Möbius Concepts in Architecture

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Abstract

The familiar one-sided Möbius Band has inherent properties that can be expressed in terms of architectural form, surface, and space. This paper first investigates the basic mathematical form of the Möbius Band and its extension to a Prism and then translates these properties into a series of architectural entities differentiated on the basis of human scale.

1. Introduction

Over the years, many architects have based their forms on the Platonic solids. Today we can begin to also investigate more advanced surfaces and solids based of pure mathematical descriptions. For example, imagine if a surface or spaces could be constructed with a continuous twist in it. This would generate a paradoxical geometry similar to that of the Möbius Band.

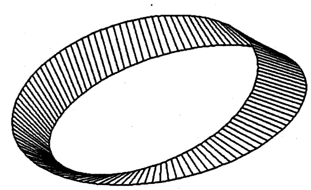

Figure 1: *The Möbius Band.*

The Möbius Band is an example of one-sided surface in the form of a single closed continuous curve with a twist. A simple Möbius Band can be created by joining the ends of a long, narrow strip of paper after giving it a half, 180°, twist, as in Figure 1. An example of a nonorientable surface, this unique band is named after August Ferdinand Möbius, a German mathematician and astronomer who discovered it in the process of studying polyhedra in September 1858. But history reveals that the true discoverer was Johann Benedict Listing, who came across this surface in July 1858 [1].

Since then, the Möbius Band has been used in various realms including mathematics, art, engineering, science, magic, music and literature, either in its true form or as a metaphor. The evolution of the Möbius Band as an art form was first seen in the "Endless Ribbon", Figure 2a, a granite sculpture by Max Bill [2] in the year 1935. M.C.Escher [3] treated the Möbius as a paradoxical object and painted a number of variations of the band, the most popular one being the "Möbius Strip II" with the nine red ants that seem to crawl forever. Many other artists sought art in the form of this band including Brent Collins [4], Helaman Ferguson [5], Cliff Long [6], Charles Perry [7], John Robinson [8], Keizo Ushido [9] and Robert Rathbun Wilson [10]. More recently, the Möbius Band has also appeared in the form of three-dimensional virtual computer-aided sculptures like the "Möbius Helix", Figure 2b, by Tom Longtin [11]. In all of these example the Möbius form is explicitly translated into the art itself and is easily seen.

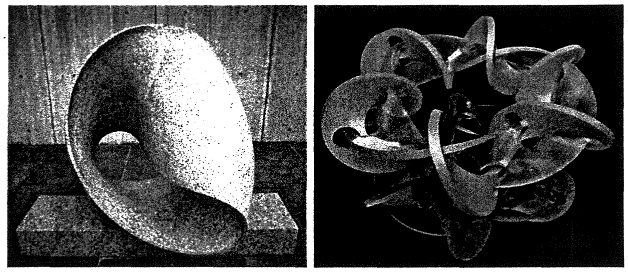

Figure 2: *Möbius in art: (a) Max Bill's "Endless Ribbon" and (b) Tom Longtin's "Möbius Helix".*

In terms of architecture, Peter Eisenman [12] pioneered the Möbius form by roughly translating it into the "Max Reinhardt Haus" building, Figure 3a. He slices the form at the ground, thus failing to achieve the visual continuity of the Möbius as a whole. The concept of the Möbius Band also appears to be used in the "Möbius House Het Gooi", Figure 3b, by UN Studio. According to UN Studio's Ben Van Berkel [13], the mathematical model of the Möbius is not literally transferred to the building, but is conceptualized and can be found in architectural ingredients, such as the light, the staircases and the way in which people move through the house. Others who were inspired by the idea of the Möbius Band include architects Zaha Hadid [14], Stephen Perrella [15] and Gonzalo Valez Jahn [16] and more recently, engineer Helmut Cerovsek [17] and computer scientist and artist Carlo Sequin [18].

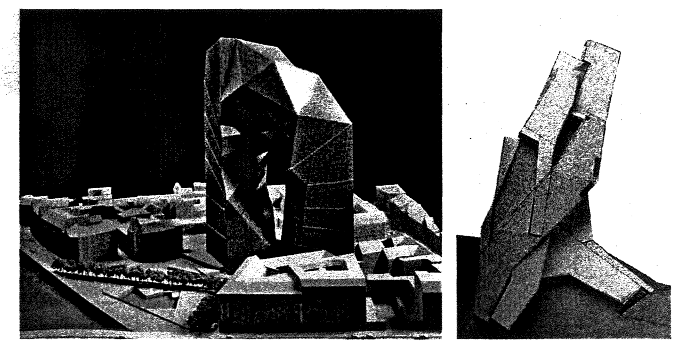

Figure 3: *Möbius in architecture, (a) Peter Eisenman's " Max Reinhardt Haus" (b) Plan of UN Studio's " Möbius House Het Gooi".*

