

Sculpture from Symmetrically Arranged Planar Components

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Abstract

The extended face planes of symmetric polyhedra form a set of interconnected canvases that can be used as the basis for a wide range of sculptural forms. A special-purpose software tool is described which allows the user to select families of such planes, to draw within them, to maintain constraints concerning points lying on their intersections, to view and modify the results interactively in real time, and to output the result for production either as 3D rapid prototyping or 2D laser-cutting, etc. A range of examples demonstrate the versatility of the approach.

1. Introduction

Artists typically self-impose design constraints, challenging themselves to explore new possibilities in a restricted genre. I find myself fascinated by the sculptural possibilities inherent in symmetrically arranged intertwined planar components. Several of my sculptures are 3D forms assembled from flat parts that lie in symmetrically arranged planes. For example, Fig. 1 shows *Twisted Rivers, Knotted Sea*, a four-foot diameter steel sculpture based upon the intersections of the extended face planes of an icosahedron.

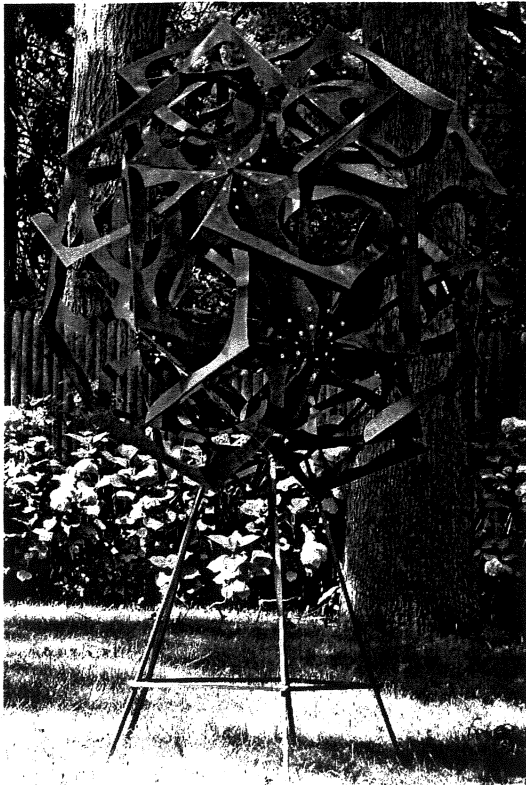


Figure 1: *Twisted Rivers, Knotted Sea*, steel, 4' diam., 2001

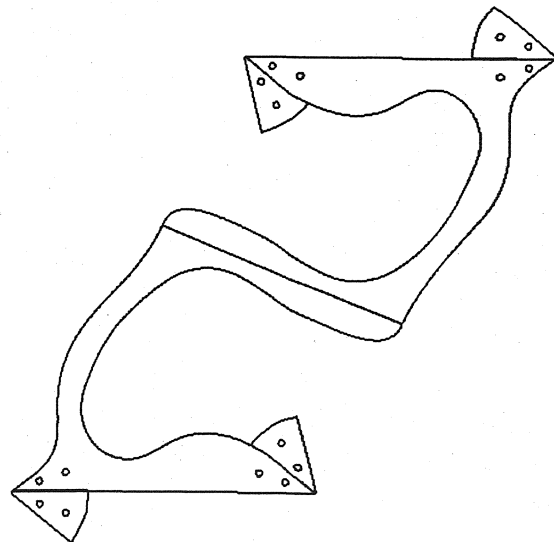


Figure 2: Layout for the 30 components of Fig. 1

The sculpture in Figure 1 is assembled from thirty identical curved units. Fig. 2 shows the flat layout for each of the thirty components, before folding the metal. I designed this sculpture by visualization, hand calculation of lengths and angles, and a series of paper models. Then I laid out the flat shape in a standard drawing program and sent the file to a laser-cutting service where the steel was cut on a computer-controlled laser cutter. I folded the parts to the proper angles in a hand brake then sent them out for the blue powder coating. Finally, I wove the thirty parts together and fastened them with stainless steel bolts. Ten pieces meet at each of the twelve junctions where the bolt patterns are visible. The twenty openings are three-fold "whirlpools". If one studies the sculpture in person, it is apparent that groups of three C-shaped curves each lie in a common plane. There are 20 such planes—the planes of an icosahedron.

