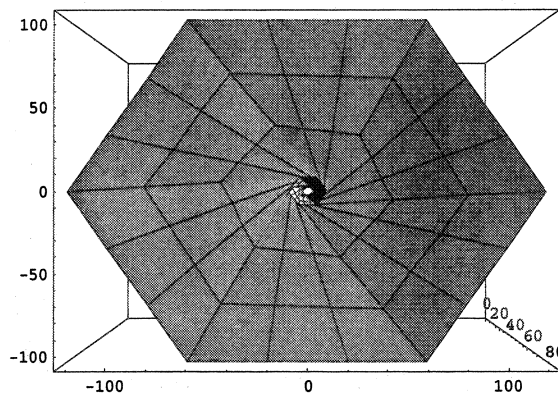


Figure 5. Glass roof of umbrella type. Ground plan

Example of Transformation of a Regular Hexagonal Prism into an Hexagonal Roof

In this example, it has been obtained a very short roof, that is formed by a series of pieces, namely non-plane quadrilaterals.

Similar to the preceding case, the original regular hexagonal prism has been divided in some regions. Again by working on the length, side and rotation between basis with respect to the axis of the prism, we got the modeled roof (Figures 6, 7 and 8).

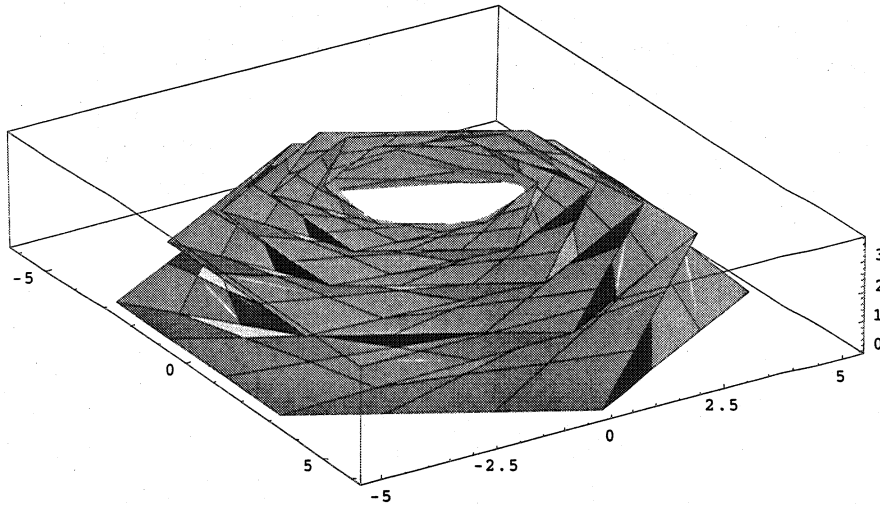


Figure 6. Hexagonal roof. Perspective

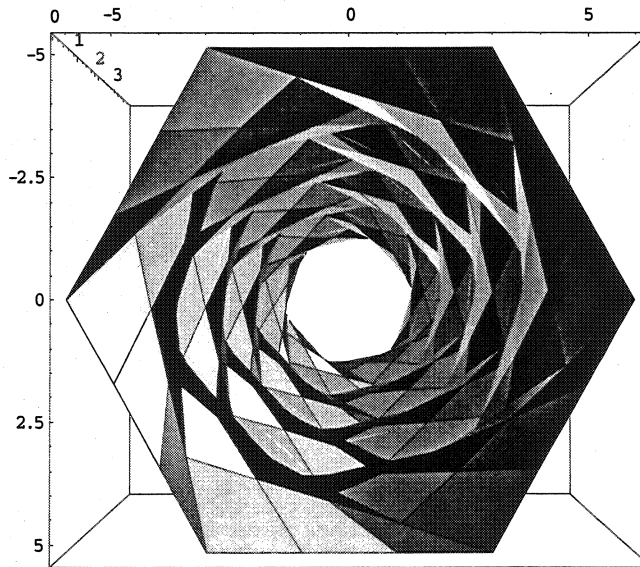


Figure 7. Hexagonal roof. View from the inside

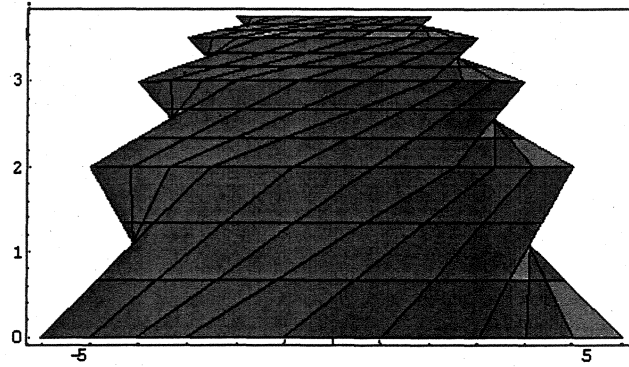


Figure 8. Hexagonal roof. View from front

The algorithm and its implementation are the same, as shown before, and the figures that follow are perspectives from the 3D model used to configurate the roof.

Conclusions

The operations of lengthening, expansion and torsion are easily done in the modelling process by using Lagrangian interpolation. This is due to the fact that the parametric equations of the surfaces obtained are linear functions of the coordinates of the interpolation points.

By using these operations we can get very satisfactory solutions, both aesthetic and functionally, for forms that are used in architecture and civil engineering.

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