The Möbius Band has several interesting properties that can be interpreted into architecture. Some of them can be achieved spatially while others can be achieved in terms of form and structure. The infiniteness and paradox of the Möbius can be demonstrated in terms of an enclosure in which one would walk around and feel the spatial twist without having to walk upside down. The continuity, twist and visual dynamism can be generated in terms of form and space where a Möbius Band would split into a flat surface, on which one could walk, and a twisted Möbius surface that could be treated as a wall or a ceiling or even the floor at certain instances. Another unique property of the band that would be very interesting when expressed in architecture is the concept of transformation, the event of the inside becoming the outside and vice versa. Considering these properties we proceeded to generate a series of variations of the Möbius Band and the Enclosure.

2. Generation and Development of Möbius Concepts

2.1. Digital Sketches of a Möbius Band. A simple Möbius Band is digitally generated, by joining together a set of lines or axes, twisting uniformly as they travel in a circular or elliptical path. The computer plays the role of a sketchbook, with which the initial sketches and design ideas are developed. Various routines are written using the programming language AutoLISP within AutoCAD, to generate the desired sketches using a variety of parameters. In this way, there is total control of the twisting and smoothness and form of the Möbius.

The basic path or orbit, be it a circle or an ellipse, is generated by a desired number of segments, the center of the segment becomes the axis for the twist from which the edge points are computed. The axis is twisted at a constant rate with respect to its center point. These edges joined together evolve into a smooth Möbius Band. Figure 4 displays pure Möbius Bands along an elliptical path with a 180° twist, and a 540° twist.

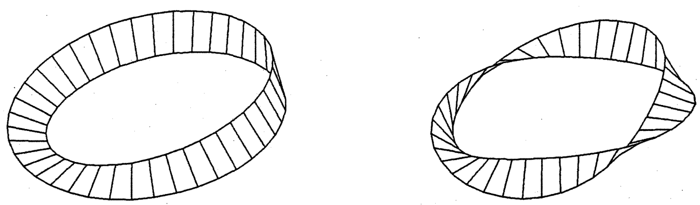

Figure 4: *Möbius Bands with various degrees of twist.*

2.2. The Möbius Surface. The Möbius Band, in its most pure form, when applied in a built architectural form, would be impossible to walk on due to the twist. The natural Möbius band includes a very small flat area, unless the twist is not applying in a constant manner. In order to walk along the band, let us consider splitting the band thickness into two parts. One part goes on as a Möbius while the other continues as a flat surface, as shown in Figure 5. Let us call this flat surface as the Flat Band. As one walks along this Flat Band, the Möbius Band is experienced without falling down. At certain intervals of travel, one encounters the Möbius floor that would rise to become the Möbius wall and eventually the Möbius ceiling or go back to the Möbius floor.

This concept expands the available flat surface of a Möbius, but does not allow continuous movement along the band because of the portion that comes down to meet the flat portion, but a very large portion of the band is available for either a floor surface or at a smaller scale a seating area.

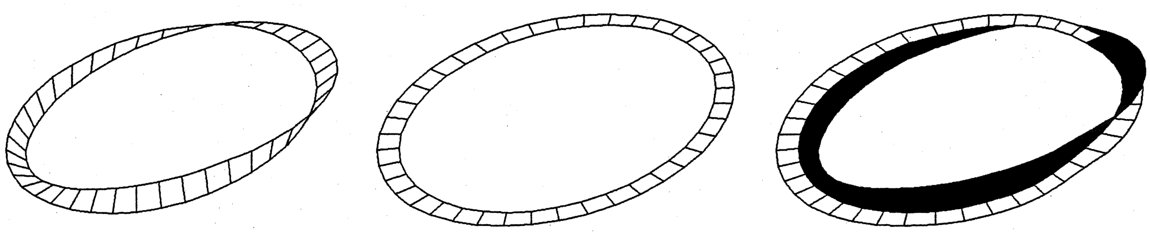

Figure 5: *Möbius Band split into two to get a combined Möbius Floor.*

The Flat Band could also be treated as a separate entity. In this case, it would slide into the Möbius to create a usable surface that would not be possible otherwise, as seen in Figure 6. Again, the Möbius twist could be double or even tripled which would make the Möbius go under the floor at certain intervals. This could serve as a structural support for the floor above.