Figures 3 and 4 show a second example, *Fire and Ice*. This is a 24" diameter hollow form, assembled from sixty identical pieces of red oak and ten interwoven loops of brass. These oak components were also hand designed using visualization and paper models, and laid out in a general-purpose drawing program. I then used the file to guide the cutting on a computer-controlled router. The two long edges were then beveled at 45 degrees, and the short edge at 72 degrees. They fit together as they do because the 60 pieces lie by pairs in the 30 planes which are extensions of the faces of a rhombic triacontahedron.

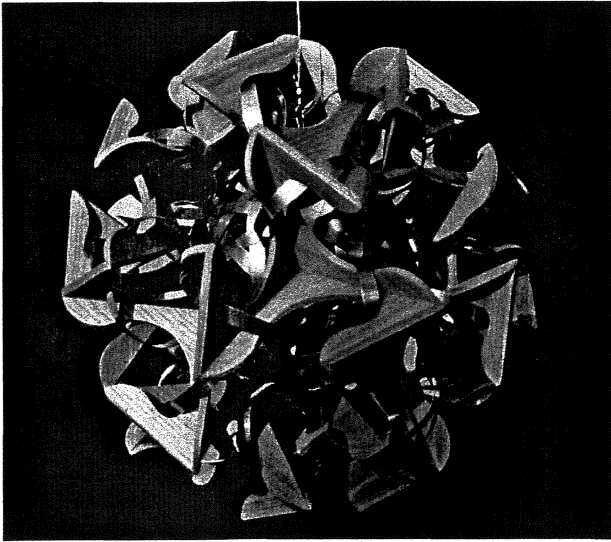


Figure 3: *Fire and Ice*, oak and brass, 24" diam., 1997

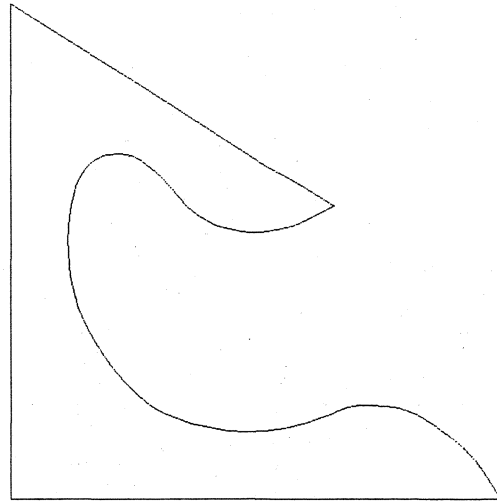


Figure 4: Layout for 60 wooden parts of Fig. 1

Fig 5 shows *Whoville*, a more complex example in which the planes of the sculpture's components lie in the planes of three concentric polyhedra, an icosahedron, a dodecahedron, and a rhombic triacontahedron. (At one type of joint, along the 5-fold arms, the form incorporates a gradual twist from one plane to another, rather than a fold.) This was designed by hand with the aid of paper models. For this sculpture, I cut, drilled, and textured the metal by hand, then assembled the parts with rivets.

These three examples are members of a rich family of possible forms that involve symmetrically arranged planar components. Only years after creating these works and others, did I gradually awaken to the fact that I could use a special-purpose piece of software to aid me in the design, layout, and rapid prototyping of new sculptures in this category. After deciding what features I wanted and planning a design general enough for use in many new forms, I wrote the program described in the following section.

