Mathematics in Literature

A Continued Exploration of the Use of Mathematical Concepts In the Works of Jorge Luis Borges

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Abstract

The fiction of the great Argentinean writer, Jorge Luis Borges, lies at one of the rare points of intersection between classical literature and mathematics. Several of Borges' short stories have a structure that is based on a mathematical concept, such as the infinite, recursion, duality or chance. Through these stories, Borges allows the same beauty that the mathematician experiences through her understanding these concepts, to be experienced by a larger nonmathematical audience. In my first paper on Borges' fiction, The Poetry of Infinity (Bridges, 2001), I examined Borges' use of mathematical ideas as metaphors in his fiction. I looked at four of his most famous stories, *The Library of Babel, The Garden of Forking Paths, The Book of Sand* and *The Circular Ruins*. This current paper is a continuation of my exploration of Borges work, and in it I examine several more of his stories and the mathematics they contain. The mathematical concepts discussed will include duality, recursion, probability, as well as more basic ideas like the consistency of the laws of arithmetic, the use of variables and the need for mathematical abstraction.

Jorge Luis Borges

Jorge Luis Borges is one of the most important and influential of all Latin American writers. He was born in Argentina in 1889 to an aristocratic family and was educated in Europe after the age of fourteen. He was also a poet and essayist but is most famous for his short stories, of which over a hundred have been published. In 1961, he was awarded the International Publishers Prize for *Ficciones*, a collection of short stories, some of which are discussed below. Throughout his life, he was awarded many other prizes including the most prestigious cultural awards from both Spain and Mexico, as well as many honorary doctorates, including those from Columbia University, Oxford and Cambridge. Many of his short stories include mathematical ideas, especially Infinity and its many seemingly paradoxical properties. In this paper, we will explore several more of his stories and the mathematics they contain.

The Lottery of Babylon

This is the story of a lottery that was started benignly enough in Babylon as a way for its creators to make money, but which takes an apparently sinister turn. Originally, participants would purchase tickets and a drawing would be held to determine the lucky ticket, and the owner would be rewarded with silver. Eventually, however, the public became bored, so the creators of the lottery tried to spice things up by adding the occasional unlucky ticket, whose owner would be required to pay a fine. This created much renewed interest in the lottery, but a company had to be formed to make sure that the losers paid their

fines. When the losers could not pay, the Company would put them in jail, and eventually the shortcut was taken of simply writing the number of days to be spent in jail on the losing ticket.

> That shorthand, as it were, which went virtually unnoticed at the time, was of utmost importance: It was the first appearance of nonpecuniary elements in the lottery. ([1], p. 102)

In time, the lower castes complained that due to poverty, they were excluded from the lottery so

> the Company was forced to assume all public power...the Lottery was made secret, free of charge and open to all....The sale of lots was abolished...and every free citizen automatically took part in the sacred drawings...A lucky draw might bring about a man's elevation to the council of Magi or the imprisonment of his enemy....an unlucky draw: mutilation, dishonor of many kinds, death itself. ([1], p. 103)

Sometimes bets would be combined, which was difficult, but

> the members of the Company were (and still are) all-powerful and clever. ([1], p. 103)

The Babylonians began to produce a general theory of gaming from which arose the following conjecture:

> If the Lottery is an intensification of chance, a periodic infusion of chaos into the cosmos, then is it not appropriate that chance intervene in every aspect of the drawing, not just one? Is it not ludicrous that chance should dictate a person's death, while the circumstances of that death....should not be subject to chance? ([1], p. 104)

Further reforms were made so that the Lottery determined every aspect of each person's life. Now

> the number of drawings is infinite. No decision is final; all branch into others. ([1], p. 105)

Now that every aspect of a person's life is subject to chance, each life could be represented by an infinite *tree*. A *tree* is a type of *graph* which contains no *circuits*. In this case, each *vertex* in the *tree* represents a moment when two or more possibilities exist; each possibility represented by an *edge* leaving that *vertex*. A *path* in a *tree* is a collection of consecutive *edges* in the *tree*, so in this case, each *path* in the *tree* represents a possible life the person could have.

Suppose for a moment that at each moment in a person's life when more than one possibility existed, that there would be only two possibilities. With say, only three such moments, there would be $2 \cdot 2 \cdot 2 = 8$ different *paths*. With four moments there would be $2 \cdot 2 \cdot 2 \cdot 2 = 16$ *paths*. With n steps, we would have

$$2 \cdot 2 \cdot 2 \cdots 2 = 2^n$$

paths. Since it is assumed that there would be (countably) infinitely many moments at which choices would arise, we let n go to *infinity* and find that the number of possible infinite *paths* in the *tree*, and hence the number of possible lives a person could have, is *uncountably infinite*. Thus the probability of any particular life (sequence of events) occurring is zero.