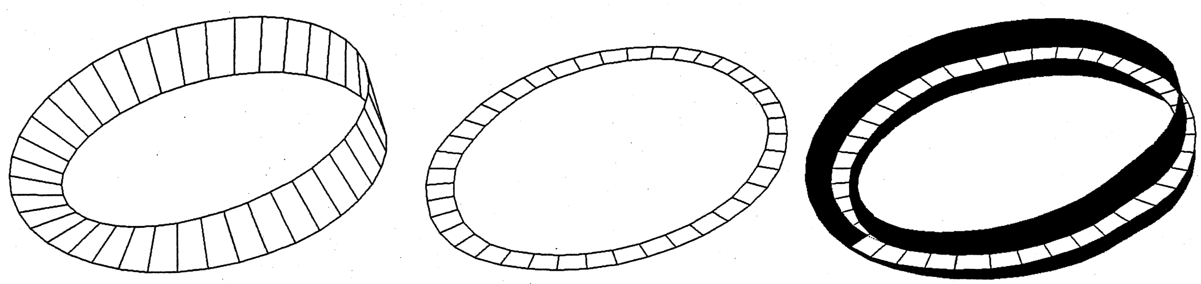

Figure 6: *Möbius Concept where the floor slides in and out of the Band.*

A similar attempt to investigate such a concept was made by Vesna Petresin and Laurent-Paul Robert [18] where intertwining Möbius Bands were suggested. In their concept, only a small portion of the Band was flat enough to be architecturally viable.

2.3. The Möbius Enclosure. According to Charles Joseph Matthews [19], the Möbius Band could be considered as a three-dimensional surface without any thickness. When thickness is applied to the Möbius, it becomes a twisted prism. Let us call this twisted prism as the Möbius Enclosure. The cross-section of this Enclosure could be a triangle, square or polygon of any number of sides, even or odd. The number of twists could also be more than one. The combination of these two parameters could lead to interesting Möbius forms.

A series of Möbius forms that would be architecturally feasible is investigated using the parameters of the number of twists and the number of sides of the cross-section under consideration. Instead of axes, frames are generated that twist along the twist of a Möbius Band as in Figure 7a. Each frame is extruded to join the vertices of the next frame and this procedure follows till a Möbius Enclosure is formed as in Figure 7b. The assignment of color for alternate surfaces acts as a guide to follow each side along the twist. In the case of a section with odd number of sides, each side is divided into halves, alternately colored as in Figure 7c. When translated into an architectural entity, the color would materialize into opaque, translucent or transparent surfaces alternately to highlight the twisting motion.

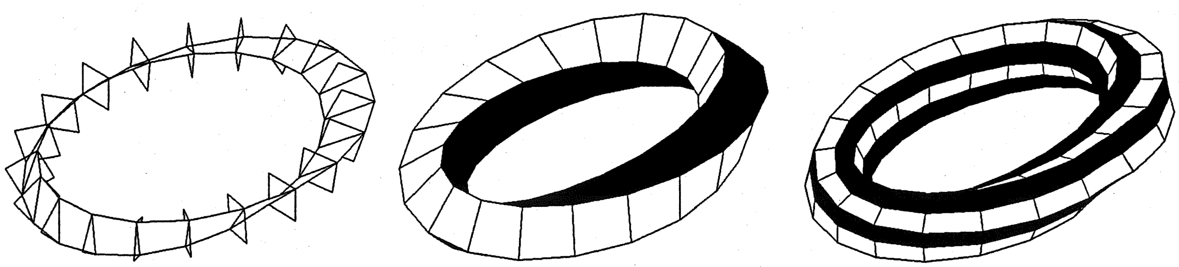

Figure 7: *Möbius Enclosure:(a) twisted frame (b) sides of frame is even (c) sides of frame is odd.*

2.4. Inside The Möbius Enclosure. Once an empty enclosure is generated in the form of the Möbius, let us consider the concept of a floor in this twisted enclosure. Here, the horizontal axis (in case of a square, the horizontal diagonal) of the frame with maximum vertical height is considered to be the floor level. The height of the floor remains constant as it travels through the desired circular or elliptical path as in Figure 8a. The floor surface width would also increase or decrease as it progresses through the Möbius.

Another variation of this concept would be to change the floor level at regular intervals. This would lead to a series of ramps within the Möbius Enclosure. The biggest and the smallest frames determine the start and end of each ramp, which could be more than one in each case. An example of this concept is demonstrated in Figure 8b and 8c where the section is a square and ramps go up and down alternatively. In this case, the floor surface width is constant.