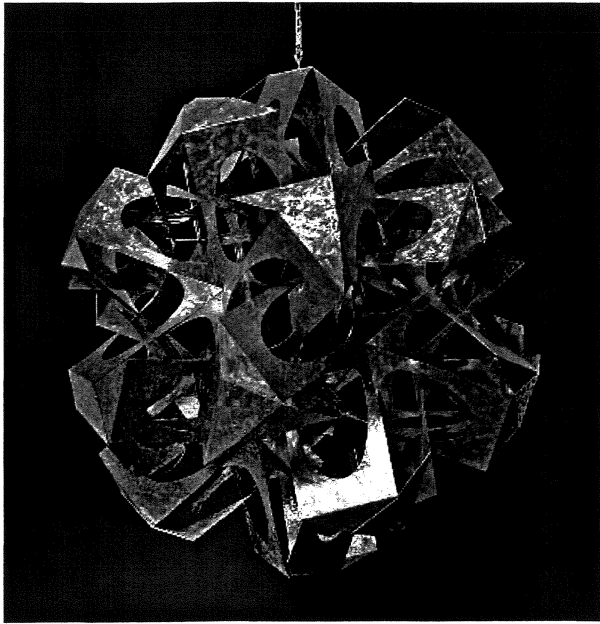


Figure 5: *Whoville*, aluminum, 35" diam., 1997

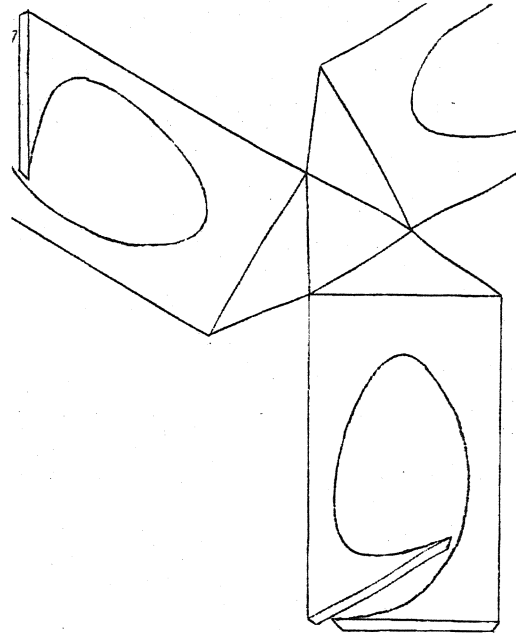


Figure 6: Template for paper model of *Whoville*

2. A Sculpture Design Program

My goal in writing this software is to have a tool which can help me design, visualize, and produce large sculptures assembled from symmetrically arranged flat pieces. The program's editing tools allow me to design and position different families of components lying in planes that I specify. At the same time, the program shows me a rotating image of the 3D construction, which changes instantly in response to my editing any of the planar components. If I wish to produce a physical model by stereolithography, fused deposition modeling, or other 3D rapid prototyping (RP) technique, the program outputs the STL file for fabrication. When I have a design finalized, the metal, wood, or plastic components can be cut by machinery such as laser cutters or water-jet cutters, guided by the 2D layouts the program produces as DXF files. The software handles the mechanics of replication, maintenance of symmetry, plane intersections, lengths, dihedral angles, and so forth. I can focus on the design issues.

Figures 7 and 8 show the user-interface. As illustration, *Twisted Rivers*, *Knotted Sea* is reproduced. Compare this to the actual sculpture in Fig. 1 and the original hand-drawn layout in Fig. 2. The 3D-view window appears when the program is started. It always shows a (rotating) 3D view of the sculpture being designed. A menu item in that window is used to create new 2D design windows. The user can create any number of 2D windows, specifying the orientation and distance from the origin for each. In this illustration, there is only one 2D window, as this sculpture only incorporates one set of planes. The planes were created by specifying them to be "icosahedral" meaning there are 20 instances of each plane and each has 3-fold symmetry. Other options are "dodecahedral" with twelve 5-fold planes, "triacontahedral" with thirty 2-fold planes, and "hexecontahedral" with sixty planes.

In the background of the editor window, one sees "guide lines"—the intersections of this one representative plane with the other 19 in its family. (This arrangement of lines is known as the stellation pattern of the icosahedron. [2]. Of the 19 other icosahedral planes, only 18 show up as lines, because the plane of the opposite face is parallel, so doesn't intersect this face) Over these guide lines, the user can draw with the mouse, insert, delete, and move points, connect points with edges, etc. to create a 2D layout. Each point can be specified as attached to zero, one, or two guidelines. The user's drawing is automatically replicated to maintain rotational symmetry in this plane. The toolbar controls the mouse function, and with sliders the view can be zoomed, rotated, and panned.