> The ignorant assume that infinite drawings require infinite time; actually, all that is required is that time be infinitely subdivisible, as in the famous parable of the Race with the Tortoise. ([1], p.105)

This is a reference to Xeno's paradox of Achilles and the tortoise which Borges discusses at some length in his essay *Avatars of the Tortoise*.

Finally,

> [The Company's] silent functioning, like God's, inspires all manner of conjectures. One scurrilously suggests that the Company ceased to exist hundreds of years ago, and that the sacred disorder of our lives is purely hereditary, traditional; another believes the Company is eternal. Another...says that the Company never existed and never will. Another....argues that it makes no difference whether one affirms or denies the reality of the shadowy corporation, because Babylon is nothing but an infinite game of chance. ([1], p. 106)

The God's Script

Recursion is a common theme in many of Borges' stories and is used in a most poignant way in his story *The Circular Ruins*, which I discussed in The Poetry of Infinity. In *The God's Script*, the narrator is a priest imprisoned in a vault. One night, he has a disturbing dream.

> I dreamed there was a grain of sand on the floor of my cell. Unconcerned, I went back to sleep; I dreamed that I woke up and there were two grains of sand. Again I slept; I dreamed that now there were three. ([1], p. 252)

He continues falling back asleep in the previous dream only to dream that he awakes to an additional grain of sand.

> Thus the grains of sand multiplied, little by little, until they filled the cell and I was dying beneath that hemisphere of sand. ([1], p. 252)

So through a sequence of dreams within dreams, he has dreamed an enormous pile of sand that fills the space in his cell. He realizes that he is dreaming and uses all his effort to wake himself up so that he will not suffocate.

> But waking up was useless----I was suffocated by the countless sand. Someone said to me, *"You have wakened not out of sleep, but into a prior dream, and that dream lies within another, and so on, to infinity, which is the number of grains of sand. The path that you have taken is endless, and you will die before you have truly awakened."* ([1], p. 252)

Just as the pile was created through recursive dreams, it must be undone by recursive awakening into each previous dream. Here Borges has a logistical problem: Since it seems to have taken finitely many recursive steps (dreams) to create the pile of sand, it seems that it should also take the same number of finite awakenings to remove all the sand. Somewhere Borges has changed the process from finite to infinite.

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Recursion through dreams occurs again in this story, in which Borges, the narrator, enters a hotel lobby for the first time, only to find that his own name has already been signed in the register. He runs up to the room assigned to the man who signed the register in his name where he comes face to face with an older version of himself, lying in a bed after swallowing a bottle of pills. The younger Borges (the narrator) is sixty-one, while the older is eighty-four. Each of them assumes it is he who is dreaming, and they begin to argue about who is dreaming whom.

"Then…" I asked fearfully, "all this is a dream?"

"It is, I am sure, my last dream." He gestured toward the empty bottle on the marble nightstand. "You, however, shall have much to dream, before you come to this night."

"I am dreaming you in room 19."

"Who is dreaming whom? I know I am dreaming you----I do not know whether you are dreaming me."

"I am the dreamer," I replied, with a touch of defiance. ([1], p. 490)

The younger Borges asks the older about the rest of his/their life and the older Borges tells him, ending with,

"…you will have utterly forgotten this curious prophetic dialogue that is taking place in two times and two places. When you dream it next, you shall be who I am, and you shall be my dream."
([1], p. 493)

Moments later, the older Borges dies.

Thus we understand that there is a recursive sequence of dreams. At any given point in the sequence, a younger Borges meets an older Borges in a mutual dream, and then one day when this younger Borges has become old, he too will meet a new younger Borges in the identical mutual dream, just before the older dies.

The Theologians

Duality is a common phenomenon in mathematics and often considered one of the most beautiful. There are many examples of it from the *Platonic solids* and their duals, to *Galois Theory*, and the duality between the *subfields* of the *splitting field of a polynomial* and the *subgroups* of its *Galois group*. For example, the cube and the octahedron are Platonic duals of each other. If we place a vertex at the center of each face of the cube, and then include an edge for every adjacent face of the cube, we will get the octahedron.
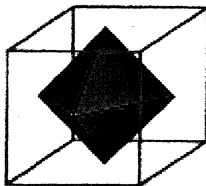

Figure 1: *The octahedron as the dual of the cube.*

Similarly if we place a vertex at the center of each face of the octahedron and add the corresponding edges, we will get a cube.
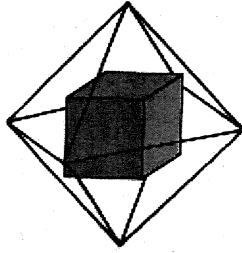
Figure 2: *The cube as the dual of the octahedron.*

The story, *The Theologians,* is about a religious sect called the *Histrioni* and their beliefs, which are based on a sort of distorted *duality* that can lead to disastrous consequences.