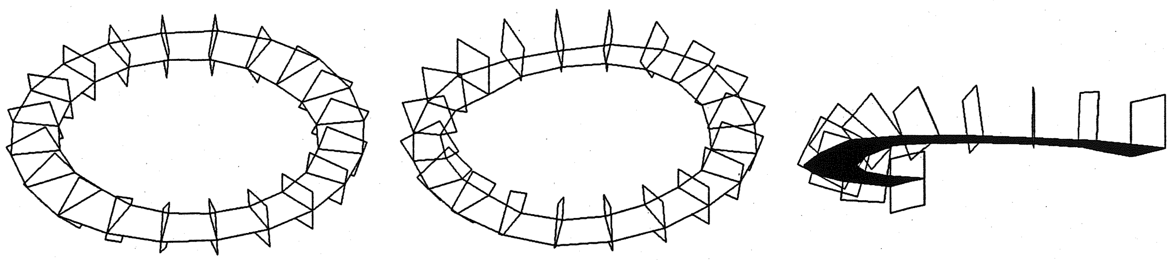

Figure 8: *Inside the Möbius Enclosure:(a) constant floor level (b) floor becomes ramp(c) section of ramp.*

3. Applications of Möbius Concepts

3.1. Scale. The translation of various Möbius concepts developed throughout the digital sketching process is performed through computer renderings that help visualize how twisted spaces and levels act in terms of architecture and human scale. A series of buildings that vary in scale are considered as apt models of experimentation.

3.2. Playgrounds and Elements of Landscape. The Möbius concept would be used as a play element such as a climber or a skating trail in a playground.

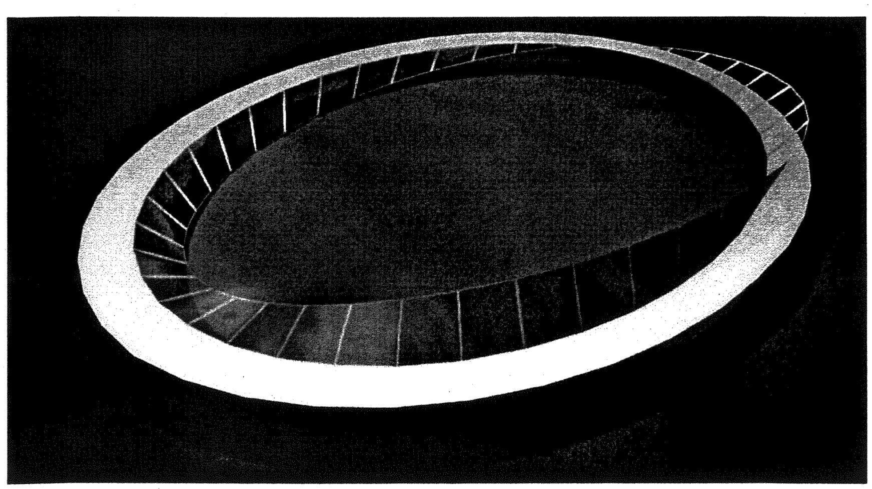

Figure 9: *Möbius Seating.*

The scale at which the Band or Enclosure used is the smallest here. It can also be used in landscaped gardens as a piece of sculpture or a seating area that would transform from a seat to a canopy and vice versa.

3.3. The Nursery School. The Möbius Band would be an important play element in a nursery school, which when housed within a Möbius structure would make it look like a playhouse. The inner open space would act as a courtyard for the kids to play. The continuity of the Möbius will be an added advantage for the curious kids to keep wondering and come across new experiences throughout the building. Materials used on the outer skin would be mainly steel and glass. The interior strip could be of smooth plastic material. The scale would be that of a single story building. The program would include classrooms, staff rooms, administrative and service areas apart from play areas. A crude example of this is shown in Figure 10.