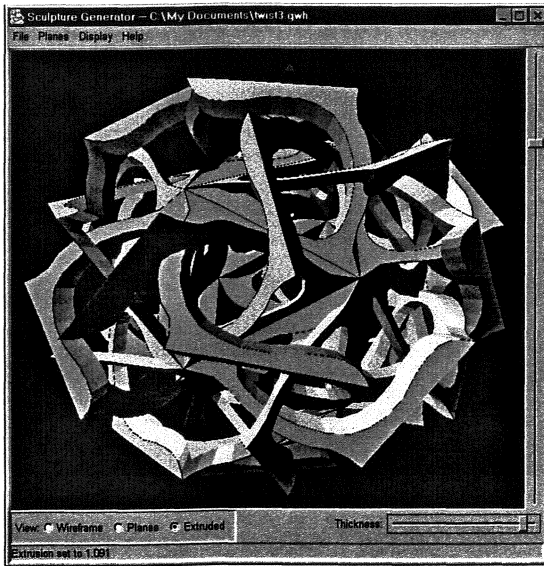


Figure 7: 3D view window

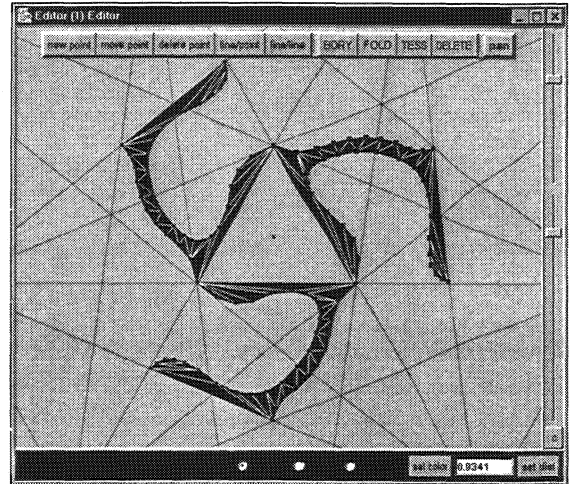


Figure 8: 2D editor window

In the 3D window, the user has the choice of three types of view: wireframe, planes, or extruded. Fig. 7 is the extruded view, which shows the sculpture as having a slight (user-specified) thickness, exaggerated here for clarity. The 2D pattern is replicated 20 times, in the planes of an icosahedron's faces. Each plane is also doubled, to an inner and outer surface, connected with a thin edge. At edges where planes meet, the representation ensures that the surface is "watertight" with no gaps. In this form, the model defines a volume, so it can be output as an STL file and fabricated by 3D rapid prototyping (RP) machinery. Figure 9 shows a plastic model produced from this STL output on a DTM Sinterstation 2500+.

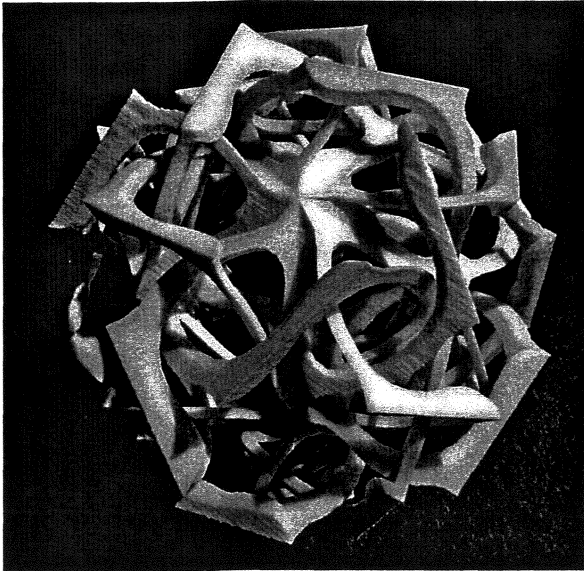


Figure 9: RP model of Twisted Rivers, 9 cm

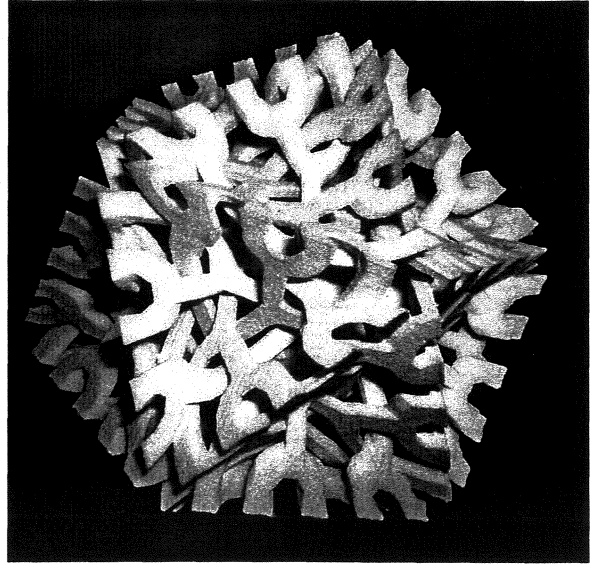


Figure 10: Tangled Reindeer, RP, 9 cm

Figure 10 shows a second RP result from the program. This is a sculpture titled *Tangled Reindeer*, which is composed of twelve planar components in the planes of a dodecahedron, though it nestles nicely inside an icosahedron. The corresponding 3D window is shown in Figure 11, and the plan for each plane is shown in the editor window of Figure 12. (The background guide lines are now the stellation pattern of the dodecahedron.) The white segments in the editor are the places where the replicated structures in the

other planes pass through this plane. The editor optionally shows these cut-through locations, so that one can see if the replicated parts will intersect each other.