> In the hermetic books, it is written that "things below are as things above, and things above as things below"; in the Zohar, that higher world is a reflection of the lower. The Histrioni founded their doctrine on a perversion of this idea. ([1], p. 204)

> [The Histrioni] imagined that all men are two men and that the real one is the *other* one, the one in heaven. They also imagined that our acts project an inverted reflection, in such a way that if we are awake, the other sleeps, if we fornicate, the other is chaste, if we steal the other is generous. When we die, we shall join this other and become him. ([1], p. 205)

In mathematics it is common to work on something indirectly, by working on its dual and then determining the effect of that adjustment back on the original object. So also in Borges' story, do followers of the *Histrioni* attempt to modify the life of the "real" aspect of a man, by doing something to the earth-bound "dual" of the man, in the following example, leading to a tragic death.

> ...a blacksmith...deluded by the Histrioni deceptions, placed a huge iron sphere on the shoulders of his small son, so that his double might fly. ([1], p. 206)

Tlon, Uqbar, Orbis Tertius

In this story, the narrator comes across the eleventh volume of *A First Encyclopedia of Tlon*, a mythical planet, that he is curious to know more about.

> I now held in my hands a vast and systematic fragment of the entire history of an unknown planet, with its architectures and its playing cards, the horror of its mythologies and the murmur of its tongues, its emperors and its seas, its minerals and its birds and fishes, its algebra and its fire, its theological and metaphysical controversies-----all joined, articulated, coherent and with no visible doctrinal purpose or hint of parody. ([1], p. 71)

From this single volume, he and a friend try to reconstruct exactly what the nature of Tlon was.

> The nations of that planet are, congenitally idealistic. Their language and those things derived from their language----religion, literature, metaphysics—presuppose idealism. For the people of Tlon, the world is not an amalgam of *objects* in space; it is a heterogeneous series of independent *acts*---the world is successive, temporal, but not spatial. ([1], p. 73)

Since in the world of Tlon, the focus is not on objects, but on acts, there are no nouns in the language of Tlon. Instead,

there are impersonal verbs, modified by monosyllabic suffixes (or prefixes) that functioning as adverbs. For example, there is no noun for our word "moon", but there is a verb which in English would be "to moonate" or "to enmoon". *To say "the moon rose above the river" they would say "Upward, behind the onstreaming it mooned."* ([1], p. 73)

Shifting the emphasis from nouns to verbs is very similar to the common shift in emphasis in mathematics from a set of functions acting on elements of a common domain, to the set of *functionals* associated with elements in the domain that in turn act on the set of functions. That is, if

$$\Gamma = \{f(x) : f : D \longrightarrow R \},$$

is the collection of all functions from D to R, then for each $x \in D$, we can create a functional,

$$\phi_x : \Gamma \longrightarrow R, \text{ so that } \phi_x(f) = f(x).$$

This create a kind of duality between the elements x of the domain D, and the functionals acting on the set Γ. Similarly in the language of Tlon, instead of the focus of subject acting on object being on the subject and the object, it is on the action itself.

The narrator also learns that on Tlon, there are two kinds of geometry: visual geometry and tactile geometry.

> Tactile geometry corresponds to our own and is subordinate to visual geometry. Visual geometry is based on the surface and not the point; it has not parallel lines, and it claims that as one's body moves through space, it modifies the shapes that surround it. The basis of Tlon's arithmetic is the notion of indefinite numbers; it stresses the concepts of greater than and less than...The people of Tlon are taught that the act of counting modifies the amount counted.... ([1], p. 76)

This is obviously similar to Einstein's *Theory of Relativity* in which the act of observation can change the thing being observed. In a similar way, things on Tlon

> grow vague or "sketchy," and to lose detail when they begin to be forgotten. The classic example is the doorway that continued to exist so long as a certain beggar frequented it, but which was lost to sight when he died. Some times a few birds, a horse, have saved a ruined amphitheater. ([1], p. 78)

The Encounter

This story contains the same shift in emphasis as discussed in the previous story. In it we learn of two famous knives that have a long history of being used in fights against each other. One day, two men who have never fought with knives before fight a spectacular battle with these two knives until one of them lay dead. Horrified and bewildered by wht he had just done, the winner, Maneco Uriarte, sobbed over the body of Duncan, the slain man.