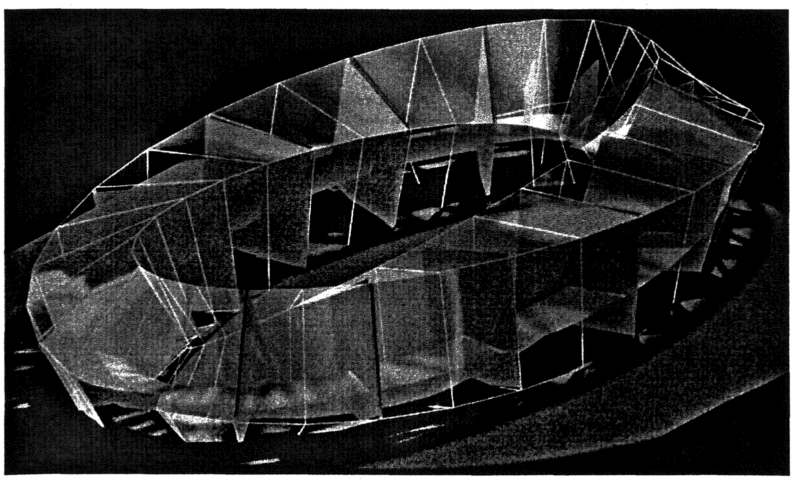

Figure 10: *Möbius School.*

3.4. The Museum. A Möbius Museum would be the best option where both the Band and Enclosure could be applied as a structure that would also act as a piece of art or a part of the building as seen in Figure 11.

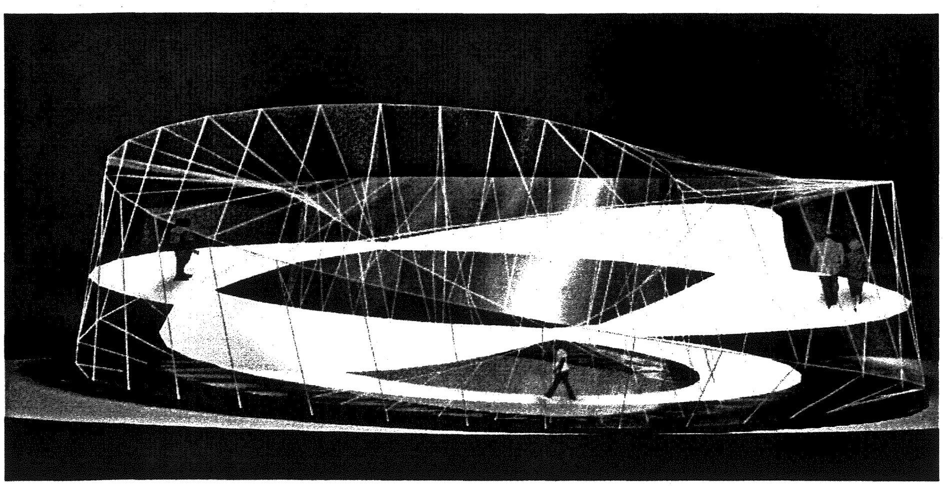

Figure 11: *Möbius Museum.*

Visitors would traverse the Band that would split up at a certain point to continue as a wall and then a ceiling and eventually the floor again. It could also transform to steps or a ramp that would lead to the floor below/above and continue as a wall there too. This would be contained in a Möbius Enclosure that would have outer facades clad with different materials (metal, glass or plastic) to signify the path of the Möbius around the outer Enclosure. The scale would be that of a two-story building with a mezzanine that would house the administrative services of the museum. Other spaces would include the museum shop, coffee shop and service areas such as parking spaces for the museum.

3.5. The Hotel. The Möbius Enclosure would be incorporated into a hotel, with rooms that would be unique in their own way (in terms of spatial experience and views) based on their location within the Möbius. The internal space could be another Enclosure that would act as circulation and interaction space. Some of the rooms (which lack good external views) would be used as service spaces including kitchen, laundry and reception area. The common spaces such as banquet halls, restaurants and recreational spaces would occupy the central space that would be covered. The center of the Möbius could be a shopping space. Another option would be a protrusion of a face of the outer Enclosure to house the common spaces leaving the central space open for outdoor recreational activities. Other details such as loading areas, staff areas and parking spaces could be integrated within and around the Möbius. The hotel would be a 3-to-4 level building. Outer facades could be mostly glass within a steel skeleton and the interiors could be filled with grand materials to reflect the quality and service of the hotel.

3.6. The Urban Connector. In an urban context, the Möbius Enclosure would act as an in-between space, i.e. it would be a space that would link a specific number of buildings (houses, in the case of a housing complex.) In a city, the Möbius would be big connecting structure that would link a number of buildings together. The Möbius will contain commercial spaces such as shopping, dining and other retail spaces. It would become an internal street within the linked buildings. It could be situated on any level that all the linked buildings share. It could be treated as a big space frame floating between buildings.

4. Conclusion

The geometry of the Möbius band has great potential as an architectural form that is difficult to visualize and investigate without the aid of digital technologies. This paper demonstrates that it is possible to develop a building that is a pure translation of the Möbius Band and it furthers a current trend in architectural forms being developed from mathematical concepts beyond mere inspiration.

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