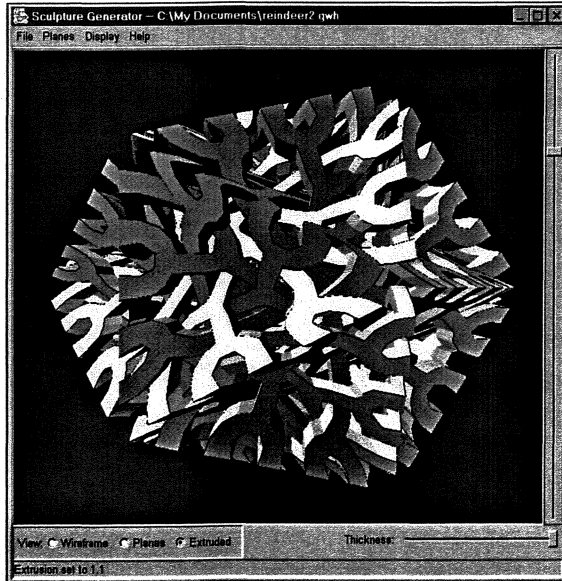


Figure 11: Tangled Reindeer, 3D view

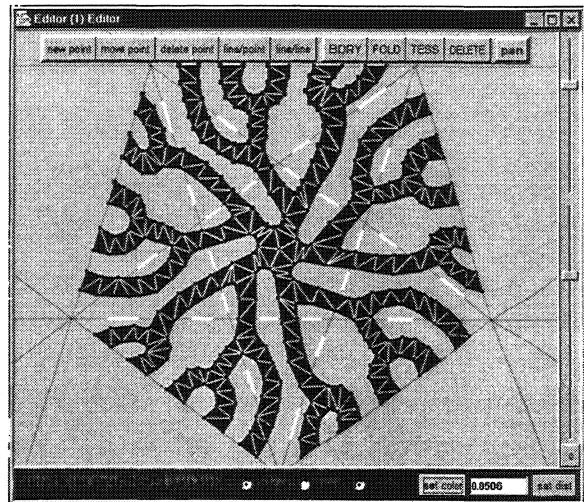


Figure 12: 2D pattern in one dodecahedral plane

The technical details of the software are beyond the scope of this paper, but an outline is as follows. Points are represented standardly as floating-point (x,y,z) coordinates, planes as normal vectors and distances, segments as pairs of points, etc. There are four types of points: "free-points" are bound to a single plane and so are draggable in two dimensions, "line-points" are bound to two planes and so are constrained to their line of intersection, "corner-points" are fixed to the intersection of three planes, and "center-points" are fixed at the center of their plane. There are three types of segments: "boundary segments", "fold lines" which lie in the intersection of two planes, and interior "tessellation segments" which simply divide polygonal regions into triangles because that is required for the STL file format. Symmetry and consistency are maintained, e.g., after a set of planes is created, the user can change their distance from the origin and points on those planes are projected to the new planes, points on the lines of intersection are projected to the new lines of intersection, and the guidelines on the editor windows move correspondingly.

At present, the extrusion process for producing STL files is simply radial in three dimensions from the origin. The outer vertices and outer surfaces are generated a few percent further from the origin than the inner surface. A slider in the 3D window controls this percentage, which is 9%–10% in Figures 7 and 11. When extruding, the handedness of one set of triangles is reversed to maintain orientation, and an edge surface is created by generating a new pair of triangles for each boundary segment. This procedure has the advantages of being simple and preserving planarity. However it is inadequate in certain cases (not shown in this paper) when there are two families of planes of different distances and the outer surface of one meets the inner surface of the other. It also has the limitation that different families of planes are extruded to different thicknesses. The further a plane is from the origin, the thicker its extruded version. However, this is not a fatal fault for a model intended as a prototype of the actual sculpture. More general extrusion processes without these limitations are planned for a future version of the software.

In Figures 8 and 12, only a few points approximate each curve, so the polygonal nature of the curves can be seen easily. For production, a more refined curve with many points could be drawn. Instead, I take the program's 2D file output and convert it from a segment representation to a curve representation (cubic splines) in a standard drawing program. This second stage of the layout process is beyond the scope of this design program. In this second stage, tabs and bolt holes might also be added, as seen in Figure 2.