> The act he had just committed overwhelmed and terrified him. ([1], p. 367)

Over time, stories of similar spectacular knife fights between men who had not fought with knives before are recalled. Suddenly, the narrator realizes,

Maneco Uriarte did not kill Duncan; it was the weapons, not the men who fought...The two knew how to fight---the knives, I mean, not the men, who were merely their instruments---and they fought well that night. They had sought each other for a long time, down the long roads of the province, and at last they had found each other....In the blades of those knives there slept, there lurked, a human grudge. ([1], p. 369)

So the emphasis has been shifted from the subjects using the objects, to the objects using the subjects. Instead of men able to fight each other using a variety of different weapons, we have knives using different men over time to fight each other. As in the last story, this is similar to the idea of functions versus functionals. Normally we would consider the men fixed, and the weapon each chose as variable. In this story, the weapons are fixed and it is the men who employ them that vary.

Things last longer than men. Who can say whether the story ends here; who can say that they will never meet again. ([1], p. 369)

Death and the Compass

This is the story of Erik Lonnrot, a detective who prides himself on his powers of reasoning, the fiendish criminal Red Scharlach, and the sequential murders of three Jewish men one winter in Paris. In it geometry is used to both solve a murder and lead a man to his death.

The first murder occurred on December 3 in the north, at the *Hotel du Nord*. The murdered man is found dead before his typewriter which contains a slip of paper with only the words

The first letter of the Name has been written. ([1], p. 149)

Lonnrot assumes the message is referring to the name of God.

The second murder occurs on January 3, in the west, in the doorway of an old paint factory. On the wall, there were some red and yellow rhombuses and the chalked words

The second letter of the Name has been written. ([1], p. 150)

The third murder occurs on February 3 in the east, outside a tavern on the *rue du Toulon*. The murdered man was last seen in the company of two harlequins, one of whom had scrawled the sentence

The last letter of the Name has been written. ([1], p. 151)

The police department receives a letter predicting that on March 3 there will not be a fourth murder, noting that the paint factory in the west, the tavern on the *rue du Toulon*, and the *Hotel du Nord* were

the perfect points of a mystical, equilateral triangle. Erik Lonnrot studied the letter. The three locations were indeed equidistant. Symmetry in time (December 3, January 3, February 3), symmetry in space as well. ([1], p. 152)

Erik Lonnrot considers the riddle for some time, and then decided that he has solved it. On March 3, he promises his boss that the murders will be behind bars shortly, and travels south to the *Villa Triste-le-Roy*. He enters the house and on the uppermost floor, is suddenly caught by two men and brought before Red Scharlach. He cries out,

"Scharlach---*you* are looking for the secret name?" ([1], p. 154)

Scharlach replies,

> "No, I am looking for something more fleeting and more perishable than that---I am looking for Erik Lonnrot. Three years ago in a gambling den on the *rue du Toulon*, you arrested my brother and saw that he was sent to prison. My men rescued me from a shootout in a coupe, but not before I'd received a policeman's bullet in my gut. For nine days and nine nights I lay between life and death in this villa...and I swore to weave a labyrinth around the man who imprisoned my brother. ([1], p. 154)

Red Scharlach explains how he had dropped clues for Lonnrot, knowing how he loved to reason.

> "One sign in the north, two more in the east and west, demand a fourth sign in the south---after all, the Tetragrammaton, the Name of God, YHVH, consists of four letters. I sent the equilateral triangle, knowing you would add the fourth missing point, the point that makes a perfect rhombus. I have done all of this, Erik Lonnrot, ...in order to draw you into the solitudes of *Triste-le-Roy*." ([1], p. 156)

Lonnrot considers his words then says,

> "There are three too many lines in your labyrinth. I know of a Greek labyrinth that is but one straight line. So many philosophers have been lost along that line that a mere detective might be pardoned if he became lost as well. When you hunt me down in another avatar of our lives, Scharlach, I suggest that you fake (or commit) one crime at A, a second crime at B, eight kilometers from A, then a third crime at C, four kilometers from A and B, and halfway between them. Then wait for me at D, two kilometers from A and C, once again halfway between them." ([1], p. 156)

Lonnrot is suggesting an infinite sequence of murders, each one halfway between point A and the point of the previous murder. The sequence of points lie along a line and converge to A.

The last three stories address more basic mathematical ideas like mathematics as abstraction, consistency in mathematics, and the idea of the variable.

Funes, His Memory

The main character of this story, Funes, has an incredible memory that allows him to perceive every detail of every specific thing, but which does not allow him to generalize so that he is unable to think about categories of things.