Furthermore, the components might be reorganized, again illustrated in Figure 2, where components from two different planes are joined into one piece, which is folded after cutting. (The bisecting fold lines are the outermost parts seen in Figure 1. An alternative would be to have a joining mechanism, e.g., bolts, along these outer seams, but I did not consider that visually appropriate.) Such aspects of the design are not within the scope of the program at this time, because the best choice for physically grouping components, e.g., tabs, folds, welds, bolts, etc., are individual to each sculpture.

The software is coded in *Java*, with *OpenGL* for the 3D graphics window. (At present there is no standard binding for using *OpenGL* with *Java*. I chose the *GLJava* binding, which appears to be popular and stable.) I have been running the program on a PC under Windows, but as it is written in *Java*, it should be directly transportable to other machines and operating systems. It responds instantly, changing the 3D view appropriately when any 2D view is edited.

3. New Acrylic Sculptures

This section shows four examples made from laser-cut acrylic plastic, i.e., *Plexiglas*. In each case, the completed sculpture is shown at left and a single component at right. To make wooden or Plexiglas models, the 2D layout is output as a DXF-format geometry file used to cut the shapes. After cutting, the parts that mate are mitered so they can butt at the proper dihedral angle. (The dihedral angles are output separately by the program. For a metal sculpture, these angles are the fold angles, or could be used to set jigs that hold pieces in relative position for welding or brazing.) Then the parts are then intricately interwoven (which usually took several hours) and glued with a solvent cement.

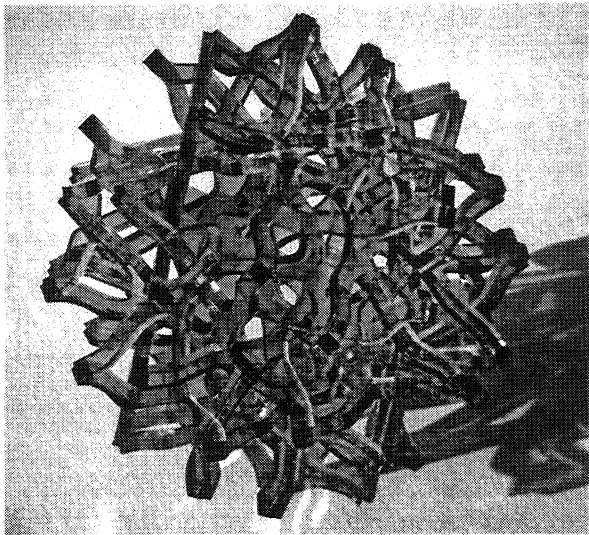


Figure 13: *Deep Sea Tango*, 15 cm

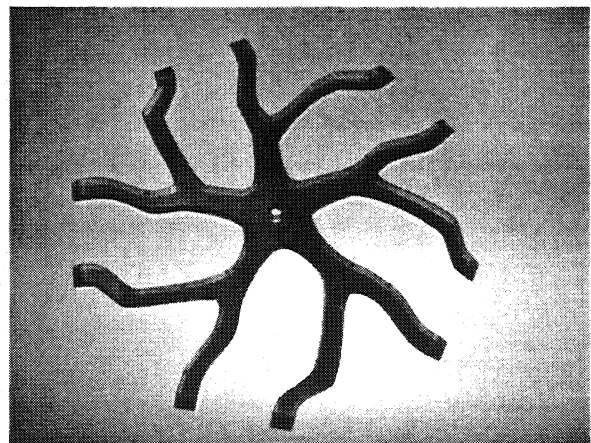


Figure 14: One component of Fig. 13

Figure 13 shows *Deep Sea Tango*, which can be thought of as twelve starfish-like forms "holding hands" in a complex ballet position. It is assembled from twelve identical parts, each as in Figure 14, which only touch each other at their tips. Because of the transparent material, it is difficult to capture in a photograph how the many arms just manage to pass by each other at various angles. The design is based on the great dodecahedron, and conceptually very similar to *Tangled Reindeer*, (cf. Figures 10-12) but twelve rigid parts shaped like Figure 14 could not be assembled because they would block each other's movement. These parts (Figure 14) have much less branching, but even so, it is just barely possible to assemble them.