> With one quick look, you and I perceive three wineglasses on the table; Funes perceived every grape that had been pressed into the wine and all the stalks and tendrils of its vineyard. ([1], p. 135)

He invents a numbering system that is precisely the opposite of our own. Each number has its own name that bears no logical relation to any other number.

> Instead of seven thousand thirteen (7013), he would say for instance, "Maximo Perez"; instead of seven thousand fourteen (7014), "the railroad".....Every word had a particular figure attached to it, a sort of marker; the latter ones were extremely complicated....I tried to explain to Funes that his rhapsody of unconnected words was exactly the opposite of a number system. I told him that

when one said "365" one said "three hundreds, six tens and five ones," a breakdown impossible with.... "a poncho full of meat." ([1], p. 136)

But it is precisely because our number system in particular, and mathematics in general, utilizes systematic abstraction, that we are able to quickly visualize the structure of different mathematical objects and clearly see their relation to one another. Without generalization, there would be no mathematics.

> I suspect, nevertheless that he was not very good at thinking. To think is to ignore (or forget) differences, to generalize, to abstract. In the teeming world of Ireno Funes there were nothing but particulars----and they were virtually immediate particulars. ([1], p. 137)

Blue Tigers

The laws of arithmetic were probably first noticed in the act of counting objects. For example, we all take for granted the familiar rules such as

$$if\ a = b,\ then\ a + c = b + c,$$

or

$$a + 0 = 0 + a = a.$$

And we depend on the consistency of these rules and believe them to be part of the foundation of reality. In the story, *Blue Tigers*, this consistency no longer exists. The story's narrator travels along the Ganges River in search of the mythical blue tigers. After much searching he finds no tigers, but comes across some little circular blue stones. He picks up some of the stones and puts them in his pocket. When he gets home, he takes them out of his pocket.

> When I opened my hand, I saw that it held thirty or forty disks; I'd sworn that I had picked up no more than ten. I left them on the table, and turned back to get the rest out of the pocket. I didn't need to count them to see that they had multiplied. I pushed them together into a single pile and tried to count them one by one. ([1], p. 498)

He tries to count them very carefully.

> That simple operation turned out to be impossible...when any one disk was separated from the rest, it became many. ([1], p. 498)

The narrator performs many experiments, trying to mark the disks so as to keep track of them. But the marked disks disappear and then later reappear.

> What mysterious sort of space was this, which in obedience to inscrutable laws or some inhuman will absorbed the stones and then in time threw an occasional one back again? ([1], p. 501)

Confounded by the stones, he attempts to find any consistent rules he can.

> The same yearning for order that had created mathematics in the first place made me seek some order in that aberration of mathematics....I attempted to find a law within their unpredictable variations. I devoted days and nights alike to establish statistics on the changes...Naturally the four mathematical operations----adding, subtracting, multiplying and dividing----were impossible. The stones resisted arithmetic as they did the calculation as probability. ([1], p. 502)

He eventually realizes that it is futile and gives up.

> As I manipulated the stones that destroyed the science of mathematics, more than once I thought of those Greek stones that were the first ciphers and that had been passed down to so many languages as the word "calculus". Mathematics, I told myself, had its origin and now has its end, in stones. ([1], p. 502)

The Zahir

This story is about a magical coin called the Zahir, and in it Borges alludes to the idea of money as a variable.

> I reflected that there is nothing less material than money, since any coin, whatsoever is, strictly speaking, a repertory of possible futures. Money is abstract...; money is the future tense. It can be an evening in the suburbs, or music by Brahms; it can be maps, or chess, or coffee. ([2], p.159)

In either case, what the coin or the variable stand for is unknown until you make the purchase or solve the equation.

Conclusion

Mathematical ideas like recursion, the infinite, duality and chance are fundamental to much of the writing of Jorge Luis Borges, even though the nonmathematical reader may enjoy the stories without any awareness of the their mathematical content. Perhaps the delight that so many readers find in reading Borges' stories may in fact be an unconscious delight in glimpsing within them one of the many facets of the beauty of mathematics. If so, Borges has done all of us, mathematician and nonmathematician alike, a great service.

References

[1] Borges, Jorge Luis, Collected Fictions, translated by Andrew Hurley, Penguin Books, NY, 1998.

[2] Borges, Jorge Luis, Labyrinths, New Directions Publishing Corporation, Toronto, 1964.

[3] Hanes, Lorna B., *The Poetry of Infinity, Exploring Mathematical Concepts as Metaphor in the Works Jorge Luis Borges*, Bridges Conference Proceedings, 2001.