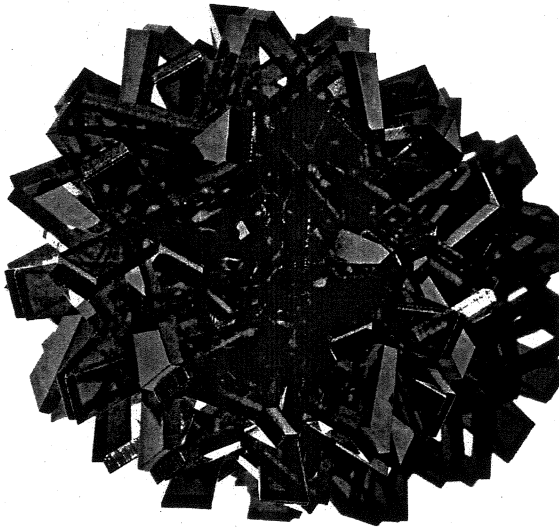


Figure 15: *Quintessence of Hedgehog*, 15 cm

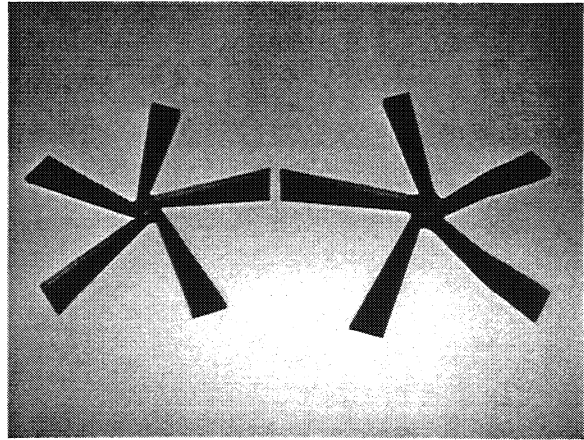


Figure 16: Two shapes of components for Fig. 15

Figure 15 shows *Quintessence of Hedgehog*, which contains twenty-four windmill-like parts, twelve in each of two slightly different 5-fold shapes. They lie in the face planes of two concentric parallel dodecahedra. The larger parts lie in the planes of the smaller dodecahedron as they have to extend further to reach the circumsphere where the two types of parts meet. The sixty junction points are each the meeting of one large arm and one small arm. (The design program allows the user to create several editor windows, each like Figure 12. The intersection lines of each plane with all the other planes are color-coded to distinguish the different families. Points on the lines of intersection of two families show up in both editors, and if dragged in one plane make the corresponding motion in the other plane.)

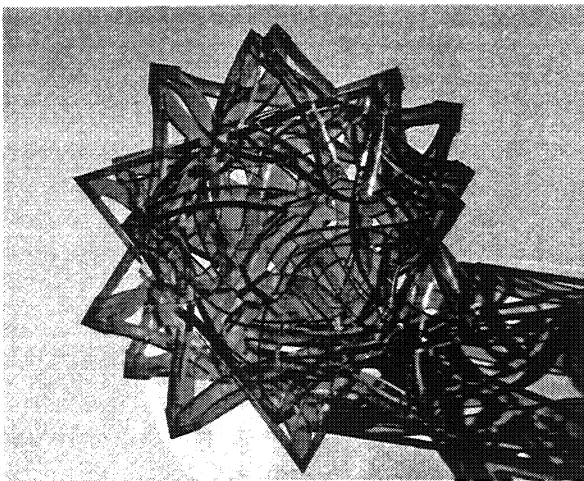


Figure 17: *This End Up*, 23 cm

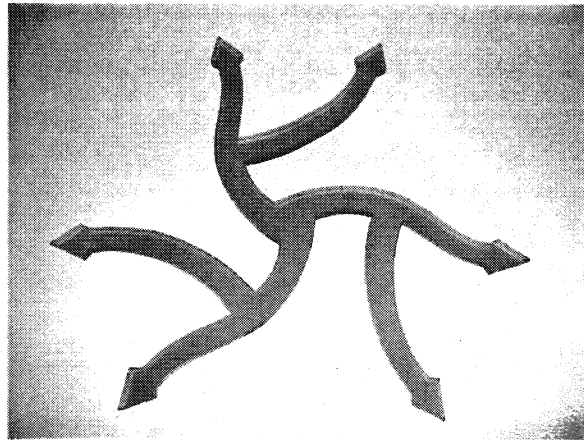


Figure 18: One component of Fig. 17

Figure 17 shows *This End Up*, which contains twenty parts of the form shown in Figure 18, lying in the planes of an icosahedron. The parts pass around each other without touching, except for where they are joined at the arrow tips in groups of four. The structure is based on the uniform compound of five regular octahedra, which is one of the stellations of the icosahedron. It took almost two hours for me and ten members of the Stony Brook University math club to assemble this. I originally thought that ten people could each hold two parts and translate them all together "from infinity" but we couldn't get the interweaving just right, and we eventually found a less systematic assembly method which did work.

Lastly, Figure 19 shows *Bouquet*, which contains thirty *S*-shaped parts as shown in Figure 20. The components lie in the face planes of a rhombic triacontahedron, but the viewer sees it as a dozen flowers that nestle inside a dodecahedron. The ends meet in groups of three, but in the interior the components make 5-fold spirals without contacting. Because of the long thin parts, this sculpture is slightly flexible.

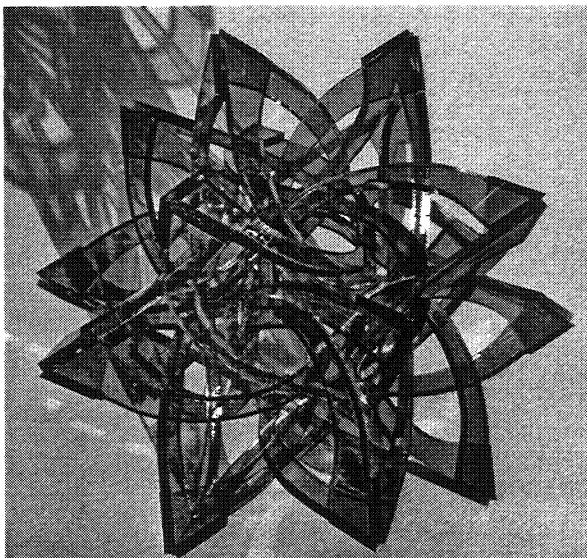


Figure 19: *Bouquet*, 23 cm

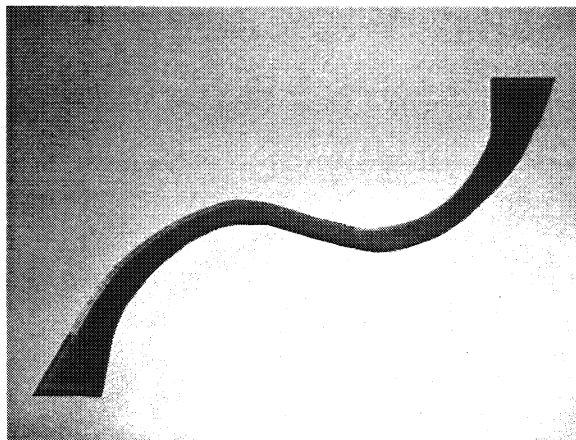


Figure 20: One component of Fig. 19

4. Conclusions and Future Work

For several years I have been exploring a space of forms with symmetrically arranged planar components. [3] I find that they can be visually and intellectually engaging, with a rich mix of simplicity, complexity, geometry, and an organic quality. (In addition, a practical benefit of this class of sculpture is in transportation: One can ship the parts of a large work as a flat package and assemble them on site.) To explore further in this family, I created a design program to virtually prototype new works. This real-time explorer of spatial ideas allows for more complexity than my previous sculptures, permitting intertwined relationships between several families of planes. It encourages experimentation while handling the computational issues of maintaining symmetry, planar intersections, and layout.

As an interactive sculpture design tool, it was inspired in part by Sequin's tool for generating sculptures in the style of Brent Collins. [4] The program presented here is in some ways analogous, but with a different family of forms. There are a number of previous programs which allow the user to design stellations, e.g., [1] and [5], but as far as I know no previous work allows freeform design in the stellation planes.

This is an ongoing project with many future directions, mainly additional sculptures. I also have a list of software enhancements to add, such as other forms of extrusions, and edge treatments for the RP results.

References

- [1] V. Bulatov, "An Interactive Creation of Polyhedra Stellations with Various Symmetries," *Bridges 2001: Mathematical Connections in Art, Music and Science*, Reza Sarhangi (editor), 2001.
- [2] H.S.M. Coxeter, P. DuVal, H.T. Flather, and J.F. Petrie, *The Fifty-Nine Icosahedra*, U. Toronto Pr., 1938, (Springer-Verlag reprint, 1982), (Tarquin reprint, with additional images, 1999).
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- [4] C.H. Sequin, "Virtual Prototyping of Scherk-Collins Saddle Rings" *Leonardo* 30:2, pp. 89-96, 1997.
- [5] R. Webb, "Stella: Polyhedron Navigator," *Symmetry Culture and Science*, 11, No. 1-4, pp. 231-268, 2000/2003.

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